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Generation of an N-qubit phase gate via atom–cavity nonidentical coupling*

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A scheme for approximate generation of an N-qubit phase gate is proposed in cavity QED based on nonidentical coupling between the atoms and the cavity. The atoms interact with a highly detuned cavity-field mode, but quantum information does not transfer between the atoms and cavity field, and thus the cavity decay is negligible. The gate time does not rise with an increase in the number of qubits. With the choice of a smaller odd number l (related to atom–cavity coupling constants), the phase gate can be generated with a higher fidelity and a higher success probability in a shorter time (the gate time is much shorter than the atomic radiative lifetime and photon lifetime). When the number of qubits N exceeds certain small values, the fidelity and success probability rise slowly with an increase in the number of qubits N. When $N \to \infty$, the fidelity and success probability infinitely approach 1, but never exceed 1.

Keywords: quantum phase gate, three-level atom, cavity QED
PACC: 0365, 4250

1. Introduction

The powerful application of some important quantum algorithms, such as Shor's prime factoring[1] and Grover's database searching,[2] has offered some convincing evidence that quantum computers have great advantages over classical computers for the effective solving and speeding up of ceratin complicated problems. It is known that the building blocks of quantum computers are one- and two-qubit logic gates,[3] in other words, any quantum computation can be decomposed into a set of these elementary gates.[4−10] So far, much attention has been devoted to the realization of these logic gates in physical systems, including trapped ions,[11] cavity quantum electrodynamics (QED),[12] nuclear magnetic resonance (NMR),[13] quantum dots,[14] and superconducting charge qubits.[15] Although fast progress has been made in manipulation of a few qubits in experiments,[16−18] the physical realization of a scalable quantum computer is still in its early stages due to the finite techniques to efficiently implement general multi-qubit gates. In principle, any multi-qubit gates can be implemented by using a set of one- and two-qubit gates; however, the direct implementation of multi-qubit gates in physical realization of quantum algorithms[1,2] and quantum-error-correction[19] consumes a shorter gate time than decomposing multi-qubit gates into elementary gates. Especially for a larger number of qubits, the procedure for the implementation of many elementary gates in obtaining a multi-qubit gate is extraordinarily complicated. Therefore, implementing multi-qubit gates directly is essential in construction of quantum-computing networks in view of shortening gate time and reducing operation complexity.

So far, fast progress towards realizing multi-qubit gates has been made based on optical lattices,[20] cavity QED,[21] NMR,[22] and so on. Among all the possible candidate systems, cavity QED, i.e., atoms interacting with a quantized electromagnetic field inside a cavity, has attracted more attention because of its superior properties:[23] (i)the long-lived internal states of the cold and localized atoms are well suited to store quantum information; (ii) the photons are fast and reliable to be used to transport quantum information over a long distance. In particular, crucial achievements, the generation of two-particle[24] or three-particle[25] entangled states and the implementation of two-qubit phase gate[26] in experiments, promote fast developments of quantum information processing via multiqubits[27−30] in cavity

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QED. The model[29] in which highly detuned cavity-field modes interact with a four-level system in an inverted-Y configuration is considered for the generation of a three-qubit gate. An N-qubit Toffoli gate is generated[30] via the nonidentical coupling of three three-level atoms with a resonant single-mode vacuum cavity field. Moreover, the gate time does not change with an increase in the number of qubits. A quantum controlled-not (CNOT) gate is generated[31] based on the large-detuning interaction of two identical two-level atoms with a single-mode cavity field. Its main feature is that, during the operation, the cavity is virtually excited, thus the efficient decoherence time of the cavity is greatly delayed. Synthesizing the ideas of the two models,[30,31] we propose a scheme for approximate generation of an N-qubit phase gate based on nonidentical coupling of the atoms with a highly detuned cavity-field mode. Its main advantages are, for example, (i) the gate time does not rise with the increase of the number of qubits and it is much shorter than the atomic radiative lifetime and photon lifetime; (ii) cavity decay is negligible; (iii) the requirements for a shorter gate time, a higher fidelity and a higher success probability can be satisfied simultaneously by appropriately adjusting the coupling constants of the atoms with the cavity; (iv) when N exceeds certain values, the fidelity and the success probability rise with the increase of the number of qubits N; thus the feasibility of the scheme increases.

This paper is organized as follows. In Section 2, we describe the atom–cavity interaction model. In Section 3, we describe how to generate an approximate N-qubit phase gate based on the nonidentical coupling of the atoms with a highly detuned cavity-field mode. In Section 4, as an example, we introduce the process for generation of a three-qubit phase gate. A discussion and summary are given in Section 5.

2. The atom–cavity interaction model

We consider that N three-level atoms simultaneously interact with a single-mode cavity field. Each atom has one excited state $|e\rangle$ and two ground states $|g\rangle$ and $|r\rangle$. In our scheme, the state $|r\rangle$ is not involved in the interaction with the cavity mode. In the rotating-wave approximation, the interaction Hamiltonian can be written as (assuming $\hbar = 1$)

$$H_{\rm i} = \sum_{j=1}^{N} g_j (S_j^+ a {\rm e}^{-{\rm i}\,\Delta t} + S_j^- a^\dagger {\rm e}^{{\rm i}\,\Delta t}), \tag{1}$$

where $S_j^+ = |e\rangle_j\langle g|$, $S_j^- = |g\rangle_j\langle e|$, $|e\rangle_j$ and $|g\rangle_j (j = 1, 2, \ldots, N)$ are the excited state and the ground state of the jth atom. $a^\dagger$ and a are the creation and annihilation operators of the single-mode cavity, respectively. g_j is the coupling constant of the jth atom to the cavity mode. $\Delta = \omega_0 - \omega$ with ω_0 and ω being the atomic transition frequency and cavity frequency, respectively. In the case of $\Delta \gg g_j$, there is no energy exchange between the atomic system and the cavity, the effective Hamiltonian is

$$\begin{aligned} H_{\rm eff} = & \sum_{j=1}^{N} \lambda_j (|e\rangle_j\langle e| aa^\dagger - |g\rangle_j\langle g| a^\dagger a) \\ & + \sum_{j,k=1, j\neq k}^{N} \lambda_{jk} S_j^+ S_k^-, \end{aligned} \tag{2}$$

where $\lambda_j = g_j^2/\Delta$ and $\lambda_{jk} = g_j g_k/\Delta$. The first two terms denote the photon-number dependent Stark shifts, and the third term denotes the cavity mode-induced dipole coupling among the N atoms, assuming that the coupling constant for the first atom with the cavity is g_1 and the coupling constants for the other atoms with the cavity are $g = g_j (j = 2, 3, \ldots, N)$. If the cavity field is in the vacuum state initially, the effective Hamiltonian becomes

$$\begin{aligned} H_{\rm eff} = & \lambda_1 |e\rangle_1\langle e| + \sum_{j=2}^{N} \lambda |e\rangle_j\langle e| \\ & + \lambda \sum_{j,k=2, j\neq k}^{N} S_j^+ S_k^- \\ & + \sqrt{\lambda_1 \lambda} \sum_{j=2}^{N} (S_1^+ S_j^- + S_1^- S_j^+), \end{aligned} \tag{3}$$

where $\lambda_1 = g_1^2/\Delta$ and $\lambda = g^2/\Delta$.

3. N-qubit phase gate

We assume that the N atoms are in the state $|e_1\rangle \prod_{j=2}^{N} |g_j\rangle$ initially. The state evolution is

$$\begin{aligned} |e_1\rangle \prod_{j=2}^{N} |g_j\rangle \rightarrow & a_{N-1} |e_1\rangle \prod_{j=2}^{N} |g_j\rangle \\ & + b_{N-1} \sum_{k=2}^{N} |g_1\rangle |e_k\rangle \prod_{j=2(j\neq k)}^{N} |g_j\rangle, \end{aligned} \tag{4}$$

where

$$a_{N-1} = \frac{(N-1)\lambda + \lambda_1 \mathrm{e}^{-\mathrm{i}((N-1)\lambda+\lambda_1)t}}{(N-1)\lambda+\lambda_1},$$
$$b_{N-1} = \frac{-\lambda\lambda_1 + \lambda\lambda_1 \mathrm{e}^{-\mathrm{i}((N-1)\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}[(N-1)\lambda+\lambda_1]}. \quad (5)$$

If the N atoms are initially in the state $|e_1\rangle \prod_{j=2}^{N} |r_j\rangle$, only the first atom interacts with the cavity mode. So the state evolution is

$$|e_1\rangle \prod_{j=2}^{N} |r_j\rangle \to c|e_1\rangle \prod_{j=2}^{N} |r_j\rangle. \quad (6)$$

If the N atoms are initially in the state $|e_1\rangle|g_k\rangle \prod_{j=2}^{N} |r_j\rangle (j \neq k)$, the state evolution is

$$|e_1\rangle|g_k\rangle \prod_{j=2,j\neq k}^{N} |r_j\rangle \to a_1|e_1\rangle|g_k\rangle \prod_{j=2,j\neq k}^{N} |r_j\rangle + b_1|g_1\rangle|e_k\rangle \prod_{j=2,j\neq k}^{N} |r_j\rangle, \quad (k = 2, 3, \ldots, N). \quad (7)$$

If the N atoms are initially in the state $|e_1\rangle|g_k\rangle \ldots |g_\eta\rangle \prod_{j=2}^{N} |r_j\rangle (j \neq k \neq \ldots \neq \eta)$, the state evolution is

$$\begin{aligned} |e_1\rangle|g_k\rangle \ldots |g_\eta\rangle \prod_{j=2,j\neq k\neq \ldots \neq \eta}^{N} |r_j\rangle \to{}& a_\chi |e_1\rangle|g_k\rangle \ldots |g_\eta\rangle \prod_{j=2,j\neq k\neq \ldots \neq \eta}^{N} |r_j\rangle \\ &+ b_\chi |g_1\rangle \left(|e_k\rangle \ldots |g_\eta\rangle + \ldots + |g_k\rangle \ldots |e_\eta\rangle\right) \prod_{j=2,j\neq k\neq \ldots \neq \eta}^{N} |r_j\rangle, \\ &(k, \ldots, \eta = 2, 3, \ldots, N), \end{aligned} \quad (8)$$

where $\chi = k + \ldots + \eta$. The evolutions of the other states remain unchanged. If the interaction time satisfies $\lambda_1 t = \pi$ and $\lambda t = l\pi$ (l is an integer). Under the condition of $\lambda \gg \lambda_1$, we can approximately obtain an N-atom phase gate.

We prepare an N-atom entangled state from an initial state $|\psi_0\rangle = (1/\sqrt{2})^N (|e\rangle + |g\rangle)_1 \prod_{j=2}^{N} (|g\rangle + |r\rangle)_j$, the corresponding fidelity F_N and the success probability P_N can be expressed as

$$\begin{aligned} F_N = \frac{1}{2^N} &\left(\sum_{i=0}^{N-2} C_{N-1}^{i} \left|a_{N-1-i}\right|^2 + C_{N-1}^{N-1} |c|^2 + \sum_{j=0}^{N-1} C_{N-1}^{j} \right)^{-1} \\ &\times \left| \sum_{i=0}^{N-2} C_{N-1}^{i} a_{N-1-i} - C_{N-1}^{N-1} c + \sum_{j=0}^{N-1} C_{N-1}^{j} \right|^2, \end{aligned} \quad (9)$$

and

$$P_N = \frac{1}{2^N} \left(\sum_{i=0}^{N-2} C_{N-1}^{i} \left|a_{N-1-i}\right|^2 + C_{N-1}^{N-1} |c|^2 + \sum_{j=0}^{N-1} C_{N-1}^{j} \right), \quad (10)$$

respectively, where $C_m^n = m!/n!(m-n)!$. Based on the universal expressions we can calculate the fidelity and success probability for an arbitrary N-qubit phase gate. The discrete values for the fidelity F and success probability P versus the number of qubits $N = 3, 4, \ldots, 20$ with $l = 5, 7$ are shown in Fig.1. When $N > 5$, the fidelity and the success probability rise slowly with the increase of the number of qubits N. For a fixed N, the fidelity and success probability induced by the larger odd number $l = 7$ are larger than that induced by the smaller odd number $l = 5$. When $N \to \infty$, the fidelity and success probability both infinitely approach 1, but never exceed 1. That is to say, the present scheme is suitable for the generation of a multi-qubit phase gate with a higher fidelity and a higher success probability.

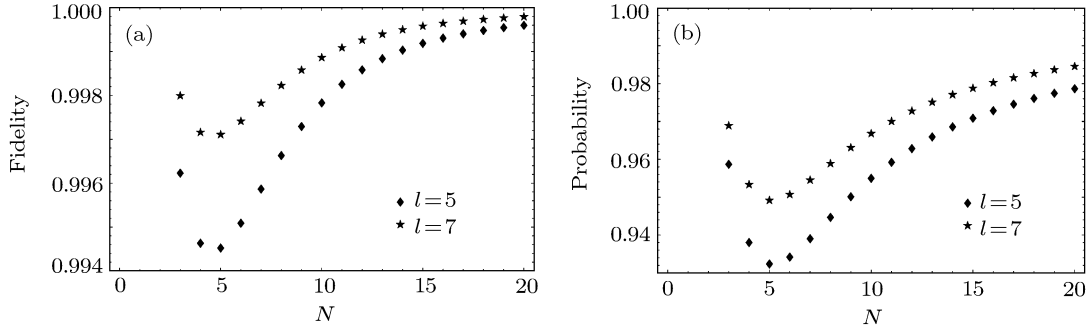


Fig.1. The discrete values of the fidelity F (a) and the success probability P (b) versus the number of qubits $N = 3, 4, 5, \ldots, 20$ when $l = 5$ and $l = 7$, respectively.

4. Three-qubit phase gate

In order to describe the present scheme explicitly, we take the case of three atoms as an example. In this case, the three three-level atoms simultaneously interact with a single-mode cavity field. Assume that the coupling constant for the first atom with the cavity is g_1 and the coupling constants for the other two atoms with the cavity are $g = g_j (j = 2, 3)$. If the cavity is initially in the vacuum state, the effective Hamiltonian is

$$H_{\text{eff}} = \lambda_1 |e\rangle_1\langle e| + \sum_{j=2}^{3} \lambda |e\rangle_j\langle e| + \lambda \sum_{j,k=2, j\neq k}^{3} S_j^+ S_k^- + \sqrt{\lambda_1 \lambda} \sum_{j=2}^{3} (S_1^+ S_j^- + S_1^- S_j^+). \tag{11}$$

If the three atoms are initially in the state $|e_1\rangle|g_2\rangle|g_3\rangle$, the state evolution is

$$\begin{aligned} |e_1\rangle|g_2\rangle|g_3\rangle \to & \frac{2\lambda + \lambda_1 \mathrm{e}^{-\mathrm{i}(2\lambda+\lambda_1)t}}{2\lambda + \lambda_1} |e_1\rangle|g_2\rangle|g_3\rangle \\ & + \frac{-\lambda\lambda_1 + \lambda\lambda_1 \mathrm{e}^{-\mathrm{i}(2\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}(2\lambda + \lambda_1)} (|g_1\rangle|e_2\rangle|g_3\rangle + |g_1\rangle|g_2\rangle|e_3\rangle). \end{aligned} \tag{12}$$

If the three atoms are initially in the state $|e_1\rangle|r_2\rangle|r_3\rangle$, only the first atom interacts with the cavity mode. So the state evolution is

$$|e_1\rangle|r_2\rangle|r_3\rangle \to \mathrm{e}^{-\mathrm{i}\,\lambda_1 t} |e_1\rangle|r_2\rangle|r_3\rangle. \tag{13}$$

If the three atoms are initially in the state $|e_1\rangle|g_2\rangle|r_3\rangle$ or $|e_1\rangle|r_2\rangle|g_3\rangle$, the state evolution is

$$|e_1\rangle|g_2\rangle|r_3\rangle \to \frac{\lambda + \lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\lambda + \lambda_1} |e_1\rangle|g_2\rangle|r_3\rangle + \frac{-\lambda\lambda_1 + \lambda\lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}(\lambda + \lambda_1)} |g_1\rangle|e_2\rangle|r_3\rangle, \tag{14}$$

or

$$|e_1\rangle|r_2\rangle|g_3\rangle \to \frac{\lambda + \lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\lambda + \lambda_1} |e_1\rangle|r_2\rangle|g_3\rangle + \frac{-\lambda\lambda_1 + \lambda\lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}(\lambda + \lambda_1)} |g_1\rangle|r_2\rangle|e_3\rangle. \tag{15}$$

Now we start to construct a three-atom phase gate. Assume that the quantum information is encoded in a subspace spanned by the atomic states $\{|e_1\rangle, |g_1\rangle, |g_2\rangle, |r_2\rangle, |g_3\rangle, |r_3\rangle\}$. After an interaction time t, the evolutions of the system are

$$\begin{aligned} |g_1, g_2, g_3\rangle &\to |g_1, g_2, g_3\rangle, \\ |g_1, g_2, r_3\rangle &\to |g_1, g_2, r_3\rangle, \\ |g_1, r_2, g_3\rangle &\to |g_1, r_2, g_3\rangle, \\ |g_1, r_2, r_3\rangle &\to |g_1, r_2, r_3\rangle, \end{aligned}$$

$$
\begin{aligned}
|e_1,g_2,g_3\rangle &\to a_2|e_1,g_2,g_3\rangle + b_2(|g_1,e_2,g_3\rangle \\
&\quad + |g_1,g_2,e_3\rangle), \\
|e_1,g_2,r_3\rangle &\to a_1|e_1,g_2,r_3\rangle + b_1|g_1,e_2,r_3\rangle, \\
|e_1,r_2,g_3\rangle &\to a_1|e_1,r_2,g_3\rangle + b_1|g_1,r_2,e_3\rangle, \\
|e_1,r_2,r_3\rangle &\to c|e_1,r_2,r_3\rangle, \qquad (16)
\end{aligned}
$$

where

$$
\begin{aligned}
a_2 &= \frac{2\lambda+\lambda_1 \mathrm{e}^{-\mathrm{i}(2\lambda+\lambda_1)t}}{2\lambda+\lambda_1}, \\
b_2 &= \frac{-\lambda\lambda_1+\lambda\lambda_1 \mathrm{e}^{-\mathrm{i}(2\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}(2\lambda+\lambda_1)}, \\
a_1 &= \frac{\lambda+\lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\lambda+\lambda_1}, \\
b_1 &= \frac{-\lambda\lambda_1+\lambda\lambda_1 \mathrm{e}^{-\mathrm{i}(\lambda+\lambda_1)t}}{\sqrt{\lambda\lambda_1}(\lambda+\lambda_1)}, \\
c &= \mathrm{e}^{-\mathrm{i}\,\lambda_1 t}. \qquad (17)
\end{aligned}
$$

If the interaction time satisfies $\lambda_1 t = \pi$ and $\lambda t = l\pi$, we approximately obtain a three-atom phase gate under the condition $\lambda \gg \lambda_1$, as

$$
\begin{aligned}
&|g_1,g_2,g_3\rangle \to |g_1,g_2,g_3\rangle, \\
&|g_1,g_2,r_3\rangle \to |g_1,g_2,r_3\rangle, \\
&|g_1,r_2,g_3\rangle \to |g_1,r_2,g_3\rangle, \\
&|g_1,r_2,r_3\rangle \to |g_1,r_2,r_3\rangle, \\
&|e_1,g_2,g_3\rangle \to |e_1,g_2,g_3\rangle, \\
&|e_1,g_2,r_3\rangle \to |e_1,g_2,r_3\rangle, \\
&|e_1,r_2,g_3\rangle \to |e_1,r_2,g_3\rangle, \\
&|e_1,r_2,r_3\rangle \to -|e_1,r_2,r_3\rangle. \qquad (18)
\end{aligned}
$$

Utilizing expression (16) to prepare an entangled state from an initial state $|\psi_0\rangle = \frac{1}{2\sqrt{2}}(|e\rangle + |g\rangle)_1(|g\rangle + |r\rangle)_2(|g\rangle + |r\rangle)_3$, we obtain the fidelity F and success probability P, as

$$
F = \frac{1}{8}\left(\frac{1}{5+|a_2|^2+2|a_1|^2}\right)|5+a_2+2a_1|^2, \qquad (19)
$$

and

$$
P = \frac{1}{8}\left(5+|a_2|^2+2|a_1|^2\right), \qquad (20)
$$

respectively.

Then we discuss the influences of the ratio $l(=\lambda/\lambda_1)$ on the fidelity F and the success probability P. According to Fig.2, the fidelity and the success probability oscillate periodically accompanying the changing of l. With the increase of l, the fidelity and the success probability increase slowly on the whole. When l reaches an appropriate ratio, the fidelity and success probability both approach 1. For example, if $l = 5$, the fidelity and the success probability are 0.996228 and 0.958678, respectively. The odd numbers l correspond to a series of oscillation maxima of the fidelity (success probability) and the even numbers l correspond to a series of oscillation minima of the fidelity (success probability). This means that a larger l does not always correspond to a higher fidelity and a higher success probability. Under certain conditions, the fidelity and success probability induced by a smaller odd number l are higher than that induced by a larger even number l. For example, when l=8, F=0.989538 (P=0.873553), while when l= 5 and 7, F=0.996228 (P=0.958678) and 0.997993 (0.968889), respectively. That is to say, in order to generate a three-qubit phase gate with a higher fidelity and a higher success probability in a shorter gate time, the choice of a smaller odd number l is suitable.

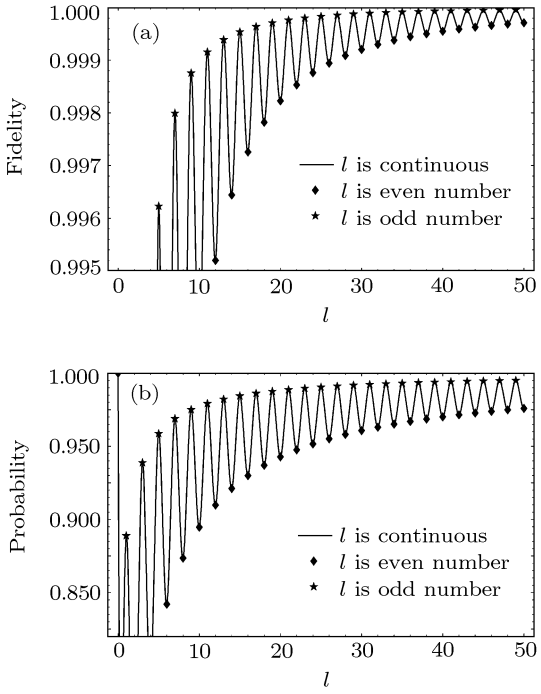


Fig.2. The curves of the values of the fidelity F (a) and the success probability P (b) versus the continuous ratio $l(=\lambda/\lambda_1)$ when $N = 3$.

5. Discussion and summary

We now discuss the physical setup and experimental parameters briefly. The previous experimental setup[26,32–34] is applicable to our scheme, which is shown in Fig.3. Consider that quantum information is stored in three circular Rydberg atomic states $|e\rangle$, $|g\rangle$ and $|r\rangle$ with principal quantum numbers 51, 50 and 49, respectively. These atoms have a long radiative lifetime (about 3×10^{-2} s) and a very strong

coupling to radiation. The atomic level structure is described in Fig.3. In our scheme, transition only occurs at the levels $|e\rangle$ and $|g\rangle$ and the corresponding transition frequency of 51.1 GHz is realizable.[26] The Rydberg atoms can be prepared in zone B of Fig.3. At first, the atoms emanate from the oven O; they undergo the velocity-select process induced by laser optical pumping and are prepared into the circular Rydberg atoms in zone B by combining laser and radio frequency excitation.[35] Then the atoms pass through the high-Q superconducting cavity C sustaining a Gaussian field mode. The cavity C is made of two superconducting niobium mirrors facing each other and enclosed by an aluminum ring with small holes. Atoms can access the cavity through the small holes. Finally, the atoms are detected by a state selective field-ionization detector D to discriminate the state $|e\rangle$, $|g\rangle$, and $|r\rangle$ after they exit the cavity C. A pulsed source S is used to inject a small coherent field in cavity C. The whole setup is cooled to around 1 K to minimize the effects of thermal field. The generation process involves nonidentical coupling of the atoms with a highly detuned cavity-field mode. So far the best cavity corresponds to a quality factor $Q = 3\times10^8$, which has a photon storage time 1×10^{-3} s.[17] With the choice of the parameters $l = 5$, $\Delta = 10g$, and $g = 2\pi \times 47 \times 10^3$ Hz,[26] the atom–cavity interaction time is $t = l\pi/\lambda = 5.3 \times 10^{-4}$ s, much shorter than the atomic radiative lifetime and photon lifetime. Therefore, the phase gate can be generated before the relaxation processes of the atom and photon set in.

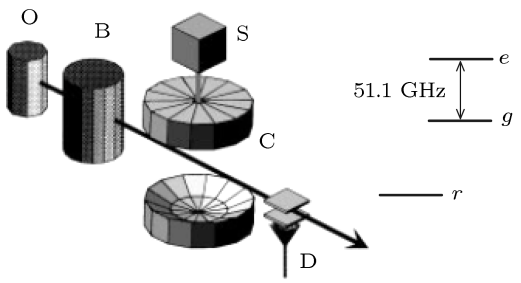


Fig.3. Experimental setup (cited from Ref.[32]) and atomic levels.

According to the above parameters, the coupling constants for the first atom with the cavity and the other atoms with the cavity should satisfy the relation $g = \sqrt{5}g_1$. In fact, the coupling strength between the atoms and the cavity is determined by the rule[32] $g = g_0 \mathrm{e}^{-r^2/\omega^2}$ with g_0, the coupling strength at the cavity center, r, the position of the atom to the cavity center, and ω, the waist of the cavity mode. Thus the above coupling condition can be satisfied by locating the first atom at the position $r = \omega(\ln\sqrt{5})^{1/2}$ to the cavity and the other $(N-1)$ atoms at the center of the cavity.

Also, it is important to point out the advantages of our scheme over the previous one in Ref.[30] based on the resonant interaction of the atoms with the cavity. In our scheme the large-detuning interaction of the atoms with the single-mode cavity reduces the effects of the cavity decay. The different dissipative mechanisms of the two schemes result in different values of the fidelity and success probability even if the ratio of the coupling constants g/g_1 takes the same values. In the scheme of Ref.[3], only if the coupling constants satisfy $g_1 \ll g$ can the fidelity and success probability approach satisfactory values; however, in our scheme, as long as the ratio $l = (g^2/g_1^2)$ takes a smaller odd number, the corresponding fidelity and success probability can reach higher values. For example, when $N = 3$ for the same ratio $(g/g_1) = \sqrt{5}, \sqrt{7}$, the fidelity and success probability in our scheme are $F = 0.996228$, 0.997993 and $P = 0.958678$, 0.968889, respectively, larger than the values $F = 0.994881$, 0.988792 and $P = 0.902115$, 0.896044 in the scheme of Ref.[30].

In conclusion, we have approximately generated an N-qubit phase gate based on nonidentical coupling of the atoms with a highly detuned cavity-field mode. In our scheme, we deal with cavity QED with 'flying' qubits. The generation process requires a large-detuning interaction of the atoms with the single-mode cavity; quantum information does not transfer between the atoms and the cavity field, and thus the cavity decay is negligible. The phase gate can be generated before the relaxation processes of the atom and the photon set in. Moreover, the gate time does not rise with the increase of the number of qubits. The fidelity and success probability for an arbitrary N-qubit phase gate can be calculated by the given expressions. The results indicate that when N exceeds certain small values, the fidelity and success probability rise with the increase of the number of qubits. When $N \to \infty$, the fidelity and success probability infinitely approach 1, but never exceed 1. That is to say, the present scheme is suitable for the generation of a multi-qubit phase gate. The requirements for a shorter gate time, a higher fidelity and a higher success probability can be satisfied simultaneously if the ratio l is chosen appropriately to be a smaller odd number. All the above mentioned advantages increase the feasi-

bility of the present scheme. We hope that the cavity QED-based phase gate scheme will help with developments in quantum information processing both theoretically and experimentally.

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