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# Zone inhomogeneity with the random asymmetric simple exclusion process in a one-lane system*

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In this paper we use theoretical analysis and extensive simulations to study zone inhomogeneity with the random asymmetric simple exclusion process (ASEP). In the inhomogeneous zone, the hopping probability is less than 1. Two typical lattice geometries are investigated here. In case A, the lattice includes two equal segments. The hopping probability in the left segment is equal to 1, and in the right segment it is equal to $p$, which is less than 1. In case B, there are three equal segments in the system; the hopping probabilities in the left and right segments are equal to 1, and in the middle segment it is equal to $p$, which is less than 1. Through theoretical analysis, we can discover the effect on these systems when $p$ is changed.

**Keywords:** zoned inhomogeneity, asymmetric simple exclusion process, one-lane system, computer simulation

**PACC:** 0250

## 1. Introduction

In physics and biology, asymmetric simple exclusion processes (ASEPs) are an important tool in modelling transport processes, like vehicular traffic and biological transport.[1,2] Using ASEPs, we can exactly analyse the nonequilibrium statistical process.[3−5] In these nonequilibrium systems, the tracks are lanes, protofilaments or actins filaments. When a local inhomogeneity (or defect) exists in the track, it will produce a great effect on the whole system. In the real world, it can lead to crowding phenomena in traffic systems, thereby inducing energy waste and environmental pollution In recent years, people have used cellular automaton (CA) traffic models to study traffic issues.[6−8] In the present paper, we use a new method (i.e. ASEP) to study traffic issues. It can conduce to a better understanding of traffic matters.

Recently, ASEPs with a single defective site in a periodic chain have been studied by Janowsky and Lebowitz.[9,10] Subsequently, the ASEP on a ring with many defective sites was considered by Tripathy and Barma.[11,12] The results show that phase segregation can exist in a wide range in the chain. In 1998, Kolomeisky studied the ASEP in an open chain,[13] followed by an extension[14] in which particles of arbitrary size (the number of sites which a particle occupies are more than 1) were studied. The phase diagrams shown in Refs.[13] and [14] are still qualitatively the same as those of the normal ASEP with shifts of phase boundaries. Recently, one has focused on the disordered ASEP.[15−25] In particular, Ebrahim Foulaadvand, Sanaz Chaaboki and Modjtaba Saalehi have studied the effect of whole sites that are due to the defects in the open ASEP.[25]

In the present paper, the effect of a zone inhomogeneity on the one-dimensional ASEP in random is studied. We define the zone inhomogeneity as a sequence of consecutive defect sites with a reduced hopping probability. There are two typical lattice geometries that are discussed here (see Figs.1(a) and 1(b)). In case A, the system includes two segments, i.e. the left segment and the right segment. The left segment is a normal TASEP, while the right segment is an inhomogeneous zone. In case B, the system is divided into three segments, i.e. the left segment, the middle segment and the right segment. Both the left and right segments are normal TASEPs and the middle segment is an inhomogeneous zone (see Fig.1(b)).

The rest of the present paper is organized as follows. In Section 2, the models of case A and case B are described. In Section 3, theoretical analysis of case A is carried out, giving rise to the phase diagram. The saturation points are defined. Case B is studied in

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Section 4. In Section 5, we present and discuss the results of the theoretical analysis and perform computer simulations. Finally, conclusions are given in Section 6.

## 2. The model

In case A, the one-dimensional lattice model includes $2N$ sites (see Fig.1(a)). Particles can be injected into the system at site 1 (i.e. left boundary) and ejected from the system at site $2N$ (i.e. right boundary). The following rules are applied to all sites in random: (i) When the site 1 is empty, a particle can be injected into the system with rate $\alpha$ ($0 < \alpha \leq 1$); (ii) A particle at site $2N$ can be ejected from the system with rate $\beta$ ($0 < \beta \leq 1$); (iii) If the site $i$+1 is not occupied, a particle at site $i$ ($1 \leq i \leq N$) can hop to the site $i$+1 with probability 1, and if the site $j$+1 is not occupied, a particle at site $j$ ($N$+1 $\leq j \leq 2N$) can hop to site $j$+1 with probability $p$. In other words, in the left segment, the hopping probability on the bulk is equal to 1, while it is equal to $p$ in the right segment (see Fig.1(a)).

Site $N$ and site $N$+1 link both segments. The left segment is a homogeneous TASEP with entrance rate at site 1 and exit rate $\beta_{\text{eff}}$ at site $N$ (see Fig.2(a)). Similarly, the right segment is, in fact, also a homogeneous TASEP with entrance rate $\alpha_{\text{eff}}$ site $N$+1 and exit rate $\beta$ at site $2N$ (see Fig.2(a)).

In case B, there are $3N$ sites in the system. In the left boundary (i.e. $1 \leq i \leq N$), particles can enter the system at site 1 with rate, and in the right boundary (i.e. $2N + 1 \leq k \leq 3N$), particles can exit from the system at site $3N$ with rate $\beta$. The following rules are applied to all sites in random: (i) A particle can enter into the system with rate $\alpha$ ($0 < \alpha \leq 1$) if the site 1 is empty; (ii) A particle at site $3N$ can exit from the system with rate $\beta$ ($0 < \beta \leq 1$); (iii) If site $i$+1 is not occupied, a particle at site $i$ ($1 \leq i \leq N$) can hop to site $i$+1 with probability 1; if site $j$+1 is not occupied, a particle at site $j$ ($N + 1 \leq j \leq 2N$) can hop to site $j$+1 with probability $p$, and if site $k$+1 is occupied, a particle at site $k$ ($2N + 1 \leq j \leq 3N$) can hop to site $k$+1 with probability 1. That is, in the left and right segments, the hopping probability on the bulk is equal to 1; while it is equal to $p$ in the middle segment (see Fig.1(b)). Site $N$ ($2N$) and site $N$+1 ($2N$+1) link the three segments. The left and the right segments are homogeneous TASEPs with entrance rate $\alpha$ ($\alpha_{\text{eff}}$) at site 1 ($2N$+1) and exit rate $\beta_{\text{eff1}}$ ($\beta$) at site $N$ ($3N$), respectively (see Fig.2(b)). Similarly, the middle segment is, in fact, also a homogeneous TASEP with entrance rate $\alpha_{\text{eff1}}$ at site $N$+1 and exit rate $\beta_{\text{eff2}}$ at site $2N$ (see Fig.2(b)).

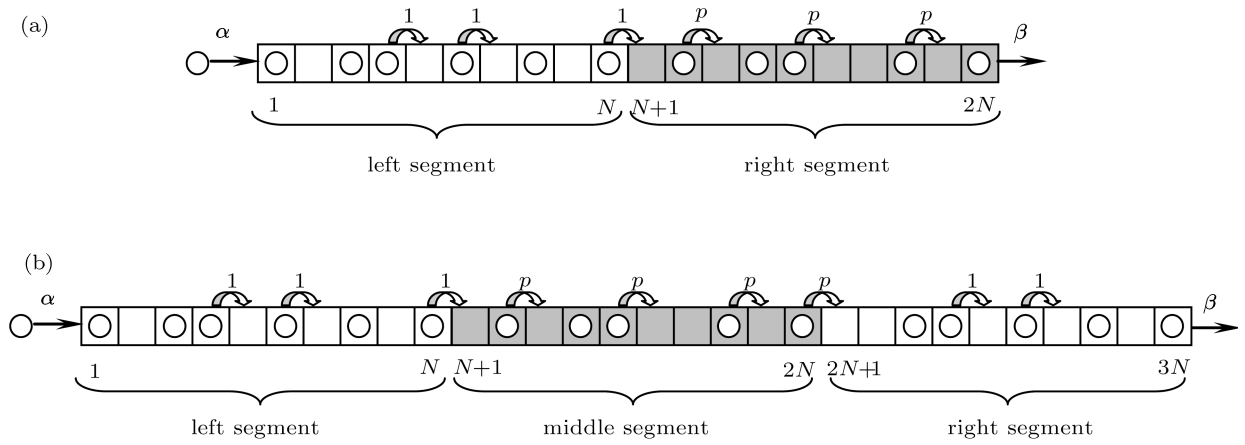



**Fig.1.** Inhomogeneous TASEP on a one-dimensional lattice. (a) Case A: two segments in the system (i.e. left segment and right segment); (b) case B: three segments in the system (i.e. left segment, middle segment and right segment).

We consider only $0 < p < 1$ in the system. When $p = 0$, the current through the system is equal to 0, for particles cannot move into the right segment. When $p = 1$, the whole system becomes a homogeneous one. For

$p > 1$, those particles dominate the system with the hopping probability equal to 1. In other words, when $p > 1$ the value of $p$ will not determine the system phase (i.e. low, high or maximum current).

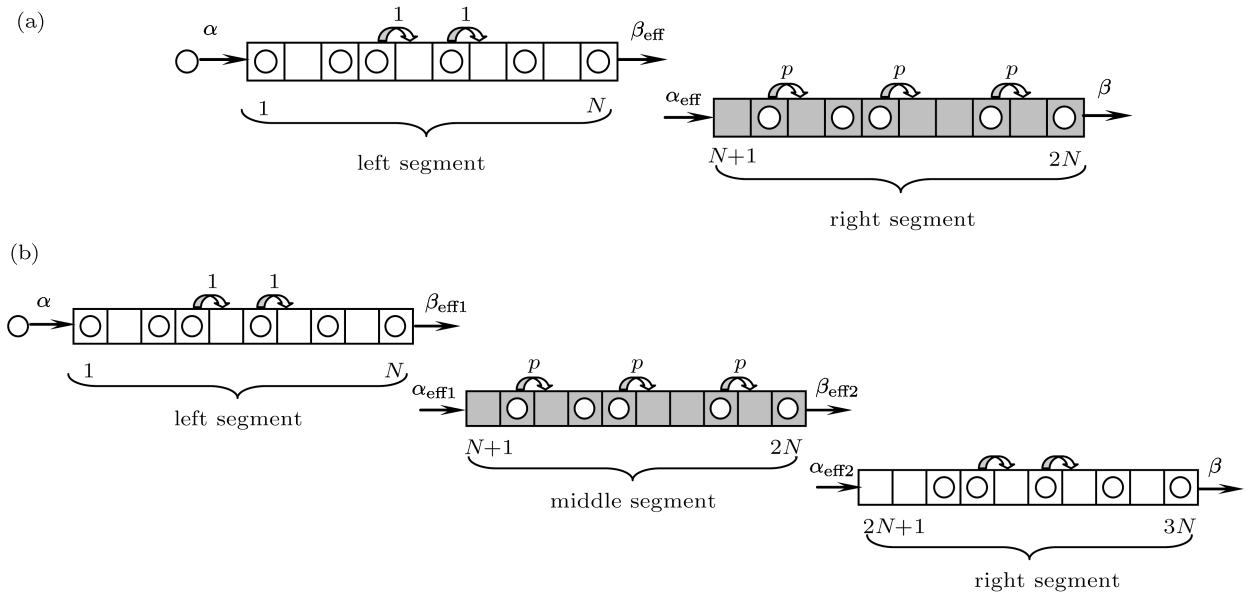



**Fig.2.** Left segment and right segment in case A and the left segment, middle segment and right segment in case B in detail.

## 3. Theoretical analysis of case A

The result of a random TASEP for general $p$ ($0 < p \leq 1$) has been presented in Ref.[25]. There are three phases (i.e. low density, high density and maximal current) in that system. When $\alpha < \beta$ and $\alpha < p/2$, the system is in the low-density (LD) phase with the current $J = \alpha(1 - \alpha/p)$. When $\beta < \alpha$ and $\beta < p/2$, a high-density (HD) phase is obtained with the current $J = \beta(1 - \beta/p)$. The conditions $\alpha > p/2$ and $\beta > p/2$ correspond to the maximal current (MC) phase with $J = p/4$. Obviously, when $p = 1$, the currents in the LD, HD and MC phases reduce into $J = \alpha(1 - \alpha)$, $J = \beta(1 - \beta)$ and $J = 1/4$, respectively.

When the system is in a steady state, the stationary current in the left segment ($J_{\rm L}$) should be equal to that in the right segment ($J_{\rm R}$), (i.e. $J_{\rm L} = J_{\rm R}$); it is this solid fact that is the basis of our theoretical analysis. This is called current conservation.

In the system, the MC phase cannot exist in the left segment for case A shown in Fig.1(a). If the MC phase exists in the left segment, the maximal current should be equal to $J_{\rm L}^{\max} = 1/4$. However, in the right segment, the maximal current should be equal to $J_{\rm R}^{\max} = p/4 < 1/4$ when $0 < p < 1$.[25] Thus, the maximal current in the left segment cannot exist, according to the rule of current conservation. Therefore, there are six possible stationary phases in the system: LD/LD, LD/HD, LD/MC, HD/LD, HD/HD and HD/MC phases. The LD/LD phase means that the left segment is in the LD phase, while the right segment is in the LD phase, too. Now we calculate these phases separately in the following.

(1) The LD/LD phase. The conditions for this phase are as follows:

$$J_{\rm L} = \alpha(1 - \alpha) = J_{\rm R} = \alpha_{\rm eff}\left(1 - \frac{\alpha_{\rm eff}}{p}\right). \quad (1)$$

Thus,

$$\alpha_{\rm eff} = \frac{p \pm \sqrt{p^2 - 4p(\alpha - \alpha^2)}}{2}. \quad (2)$$

Since the right segment of the system is in the LD phase, $\alpha_{\rm eff} < p/2$. However, as

$$\frac{p + \sqrt{p^2 - 4p(\alpha - \alpha^2)}}{2} > \frac{p}{2},$$

we can obtain

$$\alpha_{\rm eff} = \frac{p - \sqrt{p^2 - 4p(\alpha - \alpha^2)}}{2}. \quad (3)$$

Also, since $\alpha_{\rm eff} < \beta$, one can have

$$\beta > \frac{p - \sqrt{p^2 - 4p(\alpha - \alpha^2)}}{2}. \quad (4)$$

Inequality (4) is meaningful only when the term in the square root is a positive value, which implies

$$p^2 - 2p\left(\alpha - \alpha^2\right) \geq 0. \tag{5}$$

Accordingly, we have

$$\alpha \leq \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha \geq \frac{1+\sqrt{1-p}}{2}. \tag{6}$$

As $0 < p < 1$, $\alpha \geq (1+\sqrt{1-p})/2$ which violates the condition $\alpha < p/2$. Hence, $\alpha \geq (1+\sqrt{1-p})/2$ should be discarded. Moreover, the current in the left segment is less than the maximal current, that is, the following inequality should be met:

$$J_{\mathrm{L}} = \alpha\left(1-\alpha\right) < \frac{p}{4} = J_{\max}. \tag{7}$$

That means

$$\alpha < \frac{1-\sqrt{1-p}}{2}. \tag{8}$$

Thus, the conditions for the system to be in the LD/LD phase are:

$$\alpha < \frac{1-\sqrt{1-p}}{2}$$

and

$$\beta > \frac{p-\sqrt{p^2-4p\left(\alpha-\alpha^2\right)}}{2}. \tag{9}$$

(2) The LD/HD phase. This phase corresponds to the following conditions:

$$J_{\mathrm{L}} = \alpha\left(1-\alpha\right) = J_{\mathrm{R}} = \beta\left(1-\frac{\beta}{p}\right). \tag{10}$$

This is equivalent to

$$\beta = \frac{p \pm \sqrt{p^2-4p\left(\alpha-\alpha^2\right)}}{2}. \tag{11}$$

Since the right segment of the system is in the HD phase, $\beta < p/2$. However, as

$$\frac{p+\sqrt{p^2-4p\left(\alpha-\alpha^2\right)}}{2} > \frac{p}{2},$$

one can obtain

$$\beta = \frac{p-\sqrt{p^2-4p\left(\alpha-\alpha^2\right)}}{2}. \tag{12}$$

Thus, the following conditions can guarantee the LD/HD phase in the system

$$\alpha < \frac{1-\sqrt{1-p}}{2}$$

and

$$\beta = \frac{p-\sqrt{p^2-4p\left(\alpha-\alpha^2\right)}}{2}. \tag{13}$$

(3) The LD/MC phase. The following conditions should be satisfied:

$$J_{\mathrm{R}} = \alpha\left(1-\alpha\right) = J_{\mathrm{R}} = \frac{p}{4}. \tag{14}$$

That means

$$\alpha = \frac{1\pm\sqrt{1-p}}{2}. \tag{15}$$

As $0 < p < 1$,

$$\alpha = \frac{1+\sqrt{1-p}}{2} > \frac{p}{2},$$

which violates the condition $\alpha < p/2$, when the left segment of the system is in the LD phase. Hence, $\alpha = (1+\sqrt{1-p})/2$ should be discarded. Also, when the right segment of the system is in the MC phase, the condition $\beta > p/2$ should be satisfied. Thus, the LD/MC phase exists in the system when

$$\alpha = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta > \frac{p}{2}. \tag{16}$$

(4) The HD/MC phase. The conditions for this case are given by

$$J_{\mathrm{L}} = \beta_{\mathrm{eff}}\left(1-\beta_{\mathrm{eff}}\right) = J_{\mathrm{R}} = \frac{p}{4}. \tag{17}$$

That means

$$\beta_{\mathrm{eff}} = \frac{1\pm\sqrt{1-p}}{2}. \tag{18}$$

As $0 < p < 1$, $\beta_{\mathrm{eff}} = (1+\sqrt{1-p})/2 > \frac{p}{2}$ which violates the condition $\beta_{\mathrm{eff}} < \frac{p}{2}$ when the left segment of the system is in the HD phase. Hence, $\beta_{\mathrm{eff}} = (1+\sqrt{1-p})/2$ should be discarded. Thus, the system is in the HD/MC phase when

$$\alpha > \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta > \frac{p}{2}. \tag{19}$$

(5) The HD/HD phase. The corresponding conditions for this phase are

$$J_{\mathrm{L}} = \beta_{\mathrm{eff}}\left(1-\beta_{\mathrm{eff}}\right) = J_{\mathrm{R}} = \beta\left(1-\beta/p\right), \tag{20}$$

that is

$$\beta_{\mathrm{eff}} = \frac{p\pm\sqrt{p^2-4p\left(\beta-\beta^2\right)}}{2}. \tag{21}$$

Since the left segment of the system is in the HD phase, $\beta_{\mathrm{eff}} < 1/2$. However, as

$$\frac{p+\sqrt{p^2-4p\left(\beta-\beta^2\right)}}{2} > \frac{1}{2},$$

we can obtain

$$\beta_{\mathrm{eff}} = \frac{p-\sqrt{p^2-4p\left(\beta-\beta^2\right)}}{2}. \tag{22}$$

The left segment of the system is in the HD phase (i.e. $\alpha > \beta_{\rm eff}$). The solutions for the HD/HD phase can be obtained to be

$$\alpha > \frac{p - \sqrt{p^2 - 4p\,(\beta - \beta^2)}}{2} \quad \text{and} \quad \beta < \frac{p}{2}. \tag{23}$$

(6) The HD/LD phase. The conditions of existence for this phase can be written as

$$J_{\rm L} = \beta_{\rm eff}\,(1 - \beta_{\rm eff}) = J_{\rm R} = \alpha_{\rm eff}\left(1 - \frac{\alpha_{\rm eff}}{p}\right). \tag{24}$$

We cannot use the parameters $p$, $\alpha$ and $\beta$ to represent $\alpha_{\rm eff}$ and $\beta_{\rm eff}$. Thus, the HD/LD phase cannot exist in the system.

All possible stationary phases in the system can be illustrated in Fig.3(a), which includes the LD/LD, LD/HD, LD/MC, HD/MC and HD/HD phases.

The first-order transition line between the LD/LD and HD/HD phases is the LD/HD phase, while expression $\alpha = (1 - \sqrt{1-p})/2$ and inequality $\beta > p/2$ specify the straight line (i.e. LD/MC phase). The system is switched from LD/LD phase to HD/MC phase discontinuously in the left segment, but continuously in the right segment. Meanwhile, the transition from HD/MC to HD/HD phase is continuous.

The currents of LD/LD, LD/HD and HD/HD phases are only determined by the values of $\alpha$ and $\beta$, if the value of $p$ is fixed. However, when the system is in the LD/MC and the HD/MC phases separately, the currents are only related to $p$ (i.e. $J = p/4$), that is, these two phases are independent of boundary conditions.

Now, we introduce a saturation point pair $(\alpha^*, \beta^*)$ of the current, which can conduce to better understanding of the phase diagram. This point pair can be written as

$$\alpha^* = \frac{1 - \sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{p}{2}. \tag{25}$$

In Fig.3(b), we can find that with the increase of $p$, the values of $\alpha^*$ and $\beta^*$ increase, and so the regions of LD/MC and HD/MC phases decrease and the currents of both phases increase.

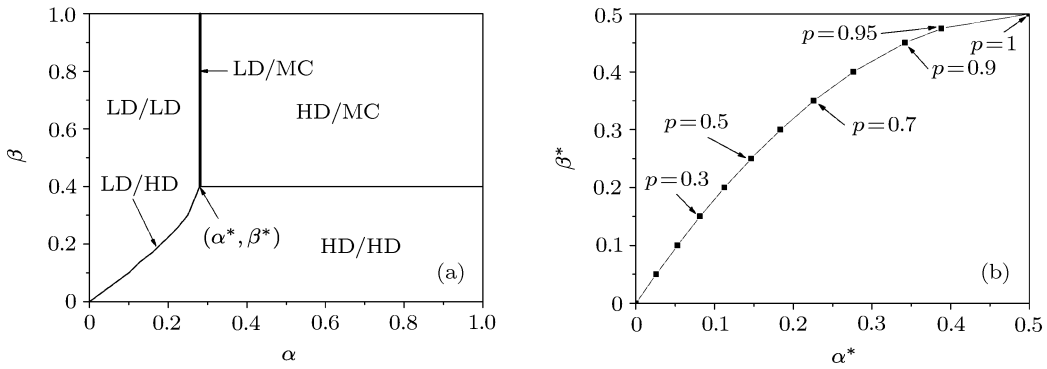



**Fig.3.** (a) Diagram of possible stationary-state phases in the system with $p = 0.8$; and (b) saturation point pair $(\alpha^*, \beta^*)$ with different values of $p$.

## 4. Theoretical analysis of case B

In this section, the phase diagram, current and density profiles will be discussed in case B (see Fig.2(b)). As above, the rule of current conservation is still used in a stationary state, i.e. $J_{\rm L} = J_{\rm M} = J_{\rm R}$ ($J_{\rm M}$ is the current of the middle segment in the system). We can calculate the phase diagram in case B. The results are as follows.

(1) The LD/LD/LD phase.

When the system is in the LD/LD/LD phase, the conditions of $\alpha$ and $\beta$ can be obtained, respectively, to be

$$\alpha < \frac{1 - \sqrt{1-p}}{2} \quad \text{and} \quad \alpha < \beta. \tag{26}$$

(2) The LD/LD/HD phase.

When the system is in the LD/LD/HD phase, we can calculate the conditions of $\alpha$ and $\beta$, respectively, to be

$$\alpha < \frac{1 - \sqrt{1-p}}{2} \quad \text{and} \quad \alpha = \beta. \tag{27}$$

(3) The LD/MC/LD phase.

Thus, the model is in the LD/MC/LD phase when

$$\alpha = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha < \beta. \tag{28}$$

(4) The LD/MC/HD phase.

Hence, the LD/MC/HD phase can exist in the model when

$$\alpha = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha = \beta. \tag{29}$$

(5) The HD/HD/HD phase.

We can obtain the corresponding conclusions about $\alpha$ and $\beta$ as follows:

$$\beta < \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta < \alpha. \tag{30}$$

(6) The HD/MC/LD phase.

According to the phase existence requirement, we can obtain the final conditions of $\alpha$ and $\beta$ to be

$$\beta > \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha > \frac{1-\sqrt{1-p}}{2}. \tag{31}$$

(7) The HD/MC/HD phase.

We can obtain the conditions of $\alpha$ and $\beta$ such that this phase exists

$$\beta = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha > \frac{1-\sqrt{1-p}}{2}. \tag{32}$$

Therefore, there can exist seven phases in case B; the phase diagram results are shown in Fig.4(a).

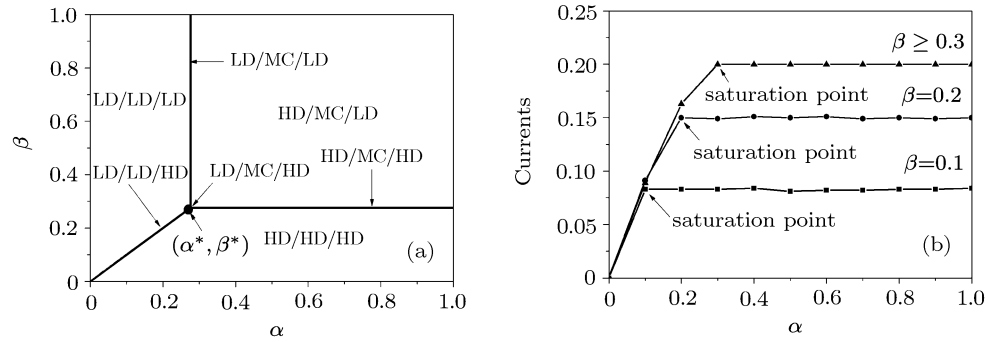



**Fig.4.** (a) Phase diagram of inhomogeneous asymmetric exclusion process with hopping probability $p$ in the middle segment when $p = 0.8$; (b) currents with fixed hopping probabilities $p = 0.8$ and varying exit rate $\beta$.

By comparison, there are some differences in the following points between case A and case B, i.e. (1) saturation point $(\alpha^*, \beta^*)$, in case A, $\alpha^* = \dfrac{1-\sqrt{1-p}}{2}$ and $\beta^* = p/2$ (see Fig.3(a)), while in case B,

$$\alpha^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{1-\sqrt{1-p}}{2}$$

(see Fig.4(a)). The reason is that in case B the left segment and the right segment have the same dynamic behaviours; (2) the numbers of stationary phases are different: there are five phases in case A (see Fig.3(a)) and seven phases in case B (see Fig.4(a)). Two new phases appear in case B (i.e. LD/MC/HD and HD/MC/HD phases); the conditions of these phases can exist in case B as follows: the LD/MC/HD phase corresponds to a point which is described by

$$\alpha^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{1-\sqrt{1-p}}{2}$$

and the HD/MC/HD phase is described by

$$\beta^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha^* > \frac{1-\sqrt{1-p}}{2}.$$

Except for the differences, there are some qualitative similarities between the two cases, i.e. (1) if the right segment in case B is ignored, case B changes to case A; (2) the LD/MC (LD/MC/LD) phase always exists between the LD/LD (LD/LD/LD) phase and the HD/MC (HD/MC/LD) phase.

## 5. Results and analysis

The theoretical and simulation results will be discussed in this section. In case A, the system includes 2000 sites (i.e. $2N$=2000) and in case B, there are 3000 sites in the system (i.e. $3N$=3000). The stationary current and the density profiles can be obtained

by the number of running time steps in each experiment $1.1\times10^9$. The first $10^8$ time steps are discarded as transients.

In Fig.5, we can see the theoretical and simulation results for the currents. They are in good agreement. We arbitrarily assume a hopping probability $p$; let it equal 0.3, 0.6 and 0.9, separately. In Fig.5(a), one can see that the current increases when the hopping probability increases, too. For example, the saturated current $J_{\rm sat} = 0.075$ for $p = 0.3$, while it increases to 0.15 for $p = 0.6$ and 0.225 for $p = 0.9$ i.e. $J_{\rm sat} = 0.15$ for $p = 0.6$ and $J_{\rm sat} = 0.225$ for $p = 0.9$. We show how we can obtain these theoretical currents. When $p = 0.3$, the saturation point, according to expression (25), corresponds to $\alpha^* = 0.082$, $\beta^* = 0.15$. Thus, the system is in the HD/MC phase when $\alpha^* > 0.082$ and $\beta^* > 0.15$. In the same way, we can obtain $\alpha^* = 0.184$ and $\beta^* = 0.3$ for $p = 0.6$ and $\alpha^* = 0.342$ and $\beta^* = 0.45$ for $p = 0.9$. When $\beta = 1.0 > \beta^*$ there are only three phases in the system, i.e. LD/LD, LD/MC and HD/MC phases (see Fig.3(a)) and when $\beta = 0.2$, the HD/HD phase can exist in the system (see Fig.5(b)). The system is in the HD/HD phase, when $p = 0.6$ and $p = 0.9$. From expression (20), we can obtain the corresponding saturation currents, which are $J_{\rm sat} = 0.133$ for $p = 0.6$ and $J_{\rm sat} = 0.156$ for $p = 0.9$.

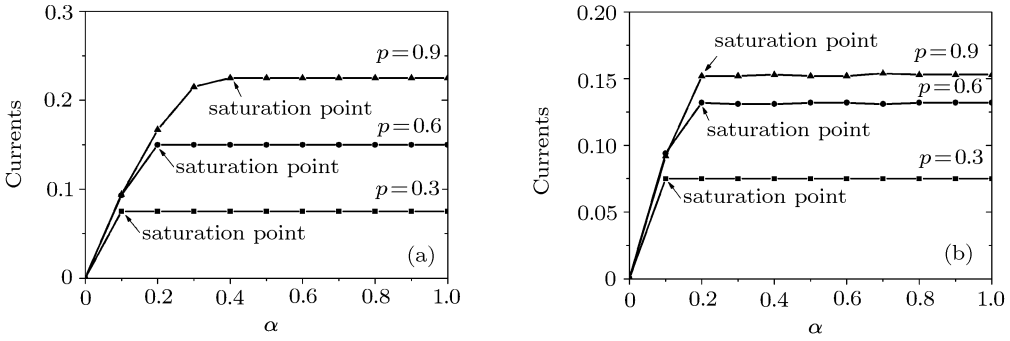



**Fig.5.** Currents with different hopping probability $p$=0.3, 0.6 and 0.9. The lines describe theoretical calculations, while symbols correspond to computer simulations. The values of exit rate $\beta = 1.0$ (a) and 0.3 (b).

In Fig.4(b), currents are calculated and plotted when $p = 0.8$ with three different values of $\beta$, and when $\alpha$ ranges from 0 to 1. When $\beta < \beta^* \approx 0.276$, with the increase of $\alpha$, the LD/LD/LD phase changes into the HD/HD/HD phase in the system (see Fig.4(a)). In the LD/LD/LD phase, the current increases as $\alpha$ increases, while the current becomes constant (i.e. $J = \beta\,(1-\beta)$) in the HD/HD/HD phase, actually only dependent on $\beta$. Similarly, when $\beta > \beta^*$, the system transfers into the HD/MC/LD phase from the LD/LD/LD phase as the value of $\alpha$ increases (see Fig.4(a)). In the HD/MC/LD phase, the current is determined by $p$ ($J = p/4$), independent of $\alpha$ and $\beta$. Using different values of $\alpha$ and $\beta$, we can obtain the hopping probability $p$ from the density profile by computer simulation (see Fig.6). In Fig.6(a), the system is in the HD/MC phase. Its explanation is as follows. When $p$ increases from 0.1 to 0.5, $\alpha^*(\beta^*)$ is still less than 0.15 (0.7), which is the entrance (exit) rate. The bulk density of the left segment decreases with the increase of $p$. However, when $p \geq 0.6$, the system is switched into the LD/LD phase (see Fig.6(b)). The bulk density of the right segment decreases with the increase of $p$ (i.e. $p$ increases from 0.6 to 1.0). In Fig.6(a) the bulk density of the right segment remains unchanged and in Fig.6(b) it is constant in the left segment. When $\alpha = 0.8$ and $\beta = 0.3$, a picture similar to that in Fig.6(a) is shown in Fig.6(c). The density profile of the HD/HD phase is indicated in Fig.6(d). With the increase of $p$, density profiles decrease in the left segment and they increase in the right segment.

In case B, the density profiles are shown in Fig.7. In the HD/MC/LD phase, when $p$ increases, both left and right segments will be affected (see Fig.7(a)). With the increase of $p$, e.g. $p \geq 0.6$, the system transfers into the LD/LD/LD phase and the density profile of the middle segment is affected (see Fig.7(b)). A similar picture is also shown in Fig.7(c). However, in Fig.7(d), the phase transition is complex with the value of $p$ varying. The system first transfers from the HD/MC/LD phase into the HD/MC/HD phase for $p = 0.8$, and then changes

into the HD/HD/HD phase.

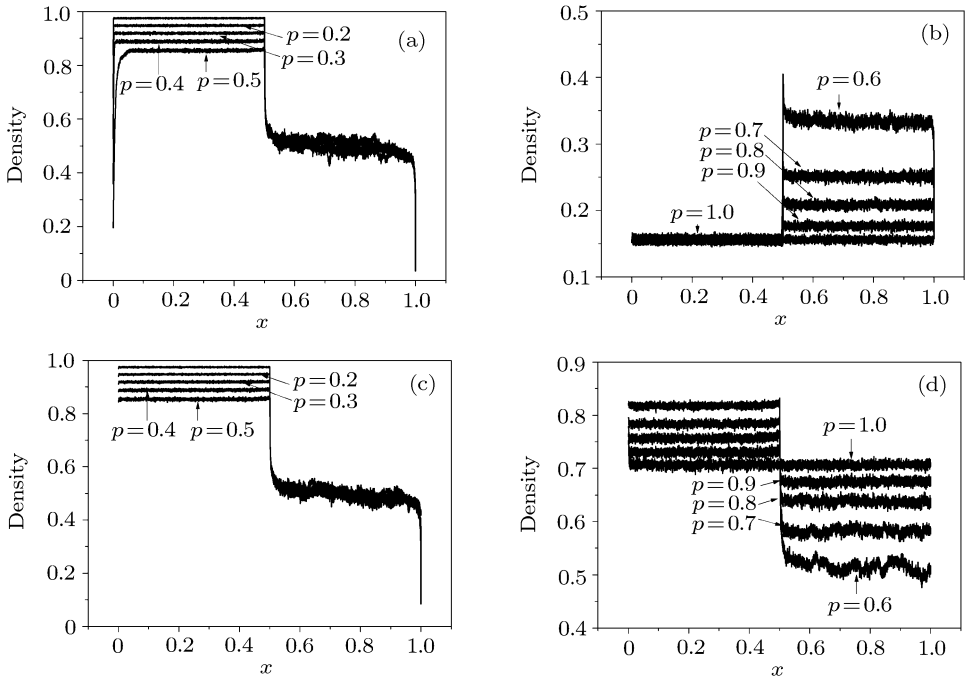



**Fig.6.** Density profiles from simulations in the stationary state with different values of $p$. $x = i/N$, is an index of sites and $N$ is the system length. (a) and (b) $\alpha = 0.15$ and $\beta = 0.7$, (c) and (d) $\alpha = 0.8$ and $\beta = 0.3$.

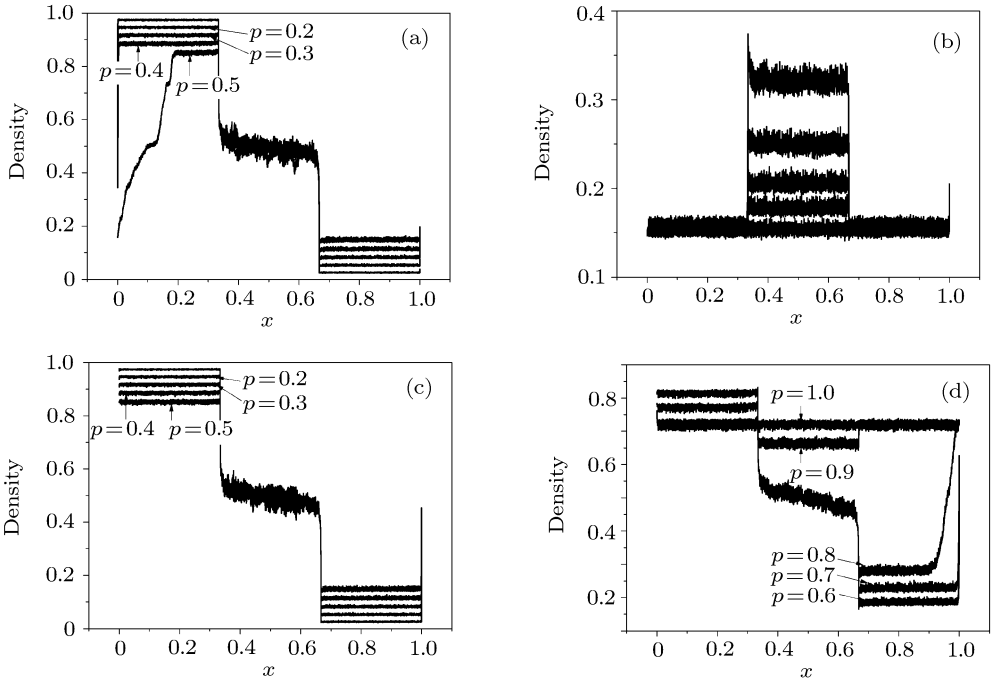



**Fig.7.** Density profile from simulations in the stationary state with different $p$. $x = i/N$, is an index of sites and $N$ is the system length. Panels (a) and (b) are for $\alpha = 0.15$ and $\beta = 0.7$, and panels (c) and (d) for $\alpha = 0.8$ and $\beta = 0.3$.

# 6. Summary

A one-dimensional TASEP with a zone inhomogeneity in random is investigated by theory and extensive computer simulations. Two typical lattice geometries are discussed. In case A, there are two segments in the system. The left segment is a normal TASEP (i.e. $p$=1), while the right segment is a TASEP with a zone inhomogeneity. In case B, both the left and the right segments are normal TASEP, while the middle segment is a TASEP with a zone inhomogeneity.

In the two cases, the phase diagrams are qualitatively the same in the shift of the phase boundary. However, there are some differences between the two cases, which include (1) saturation point ($\alpha^*$, $\beta^*$): in case A,

$$\alpha^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{p}{2}$$

(see Fig.3(a)); in case B,

$$\alpha^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{1-\sqrt{1-p}}{2}$$

(see Fig.5(a)). The reason for this is that in case B the left segment and the right segment have the same dynamic behaviours; (2) the numbers of stationary phases, which are different from each other: there are five phases in case A (see Fig.3(a)) and seven phases in case B (see Fig.5(a)). Two new phases appear in case B (i.e. LD/MC/HD and HD/MC/HD phases); the conditions of these phases which can exist in case B are as follows: the LD/MC/HD phase corresponds to a point which is described by

$$\alpha^* = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \beta^* = \frac{1-\sqrt{1-p}}{2},$$

while the HD/MC/HD phase is described by

$$\beta = \frac{1-\sqrt{1-p}}{2} \quad \text{and} \quad \alpha = \frac{1-\sqrt{1-p}}{2}.$$

The MC region in case B is larger than that in case A, which indicates that the right segment in case B can enhance the current in some situations. Density profiles and currents are simulated, and are in good agreement with theoretical calculations. It is interesting to investigate a zone inhomogeneity in the synchronous ASEP. We can extend the ASEP to the study of particles of arbitrary size.

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