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Preservation of quantum states via a super-Zeno effect on ensemble quantum computers*

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Following a recent proposal by Dhar *et al* (2006 *Phys. Rev. Lett.* **96** 100405), we demonstrate experimentally the preservation of quantum states in a two-qubit system based on a super-Zeno effect using liquid-state nuclear magnetic resonance techniques. Using inverting radiofrequency pulses and delicately selecting time intervals between two pulses, we suppress the effect of decoherence of quantum states. We observe that preservation of the quantum state $|11\rangle$ with the super-Zeno effect is three times more efficient than the ordinary one with the standard Zeno effect.

Keywords: quantum information, decoherence, Zeno effect, quantum state preservation
PACC: 0367, 8760I, 3325B

1. Introduction

Quantum computation that relies on the use of quantum properties to do calculations could, in theory, be more powerful than its classical counterpart.[1−3] Quantum coherence and quantum entanglement offer possibilities for practical applications in quantum information processing.[4,5] Quantum states of quantum mechanical systems can be used to carry information in quantum computers. Since any real quantum system unavoidably interacts with its environment, this interaction leads to rapid decay of superposition states of the quantum system, so this non-unitary evolution process causes information loss from the system to the environment, i.e., the effect of decoherence. Therefore, decoherence is one of the main obstacles to the realization of quantum computations. The problem of decoherence has attracted considerable attention in recent years. This problem has been widely studied both theoretically and experimentally.[6−15]

Some authors[16] have proposed a new strategy for state preservation that can be achieved by using a series of unitary kicks with unequal time intervals. The preservation of state using non-periodic pulse sequences is called the super-Zeno effect.[16] Theoretical studies have shown that the algorithm of state preservation with the super-Zeno effect is much more efficient than the ordinary one with the quantum Zeno effect, and does not require any symmetry properties in quantum systems.[16]

Since nuclear spins have longer coherence times, and the coherent control of nuclear spin dynamics has been well achieved with radiofrequency (RF) pulses, liquid-state nuclear magnetic resonance (NMR) is suitable for experimentally demonstrating many proposals for quantum information processing.[17,18] It is natural to use NMR quantum computers[19,20] to experimentally verify the effectiveness of state preservation with the super-Zeno effect.

In this paper, following the proposal by Dhar *et al*, we construct a sequence of NMR inverting pulses, which have delicately-selected unequal time intervals. Furthermore, we experimentally demonstrate the algorithm of state preservation by using the super-Zeno effect and the standard Zeno effect on a two-qubit NMR quantum computer, respectively, and compare the results for these two cases.

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2. Experiment

Dhar *et al* have proposed to use unitary kicks to preserve quantum states. The unitary kicks with unequal time intervals between kicks are inverting pulses, which can be constructed according to those used in Grover's quantum search algorithm.[2,16] The key to the algorithm of state preservation is to construct the inverting pulses, the effect of which can be described by a unitary operator U_P.

For simplicity, we first consider an NMR system involving two qubits. The two-qubit NMR system is described by the Hamiltonian:

$$H_0 = \omega_1 I_{z1} + \omega_2 I_{z2} + 2\pi J_{12} I_{z1} I_{z2}, \qquad (1)$$

where the first two terms describe the free precession of spins 1 and 2 about the strong static magnetic field B_0 in the z direction, and the third term represents the scalar spin–spin coupling J_{12} of the two spins. The corresponding evolution operator for a time interval t is $U_0 = \exp[-\mathrm{i}\, H_0 t]$. When the two-qubit NMR system is subjected to a sequence of n inverting pulses, the evolution operator of the system is[16]

$$W_n(t) = U_0(x_{n+1}t)U_P \ldots U_P U_0(x_2 t)U_P U_0(x_1 t), \quad (2)$$

where $x_k (k = 1, \ldots, n+1)$ are $(n+1)$ real numbers $(\sum_{k=1}^{n+1} x_k = 1)$, t is the total time interval of delays between the inverting pulses in one repetition, $x_k t$ is the time interval between the kth and $(k+1)$th pulse $(k = 1, \ldots, n-1)$, and U_P is an inverting pulse: $U_P = Q - P$ (here the projection operators P and Q for the subspaces $\boldsymbol{P}$ and $\boldsymbol{Q}$ of the Hilbert space, respectively). U_P satisfies $U_P|\psi\rangle = -|\psi\rangle$, if $|\psi\rangle \in \boldsymbol{P}$; and $U_P|\psi\rangle = +|\psi\rangle$, if $|\psi\rangle \in \boldsymbol{Q}$. Since the time of performing the unitary operation (U_P) is on the order of $1/J_{12}$, we use refocusing pulses to remove the unwanted interactions in realization of the inverting pulses, so that any evolution resulting from the unwanted interactions can be 'frozen'.[17,18] We can neglect the evolution of dynamics in performing all the inverting pulses. Therefore, here we only consider the effect of the evolution of the two spins between the inverting pulses, as shown in Eq.(2). If the initial state of the two-qubit NMR system is prepared in the state $|\Psi\rangle \in \boldsymbol{P}$, then the effect of the unitary operator $W_n(t)$ is to prevent the leakage of the quantum state in the subspace $\boldsymbol{P}$ to the subspace $\boldsymbol{Q}$.

Here we consider that the subspaces $\boldsymbol{P}$ and $\boldsymbol{Q}$ of the two-qubit Hilbert space are $\boldsymbol{P} = \mathrm{Span}\{|11\rangle\}$, and $\boldsymbol{Q} = \mathrm{Span}\{|00\rangle,\ |01\rangle,\ |10\rangle\}$, respectively. The initial state $|11\rangle$ is to be preserved, which can be created with the spatial averaging method.[20,21] For the two-qubit NMR system, the corresponding matrix of the unitary operator U_P reads

$$U_P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \qquad (3)$$

The corresponding pulse sequence of implementing the unitary operator U_P can be found in Refs.[2] and [22].

In our experiment, we use four inverting pulses with five unequal time intervals in each repetition to preserve the quantum state $|11\rangle$. The time interval $x_k t$ of the pulse sequence is optimized as follows:[16]

$$\{x_k\} = \{\beta, 1/4, 1/2 - 2\beta, 1/4, \beta\}, \qquad (4)$$

where $\beta = (3 - \sqrt{5})/8$, and $k = 1, \ldots, 5$. A quantum circuit for realizing the algorithm of state preservation by using the super-Zeno effect is shown in Fig.1. We choose the total time intervals, $t = 10\ \mu$s, $50\ \mu$s and 8 ms, between the inverting pulses for one repetition in three super-Zeno experiments, respectively. All experiments begin with the initial state $|11\rangle$ of the two-qubit NMR system. The unitary operator $W_n(t)$ $(n = 4)$ is repeatedly applied to the two-qubit NMR system M times in each experiment, where $M = 1, \ldots, 100$. In one repetition of our experiment, the total time t_1 of performing all the inverting pulses is about 10 ms. If the unitary operator $W_n(t)$ is repeatedly performed M times in one experiment, then the experiment takes the total time $T = M \times (t + t_1)$. The number of the inverting pulses used in one experiment is $N = 4M$.

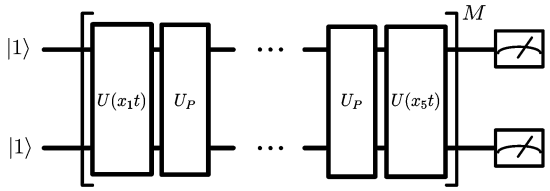


Fig.1. A quantum circuit for realizing the quantum state preservation based on the use of the super-Zeno effect. The unitary operations in parentheses are iteratively performed M times.

In NMR measurements, the intensities (i.e., the heights of the peaks) of NMR signals from the state $|11\rangle$ clearly indicate how high the populations of the spins in the state $|11\rangle$ are, after the read pulses are applied to the two qubits, respectively. Thus, we can use this simple method to investigate the variation of the population in the state $|11\rangle$ with the number of the inverting pulses. Furthermore, according to NMR signal intensities, we can estimate the leakage probability of the state $|11\rangle$ in the subspace $\boldsymbol{P}$ to the subspace $\boldsymbol{Q}$.

In order to compare the results for both the super-Zeno effect and the standard Zeno effect, we have carried out experiments based on the standard Zeno effect under the same conditions. The experimental method for the two-qubit NMR system is similar to that given in Ref.[22], while the state of the two qubits is initially prepared in the state $|11\rangle$, which is different from that in Ref.[23]. Note that in our standard Zeno experiment, we can apply a weak ensemble measurement to spin systems. The weak measurement can be realized by using a controlled-NOT gate, and gradient pulses.[23] Since Xiao and Jones have used one small-angle pulse in each repetition in their experiment, the total number of small-angle pulses used in the experiment is the times of repetitions. We have measured the intensities of NMR signals from the state $|11\rangle$ as the number N ($N = 1, \ldots, 106$) of small-angle pulses.[23] The flip angle of the small-angle pulse used in our standard Zeno experiment is about 5°.

All experiments were carried out on a Varian INOVA 600 NMR spectrometer, at a temperature of 298 K. The sample is ^{13}C labelled chloroform dissolved in deuterated chloroform, and the molecular formula is $^{13}C^{1}HCl_3$. In the experiment, we use ^{1}H and ^{13}C in the chloroform as the two qubits. The coupling constant between ^{1}H and ^{13}C is $J = 214.38$ Hz. The spin–spin relaxation times are $T_2 = 4.041$ s for the proton, and $T_2 = 0.3017$ s for the carbon. The spin–lattice relaxation times are $T_1 = 6.368$ s for the proton, and $T_1 = 18.67$ s for the carbon. All RF excitations were achieved with hard pulses.

3. Results and discussion

The experimental and calculated results are illustrated in Fig.2. We have found that the intensities of NMR signals from the initial state $|11\rangle$ for the super-Zeno effect and the standard Zeno effect decay with increasing number (N) of the inverting pulses and the small-angle pulses, respectively. A reason for this could be that realizing the unitary kick in the super-Zeno effect experiment needs a non-zero time, and that there is a non-zero time interval between the measurements in the standard Zeno effect experiment. The decay of the NMR signal intensity with the number (N) of the pulses used implies that with increasing time, the populations in the state $|11\rangle$ in the subspace $\boldsymbol{P}$ decrease. To explain our experimental observation, we have used an exponential decay $\exp(-\gamma N)$ [N: the number of the inverting pulses for the super-Zeno effect (or the small-angle pulses for the standard Zeno effect)] to fit to the experimental data. The fitting values are $\gamma = 4.5 \times 10^{-3}$, 2.5×10^{-3}, and 1.3×10^{-3} for the total time intervals $t = 8$ ms, 50 μs and 10 μs on the basis of the super-Zeno effect, respectively, while $\gamma = 5.0 \times 10^{-3}$ for the standard Zeno effect. The decay parameter (γ) in the standard Zeno effect experiment is greater than those in the super-Zeno effect experiment. Therefore, the experimental and calculated results show that the quantum state is more effectively preserved with the super-Zeno effect than with the standard Zeno effect, which is in agreement with the theoretical result given in Ref.[16]. It is interesting to note that when the total time interval between the inverting pulses in one repetition is $t = 10$ μs, the preservation of the quantum state $|11\rangle$ based on the super-Zeno effect is the most efficient in the three experiments with three different time intervals ($t = 8$ ms, 50 μs and 10 μs). This is because there are more inverting pulses used in the same total time T for evolution than those in the other cases.[16] This agrees with the result given in Ref.[16]. In addition, we do similar experiments without optimizing the time intervals to minimize the number of inverting pulses,[16] where we have used inverting pulses of 1, 2, 5, 10, 21, 42 and 83. We also find that there is a considerably slow decay of the NMR signal intensity with the number of the inverting pulses. The experimental results indicate that the super-Zeno effect can be used to effectively preserve the initial state $|11\rangle$.

Note that the experimental result for the standard Zeno effect also indicates that there is exponential decay with the number of the small-angle pulses. This can be due to our measurement being a weak ensemble measurement, and the time interval between the measurements not being zero.[23] Our measurements in our NMR experiments do not involve the reduction of wave function in the projective measurement considered in quantum mechanics. It is worth pointing out that the fitting decay param-

eter (γ) as the number of the pulses has the same order of magnitude in our experiments for both the super-Zeno and the standard Zeno effect. This shows that the state preservation with the standard Zeno effect is effective. The reason for this may be that we applied the weak ensemble measurement to NMR quantum systems. Although the small-angle pulse has the effect of artificial relaxations, the small flip angle (5° used in our experiment) means that the effect of this artificial relaxation can be weak.[23] In our experiment, the weak ensemble measurement includes the unitary kicks used in the super-Zeno effect experiment. It should be noted that in our standard Zeno effect experiment, we have employed pulsed magnetic field gradients and the controlled-NOT gate to simulate the projective measurement in quantum mechanics.

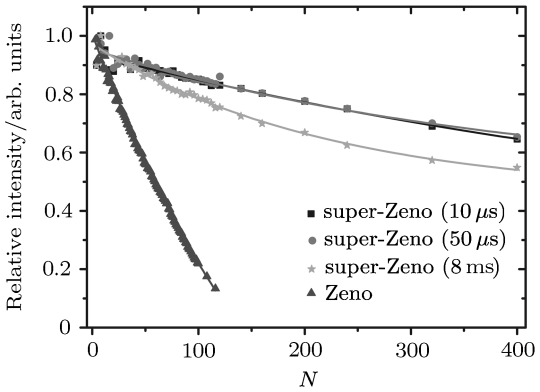


Fig.2. Results of the quantum state ($|11\rangle$) preservation by using the super-Zeno effect and the standard Zeno effect in a two-qubit NMR system. The horizontal scale is the number N of pulses used (inverting pulses for the super-Zeno effect or small flip-angle ones for the Zeno effect), while the y-axis represents the relative intensity of the NMR signals from the state $|11\rangle$, where the maximum NMR signal intensity observed in each experiment is used as the intensity of the reference of NMR signals. The different symbols are used to represent the experimental results for different cases, which are explained in the text. The smooth lines show the fitting curves to the experimental results.

Figure 2 shows that there are errors in the experimental results. These errors mainly result from inhomogeneity of the static magnetic and the RF fields, imperfect calibration of the RF pulse time (especially for the small-angle pulse) and signal decay during the acquisition.

It should also be noted that, in our experiment, the inverting pulses are used to preserve the quantum state $|11\rangle$ of the quantum system with two qubits, which may be exact only for a two-qubit system.[24] For a three-qubit system, the inverting pulses used to preserve the quantum state should have a phase of 0.677007π instead of π.[24]

4. Conclusions

In summary, we have experimentally preserved the quantum state $|\ 11\rangle$ on the basis of the super-Zeno effect, and obtained an improvement of the decay parameter of the initial state $|11\rangle$ with the number (N) of the pulses from our other experiment with the standard Zeno effect, being about 3 times smaller than that with the standard Zeno effect. Practical quantum computers require suppression of the decoherence effect to implement complicated quantum operations. Preserving quantum states with the super-Zeno effect can play an important role in multi-qubit quantum computations.

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