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Entanglement sudden death of two atoms interacting with a cavity via the two-photon process in a strong-driving-assisted system*

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We examine the entanglement dynamics between two strongly driven atoms off-resonantly coupled with a single-mode cavity via the two-photon process with the help of negativity in two different types of initial states. The results show that entanglement sudden death may occur under both the above conditions and the sudden death effect can be monitored by modulating the atom–cavity detunings. Furthermore, we also find an atomic decoherence-free subspace so that the initial entanglement between two atoms remains invariable in application.

Keywords: entanglement sudden death, negativity, atom–cavity detuning, decoherence-free subspace

PACC: 0365, 4250

1. Introduction

Entanglement, as a basic feature for distinguishing the quantum world from the classical world, has evolved from an important consequence of quantum theory to a valuable physical resource. The dynamical behaviour of entanglement in the presence of environment is a central issue in quantum computation and quantum information processing.[1−3] Recently, it has been found that two particles affected by uncorrelated reservoirs can disentangle in a finite time despite the fact that coherence is lost asymptotically. This intriguing discovery, termed 'entanglement sudden death' (ESD),[4,5] means that the entanglement of two qubits has an abrupt decay in time rather than the typical exponential decay for a single qubit. This has aroused great interest in the dynamics of entanglement, and has been observed in the lab for entangled photon pairs[6] and atomic ensembles.[7] To our knowledge ESD has been studied mainly in relation to bipartite systems,[8−12] for instance the Jaynes–Cummings (JC) model. Extensions of the JC model to more atoms and more modes, externally driven or not, and the two-photon process, have also been developed.[13−19] More recently, Yang[20] studied the entanglement dynamics of strongly driven atoms off-resonantly coupled with a cavity by two different models. In Model I, one atom was trapped in a single cavity, while the other remained outside the cavity; Model II dealt with a double-driven JC model, i.e. each atom interacted with its own cavity. The result shows that atomic ESD always occurs in the case where the atom–field coupling is larger than the atom–field detuning. Moreover, the two-photon process is known to play a very important role in atomic systems due to the high degree of correlation between emitted photons, which stimulates interest in the investigation of the two-photon JC model in theory and experiment.[21−23] Due to the realization of a two-photon maser oscillator, it is of great practical significance to study entanglement via a two-photon process. It has been shown that the atom–atom entanglement through the two-photon process is larger than that through the one-photon process in a thermal field.[24,25]

At present, in order to quantify the quantum entanglement of two-qubit states, various measures have been widely applied: the relative entropy of entanglement,[26] a measure of the 'distance' of an entangled state from the set of disentangled states; the negativity,[27] a measure of entanglement cost under operations preserving the positivity of partial transpose; and the concurrence,[28] a measure of the entanglement of formation. Nevertheless, the negativ-

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ity is simpler and more effective for our system, so we will adopt it to describe the process of entanglement evolution of two atoms. The negativity is directly related to the famous Peres–Horodecki criterion of separability,[29,30] i.e. if the partial transpose $\rho^{T_{\rm A}}$ of ρ has a negative eigenvalue, the system is entangled. According to the Peres–Horodecki criterion of separability, negativity can be defined for a two-qubit state ρ as

$$N(\rho) = \max\{0, -2\lambda_{\min}\}, \tag{1}$$

where $\lambda_{\min}$ is the minimal eigenvalue of the partial transpose $\rho^{T_{\rm A}}$ of ρ.

In this paper, we consider a system composed of two coherently driven two-level atoms trapped inside a single-mode cavity and coupled to the cavity mode by off-resonance and the two-photon process. Based on the recent advances in deterministic trapping of atoms in optical cavities, the implementation of this system in the experiment has certain feasibility. By investigating the entanglement characteristic of two atoms in two different forms of a Bell-like state, the results we obtained show that ESD can occur for both types of initial atomic state and entanglement evolution can be controlled by tuning the atom–cavity detunings. In addition, a decoherent-free subspace is found so that the initial atomic entanglement is completely protected.

This paper is organized as follows. We study the entanglement dynamics in the atom–atom subsystem in Section 2. After a brief introduction to the Hamiltonian of our model, the entanglement evolution of two important sets of entangled Bell-like states is discussed by means of negativity. We give analytical solutions of the systemic evolution and consider the influence of atom–cavity detuning on the atomic entanglement. We also find a useful atomic decoherence-free subspace. We draw conclusions from our results and give a possible physical implementation of our model in Section 3.

2. Entanglement dynamics of two atoms

We consider a pair of two-level atoms in a two-photon interaction with a single-mode cavity of high quality factor, driven additionally by an external strong classical field. The Hamiltonian of this system reads (assuming $\hbar = 1$)[31,32]

$$\begin{aligned} H = {} & \omega a^\dagger a + \omega_0 \sum_{i=\rm A,B} \sigma_i^\dagger \sigma_i \\ & + g \sum_{i=\rm A,B} \left(a^2 \sigma_i^\dagger + \sigma_i a^{\dagger 2}\right) \\ & + \Omega \sum_{i=\rm A,B} \left(\mathrm{e}^{-\mathrm{i}\,\omega_L t} \sigma_i^\dagger + \mathrm{e}^{\mathrm{i}\,\omega_L t} \sigma_i\right), \end{aligned} \tag{2}$$

where $a^\dagger$ and a denote the creation and annihilation operators of the cavity mode, respectively; $\sigma_i = |g_i\rangle\langle e_i|$ and $\sigma_i^\dagger = |e_i\rangle\langle g_i|$ are the atomic spin-flip operators down and up, associated with the upper level $|e_i\rangle$ and lower level $|g_i\rangle$ of the ith atom; g and Ω are the coupling constants of the interaction of each atom with the cavity mode and with the driving field, respectively.

In the rotating frame with respect to the driving field frequency $\omega_{\rm L}$, the systemic Hamiltonian can be expressed as

$$\begin{aligned} H_{\rm L} = {} & \delta a^\dagger a + \Delta \sum_{i=\rm A,B} \sigma_i^\dagger \sigma_i \\ & + g \sum_{i=\rm A,B} \left(a^2 \sigma_i^\dagger + \sigma_i a^{\dagger 2}\right) \\ & + \Omega \sum_{i=\rm A,B} \left(\sigma_i^\dagger + \sigma_i\right) \end{aligned} \tag{3}$$

with $\Delta = \omega_0 - \omega_{\rm L}$ and $\delta = \omega - \omega_{\rm L}/2$. We consider a resonance condition between the atoms and the external classic field $\omega_{\rm L} = \omega_0$ hereafter. In the interaction picture we then have

$$\begin{aligned} H_{\rm I} = {} & \frac{g}{2} \sum_{i=\rm A,B} \big(|+\rangle_{ii}\langle +| - |-\rangle_{ii}\langle -| \\ & + \mathrm{e}^{2\,\mathrm{i}\,\Omega t} |+\rangle_{ii}\langle -| - \mathrm{e}^{-2\,\mathrm{i}\,\Omega t} |-\rangle_{ii}\langle +|\big) a^2 \\ & \times \mathrm{e}^{-2\,\mathrm{i}\,\delta t} + \mathrm{H.c.}, \end{aligned} \tag{4}$$

where the dressed states $|\pm\rangle_i = (|g\rangle_i \pm |e\rangle_i)/\sqrt{2}$ are the eigenstates of $\sigma_{xi} = \sigma_i^\dagger + \sigma_i$ with eigenvalues ± 1, respectively. In the strong driving regime $\Omega \gg \{g, \delta\}$, eliminating the terms which oscillate with high frequencies from Eq.(4), we can obtain the effective Hamiltonian of the system under the rotating-wave approximation

$$H_{\rm eff} = \frac{g}{2} \sum_{i=\rm A,B} \left(\sigma_i^\dagger + \sigma_i\right)\left(a^2 \mathrm{e}^{-2\,\mathrm{i}\,\delta t} + a^{\dagger 2} \mathrm{e}^{2\,\mathrm{i}\,\delta t}\right). \tag{5}$$

2.1. Entangled Bell-like states $|\phi_{\rm AB}\rangle$

In this subsection, based on the above effective Hamiltonian we consider the atoms and cavity to be prepared in a Bell-like state at the initial time,

$$|\phi_0\rangle = \left(\cos\theta |ee\rangle + \sin\theta |gg\rangle\right) \otimes |0\rangle. \tag{6}$$

The state of total system governed by Eq.(5) evolves as

$$\begin{aligned}|\phi_t\rangle &= \mathrm{e}^{-\mathrm{i}\int_0^t H_{\mathrm{eff}}\mathrm{d}t}|\phi_0\rangle \\ &= -\frac{\cos\theta-\sin\theta}{2}\big(|++\rangle+|--\rangle\big)|0\rangle \\ &\quad+\frac{\cos\theta+\sin\theta}{2}|++\rangle|2\ \xi\rangle \\ &\quad+\frac{\cos\theta+\sin\theta}{2}|--\rangle|-2\ \xi\rangle, \end{aligned} \tag{7}$$

where $\xi = g(\mathrm{e}^{2\,\mathrm{i}\,\delta t}-1)/2\delta$, especially $\xi = \mathrm{i}\,gt$ with $\delta = 0$, and $|\xi\rangle$ is the squeezed vacuum state of the cavity mode. An expansion of the squeezed vacuum state in terms of Fock states is

$$\begin{aligned}|\xi\rangle &= \sqrt{\mathrm{sech}|\xi|}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{n!} \\ &\quad\times\left(-\frac{1}{2}\mathrm{e}^{\mathrm{i}\,\phi}\tanh|\xi|\right)^n|2n\rangle \end{aligned} \tag{8}$$

with ϕ being the argument of ξ.

The information about the entanglement of two atoms is contained in the reduced density matrix ρ_{AB} for the two atoms which can be obtained from Eq.(7) by tracing over the cavity field variables. The explicit matrix $\rho^{\phi}_{\mathrm{AB}}(t)$ written in the rotated basis of the atomic Hilbert space $|i\rangle_{i=1}^{4} = \{|++\rangle,|+-\rangle,|-+\rangle,|--\rangle\}$ is given by

$$\begin{aligned}&\rho_{11}=\rho_{44}=\left(\frac{\cos\theta+\sin\theta}{2}\right)^2, \\ &\rho_{14}=\rho_{41}=\left(\frac{\cos\theta+\sin\theta}{2}\right)^2 f_2(\xi), \\ &\rho_{22}=\rho_{33}=\rho_{23}=\rho_{32}=\left(\frac{\cos\theta-\sin\theta}{2}\right)^2, \\ &\rho_{12}=\rho_{13}=\rho_{21}=\rho_{24}=\rho_{31}=\rho_{34}=\rho_{42}=\rho_{43} \\ &\quad=-\frac{\cos^2\theta-\sin^2\theta}{4}f_1(\xi), \end{aligned} \tag{9}$$

where

$$f_1(\xi)=\sqrt{\mathrm{sech}|2\xi|},\quad f_2(\xi)=\sqrt{\mathrm{sech}|4\xi|}.$$

The corresponding negativity for this density matrix Eq.(9) is given by

$$N(\rho^{\phi}_{\mathrm{AB}}(t)) = 2\ \max\{0,-\lambda_{\mathrm{AB}}\}; \tag{10}$$

if $0<\theta<\pi/2$, $\lambda_{\mathrm{AB}} = [1-f_2(\xi)-(1+f_2(\xi))\sin(2\theta)]/4$, and if $\pi/2<\theta<\pi$, $\lambda_{\mathrm{AB}} = \sin(2\theta)/2$.

Figure 1 shows that the entanglement non-smoothly becomes zero and resurrects after a time interval for some values of θ in the region of $0\le\theta\le\pi/2$. This effect is just the so-called 'entanglement sudden death'. Without commenting on the physical implications here, this is due simply to the fact that λ_{AB} can take positive values. Considering a special case, i.e. the atom–cavity system under resonance condition ($\delta = 0$) as displayed in Fig.1(a), we find that the negativity between the two atoms decreases at the beginning and then vanishes all through. Hence the resonance regime can be used as an information eraser for $0\le\theta\le\pi/2$. We also see that ESD is very sensitive to initial atomic entanglement and easily occurs for initial small negativity, as shown in Fig.1(b). Comparing Fig.1(b(c)) with 1(a), for resonance the ESD is all over any initial state, while in the presence of atom–field detuning, entanglement becomes zero only for some initial states, which shows that ESD can be improved by introducing atom–field detuning. It is obvious that the values of δ/g influence the atomic entanglement from Figs.1(b) and 1(c).

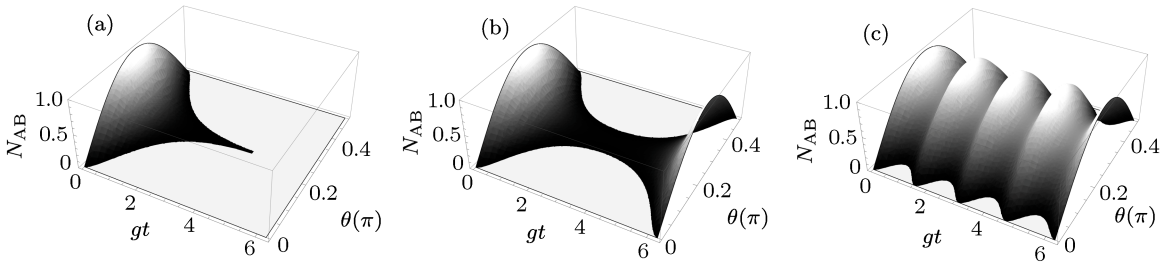


Fig.1. The negativity $N(\rho^{\phi}_{\mathrm{AB}}(t))$ as a function of θ ($0\le\theta\le\pi/2$) and gt for (a) $\delta = 0$, (b) $\delta = g/2$ and (c) $\delta = 2g$, respectively.

With the increase of δ/g the period of entanglement gradually decreases and atomic entanglement becomes stronger, which is more clearly seen in Fig.2.

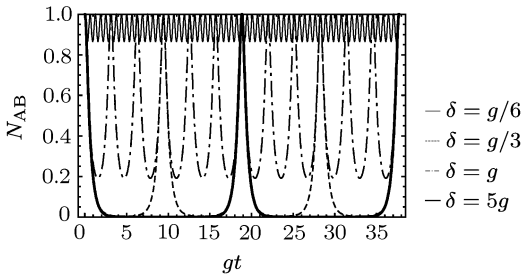


Fig.2. The negativity $N(\rho^{\phi}_{\rm AB}(t))$ as a function of gt for different values of δ with $\theta = \pi/4$.

We show the evolution of the negativity $N(\rho^{\phi}_{\rm AB}(t))$ for atoms prepared in the maximally entangled state $|\phi^{+}\rangle = \big(|ee\rangle + |gg\rangle\big)/\sqrt{2}$ (i.e., $\theta = \pi/4$). From the expression of $\xi = g(\mathrm{e}^{2\,\mathrm{i}\,\delta t} - 1)/2\delta$, we can see that the oscillation term is dependent on δ, so that the larger δ is, the more acutely the negativity oscillates (the shorter the period of negativity evolution is). Thus we find an effective way to strengthen the entanglement between two atoms, i.e. tuning the atom–cavity detunings. Figure 3 indicates that for a large detuning $\delta = 10g$ the evolutions of the entanglement between two atoms exhibit a remarkable increase. The value of negativity oscillates periodically and the amplitude is near to its initial entanglement, implying that by the modulation of detuning the atomic entanglement can be made stronger and the sudden death effect can be completely eliminated when it is not needed, as shown in Fig.3. As we all know, a larger atom–cavity detuning means that the interaction between the atom and cavity becomes smaller. In the large detuning regime, i.e. when δ is much larger than g, the cavity and the atom cannot have energy exchange; however, the atom–atom entanglement will still evolve in time as induced by interacting with the cavity field so that the entanglement becomes stronger and the ESD can completely disappear.

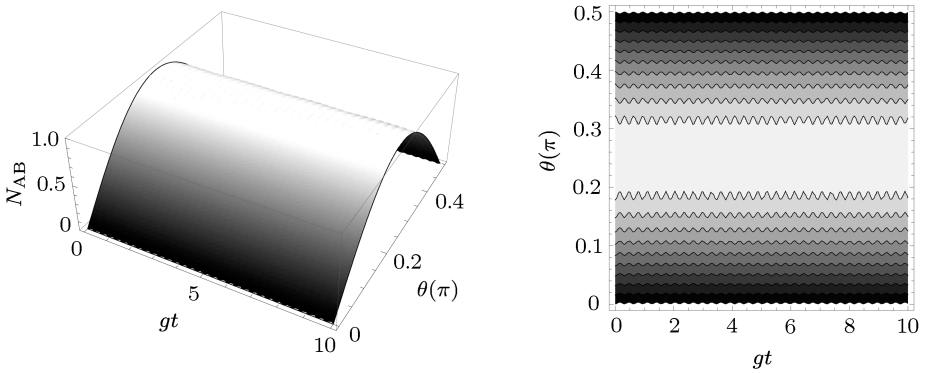


Fig.3. The negativity $N(\rho^{\phi}_{\rm AB}(t))$ as a function of θ $(0 \le \theta \le \pi/2)$ and gt for the large detuning $\delta = 10g$. The right figure corresponds to a contour plot of the $N(\rho^{\phi}_{\rm AB}(t))$. The innermost contours indicate a negativity value of 0.9, and the value drops by $\Delta N = 0.1$ between two consecutive contours.

The previous analysis reveals that δ influences the period of the ESD, as displayed in Fig.2, while Fig.4 shows that g is related to the velocity of the atomic subsystem disentanglement. When δ is fixed, the disentanglement of atoms can be delayed by a small atom–field coupling constant g. The result can be interpreted in physics that the better the atoms are coupled to the cavity mode, the more quickly the initial entanglement decays. The atomic entanglement information can be easily transferred to the cavity, thus the negativity of two atoms declines. Similarly, we can also improve the atomic entanglement by modulating the atom–cavity coupling constant, in which the period of negativity is invariable.

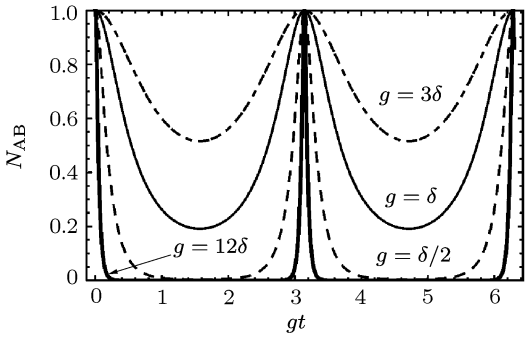


Fig.4. The negativity $N(\rho^{\phi}_{\rm AB}(t))$ as a function of δt for different values of g at $\theta = \pi/4$.

Other than the previous case, Fig.5 implies that $N_{\rm AB}$ has a novel feature that the negativity is invariant for $\pi/2 \leq \theta \leq \pi$. From the expression of negativity Eq.(10) we can see the negativity for $\pi/2 \leq \theta \leq \pi$ is only determined by the value of θ, whereas it is independent of time. Particularly, for atoms prepared in the maximally entangled state $|\phi^-\rangle = \big(|gg\rangle - |ee\rangle\big)/\sqrt{2}$ (i.e., $\theta = 3\pi/4$) the negativity is always maximum ($N_{\rm AB} = 1$). In fact, we can see from Eq.(7) that $|\phi^-\rangle$ does not evolve with time because of

$$(|+-\rangle + |-+\rangle) \otimes |0\rangle = (|gg\rangle - |ee\rangle) \otimes |0\rangle, \quad (11)$$

namely it is a dark state for the effective Hamiltonian Eq.(5). If atomic initial states are prepared in the rotating basis, the results become simpler and more evident. We can observe that either the states of the cavity field are squeezed since $|\pm\pm\rangle \otimes |0\rangle \rightarrow |\pm\pm\rangle \otimes |\pm 2\xi\rangle$, or there is no evolution at all for the states $|\pm\mp\rangle \otimes |0\rangle$, indicating that there exists an invariant subspace spanned by $|+-\rangle$ and $|-+\rangle$ in the system dynamics. Nevertheless, for other values of θ in the region of $\pi/2 \leq \theta \leq \pi$, the systemic states are not dark states, so the vacuum field should be squeezed and the states of atoms still evolve with time. We suppose that there may exist some kind of 'entanglement conservation' for atoms and the negativity is not affected when atoms interact with the cavity field for $\pi/2 \leq \theta \leq \pi$. This is an open issue that is unexplored and we think that conservation of entanglement may not be in the sense of dynamical conservation laws which cannot yet be explained reasonably at present.

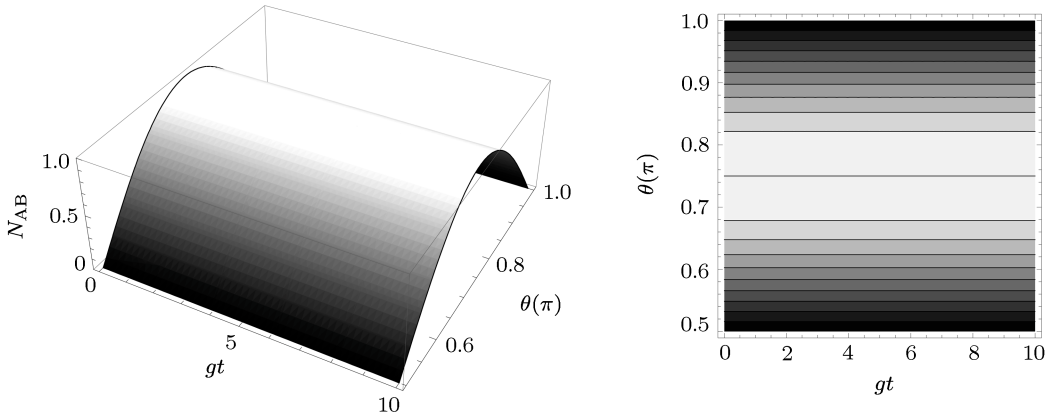


Fig.5. The negativity $N(\rho^{\phi}_{\rm AB}(t))$ as a function of θ ($\pi/2 \leq \theta \leq \pi$) and gt for the resonance $\delta = 0$. Right corresponds to a contour plot of the $N(\rho^{\phi}_{\rm AB}(t))$. The middle contour indicates a negativity value of 1, and the value drops by $\Delta N = 0.1$ between two consecutive contours.

2.2. Entangled Bell-like states $|\psi_{\rm AB}\rangle$

Next we consider another Bell-like state

$$|\psi_0\rangle = \big(\cos\theta|eg\rangle + \sin\theta|ge\rangle\big) \otimes |0\rangle. \quad (12)$$

The effective Hamiltonian Eq.(5) yields the time evolution from initial state $|\psi_0\rangle$ to

$$\begin{aligned} |\psi_t\rangle = {} & \frac{\cos\theta - \sin\theta}{2}\big(|+-\rangle - |-+\rangle\big)|0\rangle \\ & + \frac{\cos\theta + \sin\theta}{2}|++\rangle|2\ \xi\rangle \\ & - \frac{\cos\theta + \sin\theta}{2}|--\rangle|-2\ \xi\rangle. \end{aligned} \quad (13)$$

The corresponding atom–atom reduced density matrix $\rho^{\psi}_{\rm AB}(t)$ at time t reads

$$\begin{aligned} \rho_{11} &= \rho_{44} = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2, \\ \rho_{14} &= \rho_{41} = -\left(\frac{\cos\theta + \sin\theta}{2}\right)^2 f_2(\xi), \\ \rho_{22} &= \rho_{33} = \left(\frac{\cos\theta - \sin\theta}{2}\right)^2, \\ \rho_{23} &= \rho_{32} = -\left(\frac{\cos\theta - \sin\theta}{2}\right)^2, \\ \rho_{12} &= \rho_{21} = \rho_{34} = \rho_{43} = -\frac{\cos^2\theta - \sin^2\theta}{4} f_1(\xi), \\ \rho_{13} &= \rho_{24} = \rho_{31} = \rho_{42} = \frac{\cos^2\theta - \sin^2\theta}{4} f_1(\xi). \end{aligned} \quad (14)$$

By calculation we find that the negativity of $\rho_{\rm AB}^{\psi}(t)$ satisfies the equation

$$N(\rho_{\rm AB}^{\psi}(t)) = N(\rho_{\rm AB}^{\phi}(t)). \tag{15}$$

This equation reveals that the negativity is equivalent for two types of initial atomic state. It can be seen that the phenomenon of ESD can occur for both the Bell-like states. In the same way the entanglement of $|\psi^-\rangle = \big(|ge\rangle - |eg\rangle\big)/\sqrt{2}$ is preserved during the evolution of the system. Consequently, if atoms are prepared in a superposition of states $|\phi^-\rangle$ and $|\psi^-\rangle$, the state of the whole system does not evolve since the atomic subspace spanned by $|\phi^-\rangle$ and $|\psi^-\rangle$ coincides with the time-invariant subspace spanned by $|+-\rangle$ and $|-+\rangle$. The invariance of atomic entanglement can be applied in quantum computation and quantum information processing.

3. Conclusion

We have studied the entanglement dynamics of a system in which two atoms, driven by an external strong classical field, are trapped inside a single-mode cavity field and off-resonantly couple with the cavity mode via the two-photon process. Currently, there are different available routes to the implementation of our model. In the microwave regime of cavity quantum electrodynamics (QED), pairs of atoms excited to Rydberg levels cross a high-Q superconductive cavity with ignorable spontaneous emission during the interaction.[33] In the optical regime the application of cooling and trapping techniques in cavity QED[34,35] allows the deterministic loading of a single atom in a high-Q cavity, which can satisfy the requirements of atomic simultaneous injection, equal velocity, and equal coupling rate to the cavity mode. In this regime laser-assisted three-level atoms can behave as effective two-level atoms with negligible spontaneous emission.[36] Moreover, for an ultra-short laser pulse, due to the broad band of the excitation light, the two-photon process can be generated in all frequency components of the laser pulse as long as the sum of the two-photon energies equals the excited-state energy difference. Hence prospective implementation of our model is possibly based on the above systems.

Fully analytical solutions of the entanglement dynamics of two strongly driven atoms in two different initial states have been given. The results demonstrate that for two types of different initial Bell-like states, their entanglement evolutions are equivalent in our model. Therefore, entanglement sudden death can occur in both types of initial states. We also find an atomic decoherence-free subspace so that the initial atomic entanglement remains available for application. At the same time we discover an intriguing phenomenon but a rigorous interpretation in physics is currently still ambiguous. The reason for this is that 'atomic immanent entanglement conservation' may exist, which needs to be discussed in further research. Otherwise, it is worth saying that large detuning and the strongly driven regime can be used to preserve entanglement information in quantum computing and quantum information processing.

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