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Entanglement transfer between atoms in two distant cavities via an optical fibre*

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This paper presents a treatment of the entanglement transfer between atoms in two distant cavities coupled by an optical fibre. If the atoms resonantly and collectively interact with the local single-mode cavity fields and the dipole–dipole interaction between the atoms is neglected, then it shows that a complete transfer of entanglement from one pair of atoms to another can be deterministically realized. Furthermore, it also investigates the effects of dipole–dipole interaction on entanglement transfer on the condition that the interaction between the atoms and the cavity is much weaker than the coupling between the cavity and the fibre.

Keywords: entanglement transfer, dipole–dipole interaction, concurrence

PACC: 0367, 4250

1. Introduction

Entanglement is the main resource of quantum computation and quantum information processing (QIP). Usually, atoms are applied to store information while photons are always employed to carry and process information. Therefore, much effort has been devoted to the generation and manipulation of entanglement in the past decade.[1−4] One of the schemes to produce entanglement nonlocally is entanglement swapping,[5,6] which enables us to entangle two quantum systems that have never interacted directly with each other. Recently, another scheme which has been intensively studied is entanglement transfer from one quantum system to another. In particular, the entanglement transfer between the fields and qubits is of primary importance and has been widely studied.[7,8] Lee *et al*[9] investigated how entanglement can be transferred between qubits and continuous-variable (CV) systems. Serafini *et al*[10] presented strategies to enhance the dynamic entanglement transfer from continuous-variable to finite-dimensional systems by employing multiple qubits.

Recently, the system of two distant cavities coupled by an optical fibre has received broad interest,[11−13] since it is crucial for the implementation of quantum communication and quantum computation. Pellizzari[14] proposed a scheme which allows for reliable transfer of quantum information between two atoms in distant cavities coupled by an optical fibre. Serafini *et al*[15] investigated the possibility of realizing effective quantum gates between two atoms by using the same device. Yin and Li[16] proposed a multiatom and resonant interaction scheme to generate entangled states and realize logical gates in such a system. In order to realize quantum communication, the entanglement transfer between atoms trapped in distant optical cavities is also very important. Motivated by these works, we discuss the entanglement transfer from a pair of atoms to another in two distant cavities via an optical fibre. We also discuss the effects of dipole–dipole (DD) interaction[17−20] on the entanglement transfer under the condition that the interaction between the atoms and the cavity is much weaker than the coupling between the cavity and fibre.

The paper is organized as follows. In Section 2, we introduce the model under consideration. In Section 3, entanglement transfer between atoms is discussed. In Section 4, the influence of DD interaction on the entanglement transfer is investigated. In Section 5, a summary is given.

2. The model

First, we introduce the atom–cavity–fibre system. As shown in Fig.1, four identical two-level atoms 1, 2 and 3, 4 are separately trapped in two distant single-mode optical cavities A and B coupled by an optical

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fibre. The atoms resonantly and collectively interact with the local cavity fields. We assume that the size of the space occupied by the atoms in each of the cavities is much smaller than the wavelength of the cavity field. So, all the atoms in each of cavities can interact with the same field. On the other hand, the atoms in the same cavity are separated that they have no direct interaction with each other.

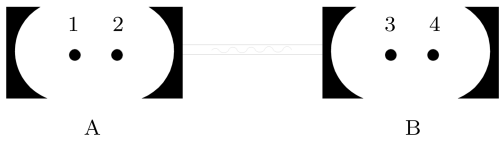


Fig.1. The atom–cavity–fibre system with atoms 1, 2 in cavity A, atoms 3, 4 in cavity B.

In the rotating-wave approximation, the interaction Hamiltonian of the atom–field system can be written as ($\hbar = 1$)

$$H_{\rm AF} = g_1(\sigma_1 + \sigma_2)a_1^+ + g_2(\sigma_3 + \sigma_4)a_2^+ + \text{H.c.}, \quad (1)$$

where a_1^+ and a_2^+ are the creation operators for the mode of cavity A and B respectively, g_1 and g_2 are the coupling constants between the cavities and the trapped atoms respectively, σ and σ^+ are the lowering and raising operators for the atom respectively.

The interaction Hamiltonian for the optical fibre coupled to the modes of the two cavities can be modelled as[14]

$$H_{\rm IF} = \sum_{j=1}^{\infty} \nu_j \{b_j[a_1^+ + (-1)^j \, {\rm e}^{{\rm i}\varphi} a_2^+] + \text{H.c.}\}, \quad (2)$$

where b_j is the annihilation operator for the mode j of the fibre, ν_j is the coupling strength between the modes of cavities and the fibre mode j, and the phase φ is due to propagation of the field through a fibre of length l: $\varphi = 2\pi\omega l/c$,[12] ω is the frequency of the cavities. Now, let $\bar{\nu}$ be the decay rate of the cavities' fields into a continuum of fibre modes. Considering the short fibre limit $(l\bar{\nu})/(2\pi c) \leq 1$, we find that only one resonant mode b of the fibre interacts with the cavity modes. The Hamiltonian $H_{\rm IF}$ can be reduced to

$$H_{\rm IF} = \nu[b(a_1^+ + a_2^+) + \text{H.c.}], \quad (3)$$

where the phase $(-1)^j \, {\rm e}^{{\rm i}\varphi}$ has been absorbed into the annihilation and creation operators of the mode of the second cavity field.

In the interaction picture, the total Hamiltonian of the atom–cavity–fibre combined system is

$$H_{\rm I} = H_{\rm AF} + H_{\rm IF}. \quad (4)$$

3. Entanglement transfer between atoms

In this section, we investigate the entanglement transfer from atoms 1, 2 to atoms 3, 4. For simplicity, we assume that there is only one excitation number in the entire system. Suppose that at the initial time cavities and fibre are in vacuum states and all the atoms in the cavity B are in ground states, while the atoms 1, 2 are in entangled states: $\cos\alpha|10\rangle_{12} + \sin\alpha|01\rangle_{12}$, where $|10\rangle_{12}$ means that the atoms 1 and 2 are in the excited state and the ground state, respectively.

The subspace with one excitation number can be spanned by the basis state vectors:

$$\begin{aligned}
|\phi_1\rangle &= |10\rangle_{12}|00\rangle_{34}|000\rangle_{\rm f}, \\
|\phi_2\rangle &= |01\rangle_{12}|00\rangle_{34}|000\rangle_{\rm f}, \\
|\phi_3\rangle &= |00\rangle_{12}|10\rangle_{34}|000\rangle_{\rm f}, \\
|\phi_4\rangle &= |00\rangle_{12}|01\rangle_{34}|000\rangle_{\rm f}, \\
|\phi_5\rangle &= |00\rangle_{12}|00\rangle_{34}|100\rangle_{\rm f}, \\
|\phi_6\rangle &= |00\rangle_{12}|00\rangle_{34}|010\rangle_{\rm f}, \\
|\phi_7\rangle &= |00\rangle_{12}|00\rangle_{34}|001\rangle_{\rm f}, \quad (5)
\end{aligned}$$

where $|n_1 n_2 n_3\rangle_{\rm f}$ denotes that there are n_1 photons in the mode of cavity A, n_2 photons in the mode of cavity B, n_3 photons in the fibre mode. In this subspace, the initial state of the system can be written as

$$|\psi(0)\rangle = \cos\alpha|\phi_1\rangle + \sin\alpha|\phi_2\rangle. \quad (6)$$

At any time t, the state of the whole system can be expanded in terms of the basis vectors Eq.(5) as

$$|\psi(t)\rangle = \sum_{i=1}^{7} A_i(t)|\phi_i\rangle. \quad (7)$$

The time evolution of the total system is governed by the Schrödinger equation

$${\rm i}\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle. \quad (8)$$

Substituting Eq.(7) into Eq.(8), we obtain the matrix form of Eq.(8):[16]

$${\rm i}\frac{\partial}{\partial t}A_i(t) = \sum_{j=1}^{7} H_{ij}A_j(t), \quad (9)$$

where H_{ij} are elements of the matrix representation for the Hamiltonian Eq.(4) in the one excitation number subspace:

$$\boldsymbol{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 & 0 & g & 0 \\ g & g & 0 & 0 & 0 & 0 & \nu \\ 0 & 0 & g & g & 0 & 0 & \nu \\ 0 & 0 & 0 & 0 & \nu & \nu & 0 \end{pmatrix}, \tag{10}$$

where we have assumed $g_1 = g_2 = g$. The eigenvalues and the corresponding eigenstates of the Hamiltonian Eq.(10) are easily derived as

$$\begin{aligned} &E_{1,2,3} = 0, \\ &E_{4,5} = \pm\sqrt{2}g, \\ &E_{6,7} = \pm\sqrt{2\nu^2 + 2g^2}, \end{aligned} \tag{11}$$

and

$$\begin{aligned}
|\varphi_1\rangle &= \frac{1}{\sqrt{2}}|\phi_1\rangle - \frac{1}{\sqrt{2}}|\phi_2\rangle, \\
|\varphi_2\rangle &= \frac{r}{\sqrt{1+2r^2}}|\phi_1\rangle + \frac{r}{\sqrt{1+2r^2}}|\phi_3\rangle - \frac{1}{\sqrt{1+2r^2}}|\phi_7\rangle, \\
|\varphi_3\rangle &= \frac{r}{\sqrt{1+2r^2}}|\phi_1\rangle + \frac{r}{\sqrt{1+2r^2}}|\phi_4\rangle - \frac{1}{\sqrt{1+2r^2}}|\phi_7\rangle, \\
|\varphi_4\rangle &= \frac{1}{2\sqrt{2}}|\phi_1\rangle + \frac{1}{2\sqrt{2}}|\phi_2\rangle - \frac{1}{2\sqrt{2}}|\phi_3\rangle - \frac{1}{2\sqrt{2}}|\phi_4\rangle + \frac{1}{2}|\phi_5\rangle - \frac{1}{2}|\phi_6\rangle, \\
|\varphi_5\rangle &= \frac{1}{2\sqrt{2}}|\phi_1\rangle + \frac{1}{2\sqrt{2}}|\phi_2\rangle - \frac{1}{2\sqrt{2}}|\phi_3\rangle - \frac{1}{2\sqrt{2}}|\phi_4\rangle - \frac{1}{2}|\phi_5\rangle + \frac{1}{2}|\phi_6\rangle, \\
|\varphi_6\rangle &= \frac{1}{2\sqrt{2+2r^2}}|\phi_1\rangle + \frac{1}{2\sqrt{2+2r^2}}|\phi_2\rangle + \frac{1}{2\sqrt{2+2r^2}}|\phi_3\rangle \\
&\quad + \frac{1}{2\sqrt{2+2r^2}}|\phi_4\rangle + \frac{1}{2}|\phi_5\rangle + \frac{1}{2}|\phi_6\rangle + \frac{r}{\sqrt{2+2r^2}}|\phi_7\rangle, \\
|\varphi_7\rangle &= \frac{1}{2\sqrt{2+2r^2}}|\phi_1\rangle + \frac{1}{2\sqrt{2+2r^2}}|\phi_2\rangle + \frac{1}{2\sqrt{2+2r^2}}|\phi_3\rangle \\
&\quad + \frac{1}{2\sqrt{2+2r^2}}|\phi_4\rangle - \frac{1}{2}|\phi_5\rangle - \frac{1}{2}|\phi_6\rangle + \frac{r}{\sqrt{2+2r^2}}|\phi_7\rangle,
\end{aligned} \tag{12}$$

where $r = \nu/g$. In terms of Eq.(12), we write the unitary matrix $\boldsymbol{S}$ which diagonalizes the Hamiltonian (10) as follows:

$$\boldsymbol{S} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ \frac{r}{\sqrt{1+2r^2}} & 0 & \frac{r}{\sqrt{1+2r^2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{1+2r^2}} \\ \frac{r}{\sqrt{1+2r^2}} & 0 & 0 & \frac{r}{\sqrt{1+2r^2}} & 0 & 0 & -\frac{1}{\sqrt{1+2r^2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2} & \frac{1}{2} & \frac{r}{\sqrt{2+2r^2}} \\ \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & \frac{1}{2\sqrt{2+2r^2}} & -\frac{1}{2} & -\frac{1}{2} & \frac{r}{\sqrt{2+2r^2}} \end{pmatrix}. \tag{13}$$

By use of the unitary matrix $\boldsymbol{S}$ and its inverse matrix $\boldsymbol{S}^{-1}$, Eq.(9) can be written as

$$\mathrm{i}\frac{\partial}{\partial t}\boldsymbol{SA} = \boldsymbol{HS}^{-1}\boldsymbol{SA}, \tag{14}$$

where $\boldsymbol{A} = [A_1, A_2, A_3, A_4, A_5, A_6, A_7]^{\mathrm{T}}$.

Because the matrix $\boldsymbol{SHS}^{-1}$ is diagonal, by solving Eq.(14) we can obtain the solution of Eq.(9) as

$$A_j(t) = \sum_{k=1}^{7}[\boldsymbol{S}^{-1}]_{jk}[\boldsymbol{SA}(0)]_k\,\mathrm{e}^{-\mathrm{i}E_k t}. \tag{15}$$

Substituting Eqs.(11) and (13) into Eq.(15), for the initial condition $\boldsymbol{A}(0) = [\cos\alpha, \sin\alpha, 0, 0, 0, 0, 0]^{\mathrm{T}}$, we have

$$\begin{aligned}
A_1(t) &= \frac{2+r^2}{4(1+r^2)}(\cos\alpha - \sin\alpha) + \frac{r^2}{2(1+r^2)}\cos\alpha + \frac{1}{4}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt)\\
&\quad + \frac{1}{4(1+r^2)}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt\sqrt{1+r^2}),\\
A_2(t) &= -\frac{2+3r^2}{4(1+r^2)}(\cos\alpha - \sin\alpha) + \frac{r^2}{2(1+r^2)}\cos\alpha + \frac{1}{4}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt)\\
&\quad + \frac{1}{4(1+r^2)}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt\sqrt{1+r^2}),\\
A_3(t) &= -\frac{r^2}{4(1+r^2)}(\cos\alpha - \sin\alpha) + \frac{r^2}{2(1+r^2)}\cos\alpha - \frac{1}{4}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt)\\
&\quad + \frac{1}{4(1+r^2)}(\cos\alpha + \sin\alpha)\cos(\sqrt{2}gt\sqrt{1+r^2}),\\
A_4(t) &= A_3(t),\\
A_5(t) &= -\frac{\mathrm{i}}{2\sqrt{2}}(\cos\alpha + \sin\alpha)\sin(\sqrt{2}gt) - \frac{\mathrm{i}}{2\sqrt{2}\sqrt{1+r^2}}(\cos\alpha + \sin\alpha)\sin(\sqrt{2}gt\sqrt{1+r^2}),\\
A_6(t) &= \frac{\mathrm{i}}{2\sqrt{2}}(\cos\alpha + \sin\alpha)\sin(\sqrt{2}gt) + \frac{\mathrm{i}}{2\sqrt{2}\sqrt{1+r^2}}(\cos\alpha + \sin\alpha)\sin(\sqrt{2}gt\sqrt{1+r^2}),\\
A_7(t) &= \frac{r}{2(1+r^2)}(\cos\alpha - \sin\alpha) - \frac{r}{1+r^2}\cos\alpha + \frac{r}{2(1+r^2)}\cos(\sqrt{2}gt\sqrt{1+r^2})(\cos\alpha + \sin\alpha).
\end{aligned} \tag{16}$$

Then at any time t, the density operator for the whole system is given by

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|. \tag{17}$$

The information about the entanglement between atoms i and j is contained in the reduced density matrix $\boldsymbol{\rho}_{Aij}$, and $\boldsymbol{\rho}_{Aij}$ can be obtained from Eq.(17) by tracing over the variables of the fields and other atoms. In the bases $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, $\boldsymbol{\rho}_{Aij}$ can be given by

$$\boldsymbol{\rho}_{Aij} = \begin{pmatrix} \sum_{k\neq i,j}|A_k|^2 & 0 & 0 & 0\\ 0 & |A_j|^2 & A_jA_i^* & 0\\ 0 & A_iA_j^* & |A_i|^2 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

It is convenient to quantify the degree of any bipartite entanglement by concurrence[21] defined as

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{19}$$

where λ_i is the ith eigenvalue in decreasing order of matrix $\tilde{\boldsymbol{\rho}}_{Aij} = \boldsymbol{\rho}_{Aij}(\sigma_y \bigotimes \sigma_y)\boldsymbol{\rho}^*_{Aij}(\boldsymbol{\sigma}_y \bigotimes \boldsymbol{\sigma}_y)$, $\boldsymbol{\sigma}_y$ is the Pauli spin matrix and the asterisk denotes complex conjugation. The concurrence varies from $C = 0$ for a separable state to $C = 1$ for a maximally entangled state. If the density matrix has the form of Eq.(18), the concurrence can be calculated by

$$C_{ij}(t) = 2|A_jA_i^*|. \tag{20}$$

We present the results of the concurrence C_{12} and C_{34} in Fig.2, which shows the influences of the parameter r on the entanglement transfer with $\alpha = \pi/4$ (namely the two atoms are initially in the maximal entangled states).

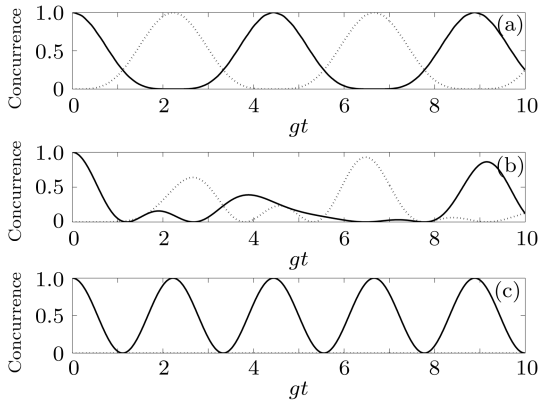


Fig.2. The entanglement transfer between the atoms 1, 2 and 3,4, where C_{12} is represented by a solid curve, C_{34} is represented by a dotted curve: (a) $r = 20, \alpha = \pi/4$; (b) $r = 1, \alpha = \pi/4$; (c) $r = 0.01, \alpha = \pi/4$.

In Fig.2 (a), $r = \nu/g = 20$, which means that the interaction between the fibre and the cavity is much stronger than the coupling between the cavity and the atoms; we can observe that C_{12} and C_{34} both range from 0 to 1 with the same oscillation period $\sqrt{2}\pi$, and the latter retards by half the oscillation period in comparison with the former. The entanglement completely transfers from atoms 1, 2 to atoms 3, 4 at the time $gt = (2n-1)\pi/\sqrt{2}$ and then comes back at the time $gt = 2n\pi/\sqrt{2}$, where $n = 1, 2, 3\ldots$. Because in the regime $r \gg 1$, the transmission time of photons through the fibre becomes much shorter than the interaction time of the atom and cavity modes. The fibre modes are almost unpopulated during the whole operation process. It is just as if atoms 1, 2 and atoms 3, 4 are resonantly coupled through a single mode of the cavity field, so the entanglement transfer from a pair of atoms to another can be deterministically realized. In Fig.2(b), when r becomes smaller, the efficiency of entanglement transfer decreases and the evolution of C_{12} and C_{34} becomes irregular. If $r \ll 1$, this denotes that the interaction between the fibre and the cavity is weak enough that the cavity A reduces to a two-atom Tavis–Cummings model and without the link of cavity B. Since the atoms in cavity B are initially in the ground states, the entanglement between atoms 3, 4 will always be zero, as shown in Fig.2(c).

If the condition $r \gg 1$ is satisfied by the coupling constants ν and g (which is not very difficult to achieve in experiment), we have $\frac{1}{\sqrt{2}}(|10\rangle_{12} + |01\rangle_{12}) \bigotimes |00\rangle_{34} \rightarrow \frac{1}{\sqrt{2}}|00\rangle_{12} \bigotimes (|10\rangle_{34} + |01\rangle_{34})$. Therefore, the perfect entanglement transfer is deterministically implemented, which may have an important application in the field of quantum communication.

4. Effects of DD interaction on entanglement transfer

In this section, we discuss the influences of DD interaction on the entanglement transfer under the condition that the interaction between the fibre and the cavity is much stronger than the coupling between the cavity and the atoms. Considering the DD interaction between the atoms,

$$H_{\rm AA} = \Omega_1(\sigma_1\sigma_2^+ + \sigma_1^+\sigma_2) + \Omega_2(\sigma_3\sigma_4^+ + \sigma_3^+\sigma_4). \quad (21)$$

In the Schrödinger picture, the Hamiltonian of the whole system has the form

$$H_{\rm S} = \omega\left(a_1^+a_1 + a_2^+a_2 + b^+b + \sum_{i=1}^{4}\sigma_i^z\right) + H_{\rm AF} + H_{\rm IF} + H_{\rm AA}, \quad (22)$$

where we have assumed that the interaction between the atoms and the cavity field and the coupling between the cavity and fibre fields are resonant. Also, $\sigma_i^z = (|1_i\rangle\langle 1_i| - |0_i\rangle\langle 0_i|)/2$ for atom i. Let us introduce three normal bosonic modes c, c_-, and c_+ by the canonical transformations:[15]

$$\begin{aligned} c &= (a_1 - a_2)/\sqrt{2}, \\ c_+ &= (a_1 + a_2 + \sqrt{2}b)/2, \\ c_- &= (a_1 + a_2 - \sqrt{2}b)/2. \end{aligned} \quad (23)$$

Then, based on the new bosonic operators, the Hamiltonian Eq.(22) can be re-expressed as

$$\begin{aligned} H_{\rm S} = \frac{1}{2}&[g_1(\sigma_1+\sigma_2)(c_+^+ + c_-^+ + \sqrt{2}b^+) \\ &+g_2(\sigma_3+\sigma_4)(c_+^+ + c_-^+ - \sqrt{2}b^+) + \text{H.c.}] \\ &+\omega\left(c^+c + \sum_{i=1}^{4}\sigma_i^z\right) + (\omega+\sqrt{2}\nu)c_+^+c_+ \\ &+(\omega-\sqrt{2}\nu)c_-^+c_- + H_{\rm AA}. \end{aligned} \quad (24)$$

It can be easily seen from Eq.(24) that the mode c resonantly interacts with the atoms but the modes $c_\pm$ nonresonantly interact with the atoms. In the regime $\nu \gg g$, the interaction of the atoms with the nonresonant modes is highly suppressed, and the system reduces to four atoms resonantly coupled through a

single-mode harmonic oscillator. So the Hamiltonian Eq.(24) in the interaction picture becomes

$$H_{\rm I} = \frac{1}{\sqrt{2}}[g_1(\sigma_1+\sigma_2)c^+ - g_2(\sigma_3+\sigma_4)c^+ + {\rm H.c.}] + H_{\rm AA}. \quad (25)$$

As in Section 3, the subspace in the single excitation regime can be spanned by the following basis state vectors:

$$\begin{aligned}|\phi_1\rangle &= |10\rangle_{12}|00\rangle_{34}|0\rangle_c,\\ |\phi_2\rangle &= |01\rangle_{12}|00\rangle_{34}|0\rangle_c,\\ |\phi_3\rangle &= |00\rangle_{12}|10\rangle_{34}|0\rangle_c,\\ |\phi_4\rangle &= |00\rangle_{12}|01\rangle_{34}|0\rangle_c,\\ |\phi_5\rangle &= |00\rangle_{12}|00\rangle_{34}|1\rangle_c, \end{aligned} \quad (26)$$

where $|0\rangle_c$ and $|1\rangle_c$ are number states of the mode c with zero and one photon, respectively. For the same initial state $|\psi(0)\rangle = \cos\alpha|\phi_1\rangle + \sin\alpha|\phi_2\rangle$, using the same method as that in Section 3, we can show that at time t, $|\psi(t)\rangle = \sum_{i=1}^{5} A_i|\phi_i\rangle$, where

$$\begin{aligned}
A_1(t) &= \frac{1}{2}(\cos\alpha-\sin\alpha)\exp({\rm i}mgt) + \frac{1}{4}(\cos\alpha+\sin\alpha)\exp(-{\rm i}mgt)\\
&\quad + \frac{\sqrt{8+m^2}+m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m+\sqrt{8+m^2})\right]\\
&\quad + \frac{\sqrt{8+m^2}-m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m-\sqrt{8+m^2})\right],\\
A_2(t) &= -\frac{1}{2}(\cos\alpha-\sin\alpha)\exp({\rm i}mgt) + \frac{1}{4}(\cos\alpha+\sin\alpha)\exp(-{\rm i}mgt)\\
&\quad + \frac{\sqrt{8+m^2}+m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m+\sqrt{8+m^2})\right]\\
&\quad + \frac{\sqrt{8+m^2}-m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m-\sqrt{8+m^2})\right],\\
A_3(t) &= \frac{1}{4}(\cos\alpha+\sin\alpha)\exp(-{\rm i}mgt)\\
&\quad - \frac{\sqrt{8+m^2}+m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m+\sqrt{8+m^2})\right]\\
&\quad - \frac{\sqrt{8+m^2}-m}{8\sqrt{8+m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m-\sqrt{8+m^2})\right],\\
A_4(t) &= A_3(t),\\
A_5(t) &= \frac{1}{\sqrt{16+2m^2}}(\cos\alpha+\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m+\sqrt{8+m^2})\right]\\
&\quad - \frac{1}{\sqrt{16+2m^2}}(\cos\alpha-\sin\alpha)\exp\left[-\frac{{\rm i}gt}{2}(m-\sqrt{8+m^2})\right], \quad (27)
\end{aligned}$$

where $m = \Omega/g$.

In terms of Eqs.(20) and (27), we present the results of the concurrence C_{12} and C_{34} in Fig.3, which shows the effects of the DD interaction between the atoms on the entanglement transfer in the regime $\nu \gg g$.

In Fig.3(a), the DD interaction is much stronger than the interaction between the cavity and the atoms; we note that C_{12} and C_{34} still both range from 0 to 1 with the same oscillation period, and C_{34} is delayed by half the oscillation period with respect to C_{12}. Besides, the time of entanglement transfer becomes longer with the increase of the strength of DD interaction. Although the coupling between the atoms and the cavity is weak, we cannot completely ignore it in practical situations. Considering its influences, we find that the concurrence C_{34} will oscillate sinusoidally with a large number of small oscillations. However, the entanglement transfer between the two pairs of atoms still can be realized. In Fig.3(b), when the strength of DD interaction is comparable to the resonant interaction, the entanglement cannot completely transfer from atoms 1, 2 to atoms 3, 4. If the DD interaction is much weaker than the coupling between the

atoms and the cavity, the results are shown in Fig.3(c), which are the same as those in Fig.2(a). That is, the DD interaction can be safely neglected when it is very weak.

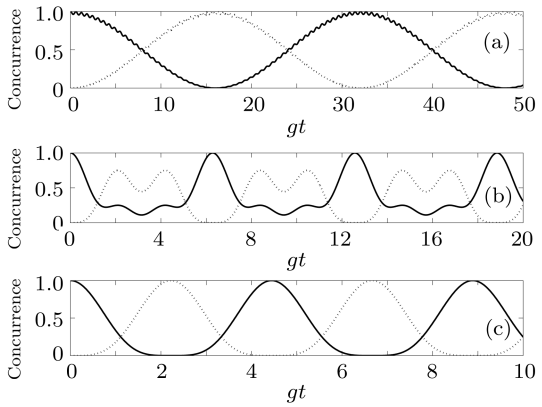


Fig.3. The entanglement transfer between the atoms 1, 2 and 3,4 with the influence of DD interaction, where C_{12} is represented by a solid curve, C_{34} is represented by a dotted curve: (a) $m = 10, \alpha = \pi/4$; (b) $m = 1, \alpha = \pi/4$; (c) $m = 0.01, \alpha = \pi/4$.

5. Summary

In this paper, we have studied the entanglement transfer between two pairs of atoms in two distant cavities coupled by an optical fibre. Supposing that the atoms collectively and resonantly couple with the local fields and the DD interaction between the atoms is ignored, we show that a complete transfer of entanglement between the two pairs of atoms can be deterministically realized when the interaction between the fibre and the cavity is very strong.

When the coupling between the fibre and cavity modes is much stronger than the coupling between the atoms and the cavity field, the entire system can be approximated as four qubits resonantly coupling to a single-mode harmonic oscillator. In this limitation, particular attention has been paid to the influences of DD interaction between the atoms on the entanglement transfer. For strong DD interaction, we note that complete entanglement transfer can still be realized, but the time is prolonged.

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