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Entanglement evolution of a two-qubit system with decay beyond the rotating-wave approximation*

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The entanglement property of two identical atoms, initially entangled in Bell states, coupled to a single-mode cavity is considered. Based on the reduced non-perturbative quantum master equation method, the entanglement evolution of the two atoms with decay is investigated beyond the conventional rotating-wave approximation. We show that the counter-rotating wave terms, usually neglected, have a great influence on the disentanglement behaviour of the system. The phenomena of entanglement sudden death and entanglement sudden birth will occur. In addition, we show that the entanglement can be strengthened by introducing the dipole–dipole interaction of the two atoms.

Keywords: quantum entanglement, beyond rotating-wave approximation

PACC: 0365, 0367, 4250

1. Introduction

Entanglement plays a central role in the field of quantum information science. It is generally considered a valuable resource for some non-classical tasks, such as quantum computation,[1−3] quantum teleportation,[4] superdense coding,[5] and quantum cryptography.[6,7] Therefore, applications of interest have triggered research on the dynamical behaviour of entanglement in order to achieve control over quantum disentanglement.[8−10]

Recently, the dynamics of entanglement in bipartite systems has attracted much attention[11−25] since the first work of Yu and Eberly,[26] where they found that entanglement may terminate abruptly in a finite time while coherence is lost asymptotically. Thereafter, this abrupt effect has been termed entanglement sudden death (ESD). Subsequently, 'entanglement sudden birth' (ESB)[27] is another highlight and further extension of the field, where they try to present an explanatory study of multipartite entanglement evolution.

In the conventional study of quantum optics, the rotating-wave approximation (RWA) is widely adopted, where counter-rotating terms of the light–matter interaction Hamiltonian, corresponding to the emission and absorption of virtual photons without energy conservation, is neglected. This is because the coupling ratio is of the order of 10^{-7}–10^{-6} in atom–field systems, and thus the RWA is justified. Recently, Meiser and Meystre[28] have investigated cavity systems with superstrong coupling, where the effect of the counter-rotating terms should be considered. This regime can be achieved in solid state systems.

Unlike the previous works, we study the disentanglement between a pair of qubits with decay, which interact with a cavity beyond the RWA. On considering real processing, atomic decay is unavoidable, which can be caused by two different physical mechanisms. The first is the transition without light radiation. In this process the energy is emitted by thermal energy or other forms. The second is the radiation transition. Photons will be emitted with the atomic transition. In our model the second is the main physical mechanism because the collision probability between the two atoms is very small. Thus a discussion on the effect of atomic decay caused by spontaneous emission on entanglement evolution is necessary.

This paper is organized as follows. In Section 2, a two-qubit model with decay is presented and the re-

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duced non-perturbative quantum master equation of atoms is derived. In Section 3, the entanglement evolution of two initially entangled atoms is investigated by using Wootters' concurrence.[29] In the last section, the conclusions are given.

2. Model

Consider two atoms A and B, which are interacting with a single-mode cavity resonantly. For simplicity, we assume that the two atoms have the same parameters. This can be described by the following Hamiltonian:

$$H = H_{\rm a} + H_{\rm f} + H_{\rm af}, \tag{1}$$

where

$$H_{\rm a} = \frac{\omega_0}{2}\sum_{i=1}^{2}\sigma_{z_i} + \Omega(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-), \tag{2}$$

$$H_{\rm f} = \omega a^+ a, \tag{3}$$

$$H_{\rm af} = \lambda\sigma(a^+ + a). \tag{4}$$

Here $\sigma = \sum_{i=1}^{2}(\sigma_i^+ + \sigma_i^-)$, $\sigma_{z_i} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$ and $\sigma_i^- = |g_i\rangle\langle e_i|$ are the atomic operators with $|e_i\rangle$ and $|g_i\rangle$ being the excited and ground states of the ith atom; ω_0 is the atomic transition frequency between the ground state and the excited state; λ is the coupling constant between atom and cavity; a and $a^\dagger$ denote the annihilation and creation operators of the cavity mode with frequency ω; Ω is dipole–dipole coupling strength between atoms.

Taking the atomic spontaneous emission into consideration, the reduced non-perturbative quantum master equation of atoms could be obtained by path integrals,[30]

$$\begin{aligned}\frac{\partial}{\partial t}\rho_{\rm a} = &-{\rm i}\mathcal{L}_{\rm a}\rho_{\rm a}\\ &-\int_0^t {\rm d}s\langle\mathcal{L}_{\rm af}\,{\rm e}^{-{\rm i}\mathcal{L}_0(t-s)}\mathcal{L}_{\rm af}\,{\rm e}^{-{\rm i}\mathcal{L}_0(s-t)}\rangle_{\rm f}\rho_{\rm a}\\ &+\frac{\gamma}{2}\sum_{i=1}^{2}(2\sigma_i^-\rho_{\rm a}\sigma_i^+ - \sigma_i^+\sigma_i^-\rho_{\rm a}\\ &-\rho_{\rm a}\sigma_i^+\sigma_i^-),\end{aligned} \tag{5}$$

where $\mathcal{L}_0$, $\mathcal{L}_{\rm a}$ and $\mathcal{L}_{\rm af}$ are Liouvillian operators defined as

$$\begin{aligned}\mathcal{L}_0\rho &= [H_{\rm a} + H_{\rm f}, \rho],\\ \mathcal{L}_{\rm a}\rho &= [H_{\rm a}, \rho],\\ \mathcal{L}_{\rm af}\rho &= [H_{\rm af}, \rho],\end{aligned} \tag{6}$$

and $< \cdots >_{\rm f}$ stands for partial trace of cavity mode; γ is the atomic decay constant; $\rho_{\rm a}$ is the reduced density matrix of two atoms.

Assuming that the cavity field is initially in vacuum state, in the standard product basis $\mathcal{B} = \{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, the non-perturbative reduced master equation of the atoms could be derived from Eq.(5),[31]

$$\begin{aligned}\frac{\partial}{\partial t}\rho_{\rm a} = &-{\rm i}\frac{\omega_0}{2}[\sigma_{z_1} + \sigma_{z_2}, \rho_{\rm a}]\\ &-{\rm i}\Omega[\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-, \rho_{\rm a}]\\ &-\lambda^2\sigma[F_1, \rho_{\rm a}] + \lambda^2[F_2, \rho_{\rm a}]\sigma\\ &+\frac{\gamma}{2}\sum_{i=1}^{2}(2\sigma_i^-\rho_{\rm a}\sigma_i^+ - \sigma_i^+\sigma_i^-\rho_{\rm a}\\ &-\rho_{\rm a}\sigma_i^+\sigma_i^-),\end{aligned} \tag{7}$$

where

$$\begin{aligned}F_1 = &f_1(|eg\rangle\langle ee| + |ge\rangle\langle ee|)\\ &+f_2(|ee\rangle\langle eg| + |ee\rangle\langle ge|)\\ &+k_1(|eg\rangle\langle gg| + |ge\rangle\langle gg|)\\ &+k_2(|gg\rangle\langle eg| + |gg\rangle\langle ge|),\\ F_2 = &f_1^*(|ee\rangle\langle eg| + |ee\rangle\langle ge|)\\ &+f_2^*(|eg\rangle\langle ee| + |ge\rangle\langle ee|)\\ &+k_1^*(|gg\rangle\langle eg| + |gg\rangle\langle ge|)\\ &+k_2^*(|eg\rangle\langle gg| + |ge\rangle\langle gg|),\end{aligned}$$

$$\begin{aligned}f_1 &= {\rm i}\frac{1-\exp({\rm i}\delta_1 t)}{{\rm i}\delta_1},\\ f_2 &= \frac{1-\exp(-{\rm i}\delta_2 t)}{{\rm i}\delta_2},\\ k_1 &= \frac{1-\exp(-{\rm i}\Delta_2 t)}{{\rm i}\Delta_2},\\ k_2 &= {\rm i}\frac{1-\exp({\rm i}\Delta_1 t)}{\Delta_1},\end{aligned}$$

and $\delta_1 = \omega_0 - \Omega - \omega$, $\delta_2 = \omega_0 - \Omega + \omega$, $\Delta_1 = \omega_0 + \Omega - \omega$, $\Delta_2 = \omega_0 + \Omega + \omega$, superscript $*$ means conjugate.

3. Disentanglement

3.1. Initial state and the entanglement measurement

In this section, we will use Wootters' concurrence to quantify the degree of entanglement.[29] For two qubits, the concurrence is calculated from the density matrix ρ for qubits A and B:

$$C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{8}$$

where λ_i are the eigenvalues of the matrix

$$\varrho = \rho_{AB}(\sigma_y^A \otimes \sigma_y^B)\rho_{AB}^*(\sigma_y^A \otimes \sigma_y^B) \tag{9}$$

arranged in decreasing order. Here ρ_{AB}^* denotes the complex conjugation of ρ_{AB}, and $\sigma_y^{A(B)}$ is the standard Pauli matrix acting in the space of qubit A (or B). The concurrence varies from $C(\rho) = 0$ for an unentangled state to $C(\rho) = 1$ for a maximally entangled state.

Here we restrict our analysis to the initial entangled states

$$|\phi\rangle = \alpha|01\rangle + \beta|10\rangle, |\psi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{10}$$

where α is real, $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\eta}$ and $\alpha^2 + |\beta|^2 = 1$. For these two entangled states, the initial atomic density matrix has an "X" structure[32] which is maintained during the evolution.[33] The reduced density matrix of the two atoms ρ_{a}, in the standard product basis $\mathcal{B} = \{|1\rangle = |ee\rangle, |2\rangle = |eg\rangle, |3\rangle = |ge\rangle, |4\rangle = |gg\rangle\}$, could be written as

$$\rho_{\mathrm{a}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{11}$$

The concurrence of ρ_{a} could be obtained

$$\begin{aligned} C_\phi(t) &= 2\max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}, \\ C_\psi(t) &= 2\max\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}\}. \end{aligned} \tag{12}$$

3.2. Numerical results and discussion

In order to study the effect of counter-rotating terms on the entanglement dynamics of the system, we investigate the entanglement evolution of two atoms by numerical calculation. For simplicity, the resonant case $\omega = \omega_0$ is considered. We choose $\gamma/\lambda = 4.35 \times 10^{-2}$.

Generally, the dipole–dipole interaction between atoms is much weaker than the interaction between atoms and cavity. Thus the case of $\Omega \ll \lambda$ is discussed at first.

1) We focus on the disentanglement of two qubits with the initial state of $|\phi\rangle$. The entanglement evolution of two decayed atoms is shown in Figs.1, 2 and 3 with different parameters. One can see that the change of C_ϕ against α^2 is symmetrical because of the symmetry of the initial state $|\phi\rangle$. When $\omega = \lambda$, Fig.1 shows that the concurrence C_ϕ changes with the initial value α^2 and the time λt. It can be seen that the concurrence decreases to zero in a finite time and the entanglement undergoes so-called ESD.

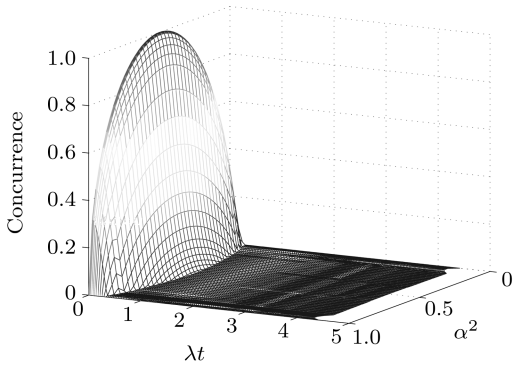


Fig.1. The concurrence C_ϕ as a function of α^2 and the time λt with $\omega = \lambda$, $\Omega = 0$.

There is a slight difference when $\omega = 3\lambda$, as shown in Fig.2. It is easy to note that there exist small fluctuations when α^2 is near 0 or 1. Particularly, when the two atoms are not entangled initially, namely $\alpha^2 = 0$ or $\alpha^2 = 1$, there is entanglement sudden birth.

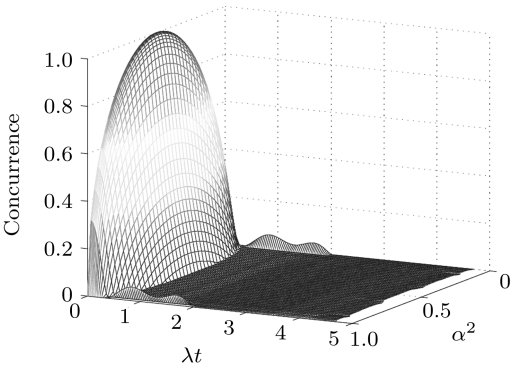


Fig.2. The concurrence C_ϕ as a function of α^2 and the time λt with $\omega = 3\lambda$, $\Omega = 0$.

As $\omega = 10\lambda$, Fig.3 reveals the time evolution of concurrence. It is shown that the entanglement decays to zero in a finite time, then revives with small amplitude and finally disappears permanently. Particularly, when the two atoms are not entangled initially, namely $\alpha^2 = 0$ or $\alpha^2 = 1$, there is also entanglement sudden birth and its amplitude is larger than that of the case $0 < \alpha^2 < 1$, then the entanglement decreases to zero eventually due to the atomic decay.

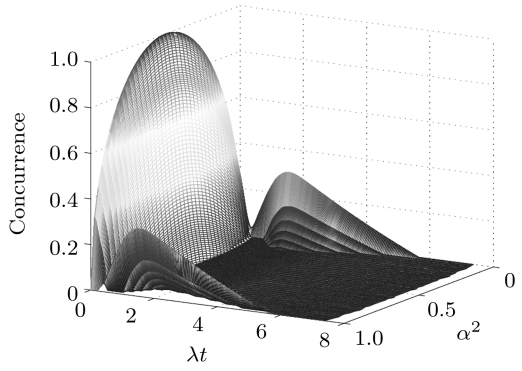


Fig.3. The concurrence C_ϕ as a function of α^2 and the time λt with $\omega = 10\lambda$, $\Omega = 0$.

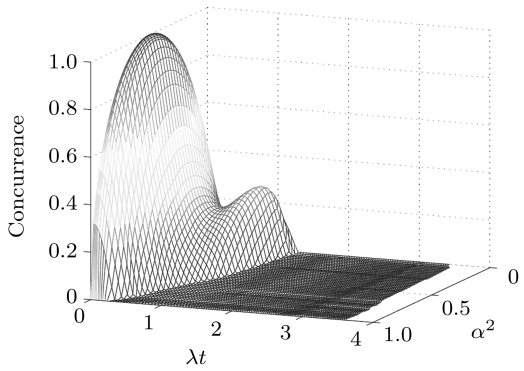


Fig.5. The concurrence C_ψ as a function of α^2 and the time λt for $\omega = 3\lambda$, $\Omega = 0$.

2) Alternatively, we focus on the disentanglement of two qubits with the initial state $|\psi\rangle$. The entanglement evolution is investigated in Figs.4, 5 and 6. The figures are not symmetrical to α^2 because the initial state of the two atoms is asymmetrical.

When $\omega = \lambda$, Fig.4 shows that the concurrence C_ψ decays exponentially to zero in almost all cases except where the value of α^2 is near 0. In the case of small α^2, there exists a small fluctuation before C_ψ vanishes permanently. Particularly, when $\alpha^2 = 0$, there exists small ESB and ESD. This effect results from the coaction of the rotating wave process and counter-rotating process on the whole system.

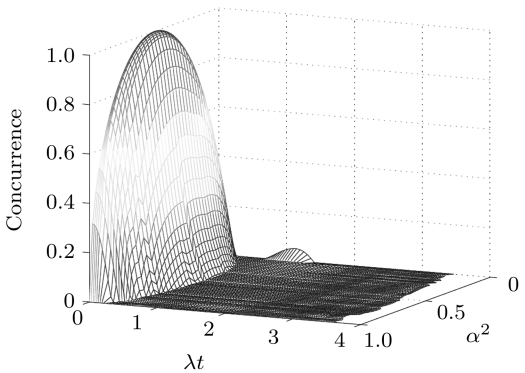


Fig.4. The concurrence C_ψ as a function of α^2 and the time λt with $\omega = \lambda$, $\Omega = 0$.

By contrast, as $\omega = 3\lambda$, Fig.5 reveals that the amplitude of the fluctuation is higher than that of $\omega = \lambda$ in the case of small α^2. It is also found that the range for the existence of the fluctuation is larger than that of the previous case. But there is no ESB in the case of $\alpha^2 = 0$.

When $\omega = 10\lambda$, the time evolution of the concurrence is plotted in Fig.6. The concurrence decreases monotonically and exponentially to zero when α^2 is near 1. The smaller the α^2 is, the more intense the fluctuation is. The concurrence disappears permanently at last because the atoms decay.

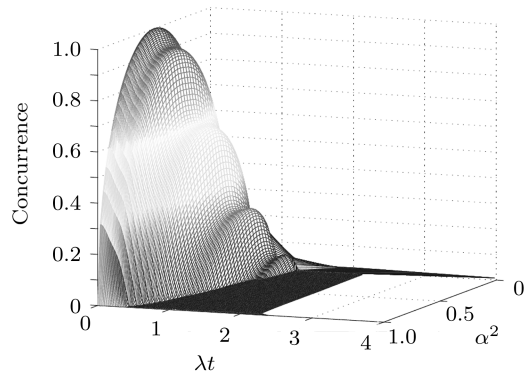


Fig.6. The concurrence C_ψ as a function of α^2 and the time λt for $\omega = 10\lambda$, $\Omega = 0$.

When the distance between the atoms is less than the wave length in the cavity, the dipole–dipole interaction of the atoms cannot be neglected. Assuming that $\Omega \ll \omega$, then the Eq.(7) can be simplified.

Considering that the dipole–dipole interaction of the atoms is weaker than the interaction between atoms and cavity, we choose $\omega = 50\ \lambda = 100\ \Omega$. The entanglement evolution of two decayed atoms with the initial state of $|\phi\rangle$ is plotted in Fig.7. It is shown that entanglement can be strengthened by introducing the dipole–dipole interaction. The time that concurrence decays to zero is prolonged, and the amplitude of ESB becomes high when the two atoms are not entangled initially.

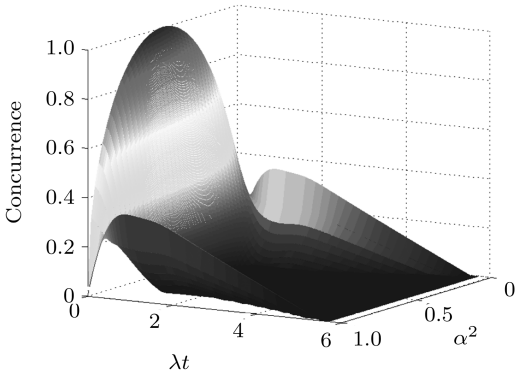


Fig.7. The concurrence C_ϕ as a function of α^2 and the time λt with $\omega = 50\lambda = 100\ \Omega$.

As for the other initial state $|\psi\rangle$, the entanglement evolution is similar to that of the case of no dipole–dipole interaction of the atoms. On the whole, dipole–dipole interaction can strengthen the entanglement.

4. Conclusions

In summary, we have investigated the disentanglement of two identical atoms with decay coupling to a one-mode cavity resonantly beyond the RWA. It is shown that entanglement evolution is dependent on the ratio of the atom–field coupling to the atomic transition frequency, and the phenomena of entanglement evolution are rich. The physical mechanism behind the phenomena is the process of emission and absorption of a virtual photon and the atomic decay. Also, the introduction of dipole–dipole interaction can strengthen the entanglement between atoms. The study of entanglement evolution beyond the RWA is a significant problem because of its importance to the field of strong coupling. It will help to deal with the practical case of the solid state system. The consideration of atomic decay is also of practical significance.

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