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Tunneling conductance in quantum wire/insulator/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor junctions

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Using the extended Blonder–Tinkham–Klapwijk (BTK) theory, this paper calculates the tunnelling conductance in quantum wire/insulator/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor (q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$) junctions. That is different from the case in d- and p-wave superconductor junctions. When the angle α between a-axis of the $d_{x^2-y^2}$ wave superconductor and the interface normal is $\pi/4$, there follows a rather distinctive tunnelling conductance. The zero-bias conductance peak (ZBCP) may or may not appear in the tunnelling conductance. Both the interface potential z and the quasi-particle lifetime factor Γ are smaller, there is no ZBCP. Otherwise, the ZBCP will appear. The position of bias conductance peak (BCP) depends strongly on the amplitude ratio of two components for $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave. The low and narrow ZBCP may coexist with the BCP in the tunnelling conductance. Using those features in the tunnelling conductance of q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junctions, it can distinguish $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor from d- and p-wave one.

Keywords: quantum wire, $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor, tunneling conductance, quasi-particle lifetime

PACC: 7450, 7210

1. Introduction

The unconventional high-T_c superconductors[1] have a complicated lattice structure with extended and/or mixed symmetry for the order parameter.[2,3] The studies of the tunnelling conductance in normal metal/insulator/unconventional superconductor junctions have attracted considerable interest[4−8] because the tunneling conductance depends heavily on the symmetry of the pair potential in the superconductor. For many of high-T_c superconductor materials, the order parameter exhibits anisotropic behavior. The mixed symmetry states that have been suggested for d-wave superconductors are $d_{x^2-y^2}+\mathrm{i}d_{xy}$ and $d_{x^2-y^2}+\mathrm{i}\,s$.[9,10] In the case of the hole-doped cuprate $YBa_2Cu_3O_{7-\delta}$, recent reports have shown evidence of a transition from a pure $d_{x^2-y^2}$ pairing symmetry in under-doped compositions to a $d+\mathrm{i}d_{xy}$ pairing symmetry in over-doped compositions.[11] In d-wave superconductor, the change in sign of pair potential induces remarkable effects, i.e., there exists a zero-bias conductance peak (ZBCP) due to the formation of zero-energy states.[12] In $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor, the existence of d_{xy} wave component in the pair potential leads to a splitting of the ZBCP in zero external field.[13] The studies for quantum point contacts (QPC) in normal metal/insulator/d-wave superconductor junctions show that the ZBCP is removed by the quasi-particle diffractions.[14,15] How will the tunnelling conductance appear for QPC in normal metal/insulator/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor (N/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$) junctions?

The injection property of quasi-particles is substantially different when the tunnelling spectroscopy is performed by using QPC. On the one hand, the transverse momentum is quantized in QPC, and therefore, the propagation direction of the quasi-particles can be changed by varying the width of the point contact. The controllability may allow us to identify the orientation of the superconducting gap from the width dependence of the conductance. Furthermore, the transverse momentum is not conserved when quasi-particles are emitted from a narrow opening because of the wave diffraction. The Andreev reflection[16] from the superconductor hence represents a response averaged over the distribution of propagation angles.[14] In a quan-

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tum wire, the wave function is a superposition of two waves with injected angles $\pm\theta$. If these two waves feel different signs of the pair potentials in the superconductor, the two Andreev reflected holes with injected angles θ and $-\theta$ cannot make a standing wave in the quantum wire since the phases of the two Andreev reflected holes are not identical. As a consequence, the Andreev reflection is prohibited. If $\Delta_{\pm}(\theta) = \Delta_{\pm}(-\theta)$ is satisfied, the two Andreev reflected holes with injected angles θ and $-\theta$ can make a standing wave in the quantum wire. Consequently, the ZBCP shows up. If the two waves feel pair potentials in the superconductor belonged to intermediate situation, for instance in p-wave (e^{θ} type) superconductor, the ZBCP may or may not appear in the tunneling conductance as shown in Ref.[17]. Is there the ZBCP for QPC in N/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junctions?

We pay attention to such a phenomenon that certain factors can bring about the shortening of the quasi-particle lifetime. For example, the degradation of superconducting properties on the surface layers of high-T_{c} superconductors caused by oxygen out-diffusion occurs at room temperature[18] and randomly distributed oxygen vacancies can destroy the structural coherence in the superconducting plane of high-T_{c} superconductors.[19] The structural incoherence can reduce the pair amplitude[20] and/or destroy the phase coherence.[21] In order to consider the finite quasi-particle lifetime, Srikanth *et al* [22] gave a phenomenological model that the modified density of states of Dynes *et al* [23] was directly put into the results of Blonder–Tinkham–Klapwijk (BTK)[24] theory. The obtained theoretical results provided a poor description of the experimental data. Plecenik *et al* [25,26] led a Γ-factor into the Bogoliubov equations (Γ is the broadening parameter characterizing the finite lifetime of the quasi-particles) and found the Γ-dependent Bogoliubov coherence factors u and v. The theoretical dependencies fit their experimental curves very well in Ref.[25]. Very recently, the finite quasi-particle lifetime effects have been widely used in the study of tunneling conductance for the electron-doped cuprate superconductors,[27] the heavy-fermion superconductors,[28] and the novel superconductor CaC_6.[29]

In this paper, we calculate the tunneling conductance by using the extended BTK theory when quasiparticles are injected into a $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor from a single-mode quantum wire, and obtain some characteristic tunneling spectra, in which the finite quasi-particle lifetime effects are considered by using the model of Plecenik *et al*.

2. Model and formulation

Consider a quantum wire/insulator/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor (q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$) junction as shown in Fig.1. The quantum wire is a one-dimensional (1D) normal metal with width w. There is a potential barrier formed by a very thin insulating layer at $x = 0$, which is described by the delta function $V(\boldsymbol{r}) = V_0\delta(x)$. We assume that the pair potential is independent of x in the $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductors,

$$\Delta(x,\theta) = \Delta_{\pm}\Theta(x). \tag{1}$$

The $\Theta(x)$ is the Heaviside function, and the $\Delta_{\pm}$ are given by[4]

$$\Delta_{\pm} = \Delta_1\cos(2\theta \mp 2\alpha) \pm \mathrm{i}\,\Delta_2\sin(2\theta \mp 2\alpha), \tag{2}$$

here Δ_+ and Δ_- stand for the pair potentials for electron-like and hole-like quasi-particles, respectively, θ is the angle between the x axis and the momentum of the quasi-particles in the $d_{x^2-y^2}+\mathrm{i}d_{xy}$ wave superconductor, Δ_1 and Δ_2 are the magnitude of the $d_{x^2-y^2}$- and d_{xy}-wave components of the pair potentials, respectively.

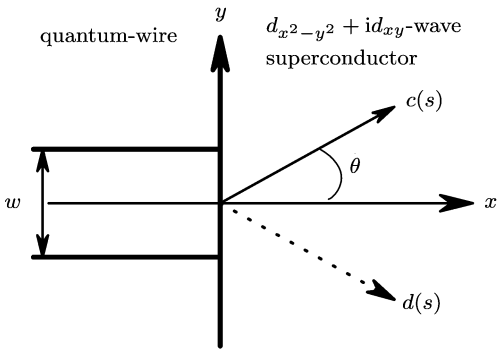


Fig.1. Quantum point contact N/I/ $d_{x2-y2}+\mathrm{i}d_{xy}$ junction. The N is a single-mode quantum wire with width w. Here $k_{\mathrm{F}}w/\pi = 1.7$.

We adopt the Bogoliubov-de Gennes (BdG) approach to study the q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction. The BdG equation is given as[30]

$$\begin{pmatrix} H_0 & \Delta(\boldsymbol{k},\boldsymbol{r}) \\ \Delta^*(\boldsymbol{k},\boldsymbol{r}) & -H_0^* \end{pmatrix}\begin{pmatrix} u_k(\boldsymbol{r}) \\ v_k(\boldsymbol{r}) \end{pmatrix} = E\begin{pmatrix} u_k(\boldsymbol{r}) \\ v_k(\boldsymbol{r}) \end{pmatrix}, \tag{3}$$

where $H_0 = -(\hbar^2/2m)\nabla^2 + V(\boldsymbol{r}) - \mathrm{i}\,\Gamma - E_\mathrm{F}$ with $\Gamma = \hbar/\tau$, τ is the lifetime of the quasi-particles between collisions, and E is the quasi-particle energy relative to the Fermi energy E_F. With general solutions of Eq.(3), the wave functions in the two regions are described by

$$\Psi_N = \left[\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{\mathrm{i}\,k_1^+ x} + a\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{\mathrm{i}\,k_1^- x} + b\begin{pmatrix}1\\0\end{pmatrix}\mathrm{e}^{-\mathrm{i}\,k_1^+ x}\right]\phi_1(y) \tag{4}$$

for one-dimensional (1D) normal metal, and

$$\Psi_S = \int_{-\infty}^{+\infty}\mathrm{d}s\left[c(s)\begin{pmatrix}u_+(s)\mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2}\\v_+(s)\mathrm{e}^{-\mathrm{i}\,\varphi_+(s)/2}\end{pmatrix}\mathrm{e}^{\mathrm{i}\,k_+ x} + d(s)\begin{pmatrix}v_-\mathrm{e}^{\mathrm{i}\,\varphi_-(s)/2}\\u_-\mathrm{e}^{-\mathrm{i}\,\varphi_-(s)/2}\end{pmatrix}\mathrm{e}^{-\mathrm{i}\,k_- x}\right]\phi_s(y) \tag{5}$$

for two-dimensional (2D) $d_{x^2-y^2}+\mathrm{i}d_{xy}$ wave superconductor. Here a, b, c and d respectively correspond to the coefficients for the Andreev reflection, normal reflection, transmission to the $d_{x^2-y^2}+\mathrm{i}d_{xy}$ wave superconductor as electron-like quasi-particles and transmission as hole-like quasi-particles; the $\varphi_\pm$ are the phases of the pair potentials $\Delta_\pm$, respectively, with $\mathrm{e}^{\mathrm{i}\,\varphi_\pm} = \Delta_\pm/|\Delta_\pm|$. Moreover

$$\begin{aligned}\phi_1(y) &= \sqrt{\frac{2}{w}}\sin\left[\frac{\pi}{w}\left(y+\frac{w}{2}\right)\right]\Theta\left(\frac{w}{2}-|y|\right),\\ \phi_s(y) &= \mathrm{e}^{\mathrm{i}\,sy}/\sqrt{2\pi},\end{aligned} \tag{6}$$

$$\begin{aligned}u_\pm(s) &= \sqrt{\frac{1}{2}\left[1+\frac{\Omega_\pm(s)}{E+\mathrm{i}\,\Gamma}\right]},\\ v_\pm(s) &= \sqrt{\frac{1}{2}\left[1-\frac{\Omega_\pm(s)}{E+\mathrm{i}\,\Gamma}\right]},\\ \Omega_\pm(s) &= \sqrt{(E+\mathrm{i}\,\Gamma)^2-|\Delta_\pm(s)|^2},\end{aligned} \tag{7}$$

where $k_1^\pm$ are the electrons (+) and holes (−) wave numbers in the wire, which are functions of the energy E of the incoming electrons from the normal metal region; $k_\pm(s)$ are the wave numbers for the quasi-particles in the superconductor and depend on the wave number along the y axis, defined by s. The Fermi wave number k_F is assumed to be equal both in the normal metal and in the superconductor. Then the wave numbers are given by

$$\begin{aligned}k_1^\pm &= \sqrt{k_1^2 \pm \frac{2mE}{\hbar^2}},\\ k_1 &= \sqrt{k_\mathrm{F}^2-\frac{\pi^2}{w^2}},\end{aligned} \tag{8}$$

$$\begin{aligned}k_\pm &= \sqrt{k(s)^2 \pm \frac{2m\Omega_\pm(s)}{\hbar^2}},\\ k(s) &= \sqrt{k_\mathrm{F}^2-s^2}.\end{aligned} \tag{9}$$

All the coefficients can be determined by matching boundary conditions at the interface:

$$\Psi_N(0) = \Psi_S(0), \tag{10}$$

$$\left(\frac{\mathrm{d}\,\Psi_S}{\mathrm{d}x}\right)_{x=0} - \left(\frac{\mathrm{d}\,\Psi_N}{\mathrm{d}x}\right)_{x=0} = \frac{2mV_0}{\hbar^2}\,\Psi_N(0). \tag{11}$$

From Eqs.(4), (5), (10) and (11), we can get

$$(1+b)\phi_1(y) = \int_{-\infty}^{+\infty}\mathrm{d}s[c(s)u_+\mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2} + d(s)v_-\mathrm{e}^{\mathrm{i}\,\varphi_-(s)/2}]\phi_s(y), \tag{12a}$$

$$a\phi_1(y) = \int_{-\infty}^{+\infty}\mathrm{d}s[c(s)v_+\mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2} + d(s)u_-\mathrm{e}^{-\mathrm{i}\varphi_-(s)/2}]\phi_s(y), \tag{12b}$$

$$[(1-2\,\mathrm{i}\,z)-(1+2\,\mathrm{i}\,z)b]\phi_1(y) = \frac{k(s)}{k_1}\int_{-\infty}^{+\infty}\mathrm{d}s[c(s)u_+\mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2} - d(s)v_-\mathrm{e}^{\mathrm{i}\,\varphi_-(s)/2}]\phi_s(y), \tag{12c}$$

$$(1-2\,\mathrm{i}\,z)a\phi_1(y) = \frac{k(s)}{k_1}\int_{-\infty}^{+\infty}\mathrm{d}s[c(s)v_+\mathrm{e}^{-\mathrm{i}\,\varphi_+(s)/2} - d(s)u_-\mathrm{e}^{-\mathrm{i}\,\varphi_-(s)/2}]\phi_s(y), \tag{12d}$$

where $z = mV_0/\hbar^2 k_1$. We have used the Andreev approximation in the boundary conditions for the derivative of the wave function, $k_1^+ = k_1^- = k_1$ and $k_+(s) = k_-(s) = k(s)$. Multiplying Eqs.(12) by $\phi^*_{s'}(y)\mathrm{d}y$ and integrating over y, we obtain

$$(1+b)p(s) = c(s)u_+ \mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2} + d(s)v_- \mathrm{e}^{\mathrm{i}\,\varphi_-(s)/2}, \tag{13a}$$

$$ap(s) = c(s)v_+ \mathrm{e}^{-\mathrm{i}\,\varphi_+(s)/2} + d(s)u_- \mathrm{e}^{-\mathrm{i}\,\varphi_-(s)/2}, \tag{13b}$$

$$[(1-2\,\mathrm{i}\,z) - (1+2\,\mathrm{i}\,z)b]\,p(s) = \frac{k(s)}{k_1}\left[c(s)u_+ \mathrm{e}^{\mathrm{i}\,\varphi_+(s)/2} - d(s)v_- \mathrm{e}^{\mathrm{i}\,\varphi_-(s)/2}\right], \tag{13c}$$

$$(1-2\,\mathrm{i}\,z)ap(s) = \frac{k(s)}{k_1}\left[c(s)v_+ \mathrm{e}^{-\mathrm{i}\,\varphi_+(s)/2} - d(s)u_- \mathrm{e}^{-\mathrm{i}\,\varphi_-(s)/2}\right] \tag{13d}$$

with

$$p(s) = \int_{-\infty}^{+\infty} \mathrm{d}y \phi_1(y)\phi^*_s(y) = \frac{2}{w}\sqrt{\frac{\pi}{w}}\frac{\cos(sw/2)}{\pi^2/w^2 - s^2}. \tag{14}$$

Form the normalization conditions

$$\int_{-\infty}^{+\infty} \mathrm{d}y \phi^*_{s'}(y)\phi_s(y) = \delta(s-s'),$$
$$\int_{-\infty}^{+\infty} \mathrm{d}y \phi^*_1(y)\phi_1(y) = 1, \tag{15}$$

the function $p(s)$ has the following property

$$\int_{-\infty}^{+\infty} \mathrm{d}s p^*_s(s)p_s(s) = 1. \tag{16}$$

Using that, from Eqs.(13) we have

$$F_2 a - (F_1 + 2\,\mathrm{i}\,z + 1)b = F_1 + 2\,\mathrm{i}\,z - 1, \tag{17a}$$

$$(F_1 - 2\,\mathrm{i}\,z + 1)a - F_3 b = F_3, \tag{17b}$$

where

$$F_j = \frac{4}{\pi^2\sqrt{\gamma_\mathrm{F}^2 - 1}}\int_{-\gamma_\mathrm{F}}^{\gamma_\mathrm{F}} \mathrm{d}q \frac{\sqrt{\gamma_\mathrm{F}^2 - q^2}}{(1-q^2)^2}\cos^2\left(\frac{\pi q}{2}\right) f_j(q), \tag{18}$$

$$q = ws/\pi, \quad \gamma_\mathrm{F} = wk_\mathrm{F}/\pi \quad \text{and} \quad j = 1,2,3.$$
$$f_1 = \frac{1 + \Gamma_+\Gamma_- \mathrm{e}^{-\mathrm{i}(\varphi_+ - \varphi_-)}}{1 - \Gamma_+\Gamma_- \mathrm{e}^{-\mathrm{i}(\varphi_+ - \varphi_-)}},$$
$$f_2 = \frac{2\Gamma_- \mathrm{e}^{\mathrm{i}\,\varphi_-}}{1 - \Gamma_+\Gamma_- \mathrm{e}^{-\mathrm{i}(\varphi_+ - \varphi_-)}},$$
$$f_3 = \frac{2\Gamma_+ \mathrm{e}^{-\mathrm{i}\,\varphi_+}}{1 - \Gamma_+\Gamma_- \mathrm{e}^{-\mathrm{i}(\varphi_+ - \varphi_-)}},$$
$$\Gamma_\pm = v_\pm/u_\pm. \tag{19}$$

The solutions for Eqs.(17) are given by

$$a = \frac{2F_3}{(F_1+1)^2 + 4z^2 - F_2F_3}, \tag{20a}$$

$$b = \frac{(1-2\,\mathrm{i}\,z)^2 - F_1^2 + F_2F_3}{(F_1+1)^2 + 4z^2 - F_2F_3}. \tag{20b}$$

By using the BTK theory, we are able to calculate the differential conductance of the q/I/$d_{x^2-y^2}$ + id_{xy} junction at temperature $T = 0$ K,

$$G_R(eV) = \left.\frac{G_S(E)}{G_N}\right|_{E=eV} = \frac{(1+F_0)^2 + 4z^2}{4F_0}(1 + |a|^2 - |b|^2), \tag{21}$$

here G_N is the conductance when $\Delta = 0$ and $a = 0$. F_0 is defined by Eq.(18) with $f_j = 1$ and the reflection coefficients are evaluated in $E = eV$, where V is the voltage.

3. Results and discussion

We shall investigate the influence of α, z, Δ_2/Δ_1 and Γ on differential conductance in q/I/$d_{x^2-y^2}$ + id_{xy} junctions. Let us first study the effects of the interface potential z. With the help of Eqs.(20) and (21), the dependence of differential conductance on the bias voltage for different values of z are shown in Figs.2 and 3 when Γ is smaller. Simultaneously, the

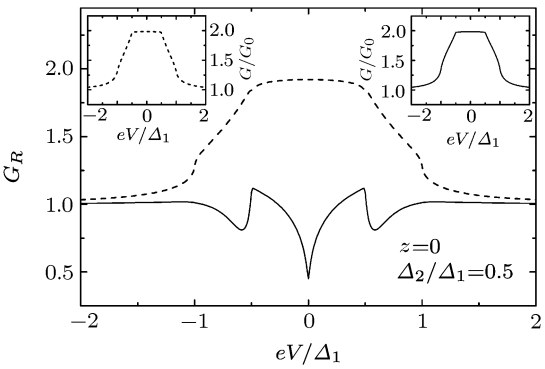


Fig.2. Normalized conductance as a function of the bias voltage in q/I/$d_{x^2-y^2}$ + id_{xy} junction (the inset for 2D N/I/$d_{x^2-y^2}$ + id_{xy} junction) at $\Gamma/\Delta_1 = 0.006$ for $\alpha = 0$ (dash), $\pi/4$ (solid). Here $k_\mathrm{F}w/\pi = 1.7$ for q/I/$d_{x^2-y^2}$ + id_{xy} junction.

differential conductance of a 2D N/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction is plotted in the inset of Fig.2. We find that the conductance in q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction varies strongly with α at $z=0$, which is distinct from that of the 2D N/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction. In the 2D N/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction, the conductance is almost independent of α at $z=0$ (see the inset). With increasing of z (as shown in Fig.3), the zero bias conductance (ZBC) is reduced for $\alpha=0$ but increased for $\alpha=\pi/4$, and the bias conductance peak (BCP) is enhanced for either of the angles. At the same time, the BCP shifts from $E=0$ and the ZBCP is absent when $\alpha=0$, and the dip outside the BCP is gradually lowered when $\alpha=\pi/4$.

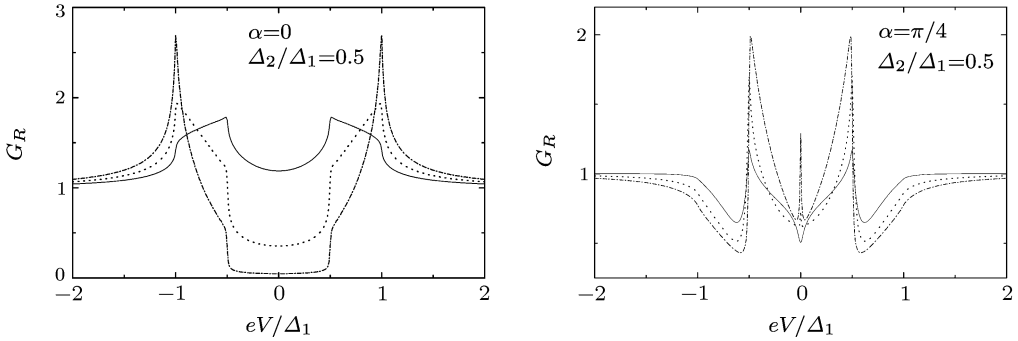


Fig.3. Normalized conductance as a function of the bias voltage in q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction at $\Gamma/\Delta_1=0.006$ for different interface potential $z=0.4$ (solid), 1 (dot), 3 (dash dot). Here $k_{\mathrm{F}}w/\pi=1.7$.

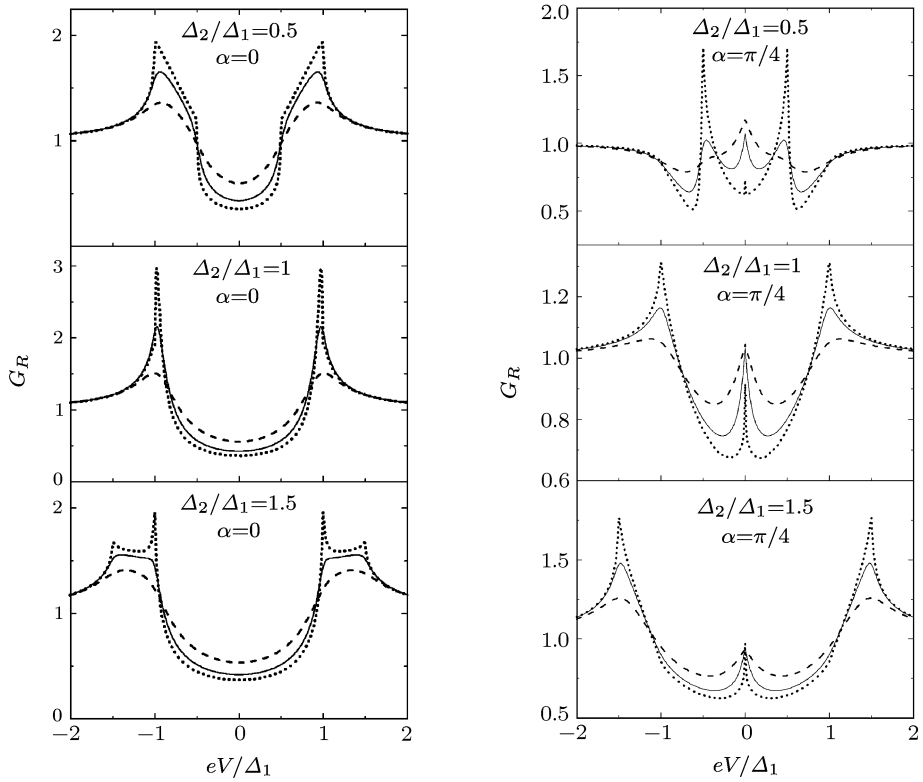


Fig.4. Normalized conductance as a function of the bias voltage in q/I/$d_{x^2-y^2}+\mathrm{i}\,d_{xy}$ junction at $z=1$ for different quasi-particle lifetime factor $\Gamma/\Delta_1=0.006$ (dot), 0.06 (solid), 0.2 (dash). Here $k_{\mathrm{F}}w/\pi=1.7$.

It is especially worth pointing out at $\alpha=\pi/4$ that there is no ZBCP in the tunnelling conductance for smaller z, or else, the ZBCP will appear, which shows that the ZBCP may or may not appear in the tunnelling conductance of q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction. We notice that in the case of $\alpha=$

$\pi/4$, the two waves with injected angles $\pm\theta$ feel pair potentials in the $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor belonged to intermediate situation just right, here $\mathrm{Re}[\Delta_\pm(\theta)] = -\mathrm{Re}[\Delta_\pm(-\theta)]$, $\mathrm{Im}[\Delta_\pm(\theta)] = \mathrm{Im}[\Delta_\pm(-\theta)]$, $\mathrm{Re}[\Delta_\pm(0)] = 0$, $\mathrm{Im}[\Delta_\pm(\pi/4)] = 0$, and $\mathrm{Re}[\Delta_\pm(\pi/2)] = 0$.

Secondly, we investigate the effects of amplitude ratio Δ_2/Δ_1 and quasi-particle lifetime factor Γ. Figure 4 shows that for different values of Δ_2/Δ_1 at $\alpha = 0$, the shape of tunnelling conductance is obviously not alike. As Γ increases, the ZBC is raised, the BCP is reduced, and the tunnelling conductance will become smoother by degrees. In comparison with the case of $\alpha = 0$, several noticeable changes are shown at $\alpha = \pi/4$. First, the BCP shifts outward evidently with increasing of Δ_2/Δ_1, and corresponding $\Delta_2/\Delta_1 = 0.5$, 1 and 1.5, the BCP are respectively located at 0.5, 1 and 1.5 exactly, this fact displays that the position of the BCP depends highly on the amplitude ratio Δ_2/Δ_1. Second, the BCP is lowered gradually and vanishes finally with the increase of Γ when $\Delta_2/\Delta_1 = 0.5$, indicating that the influence of Γ on the BCP is contrary to that of z. Third, the low and narrow ZBCP may coexist with the BCP in tunnelling conductance of q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction. In addition, it can be found from Fig.4 that the change of the ZBC with Γ and Δ_2/Δ_1 is nonlinear for $\alpha = \pi/4$, as shown in Fig.5.

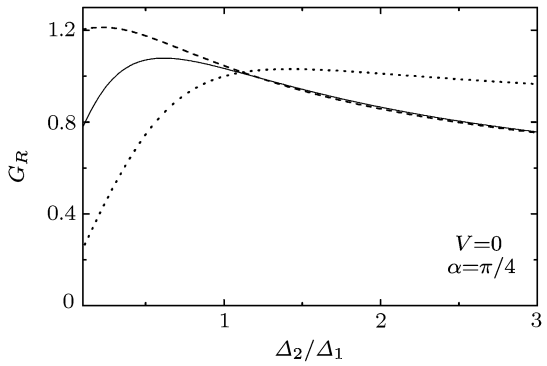


Fig.5. Zero bias conductance as a function of the amplitude ratio Δ_2/Δ_1 in q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junction, in which z and Γ/Δ_1 are the same as those in Fig.4 for $\alpha = \pi/4$. Here $k_\mathrm{F}w/\pi = 1.7$.

Finally, we present the tunneling conductance of q/I/d-wave superconductor junction at $\alpha = \pi/4$ and $z = 0$. Figure 6 shows that the result agrees with Refs.[14] and [17] for $\Gamma = 0$. As Γ increases, the ZBC is raised, but there is not the ZBCP in the tunnelling conductance.

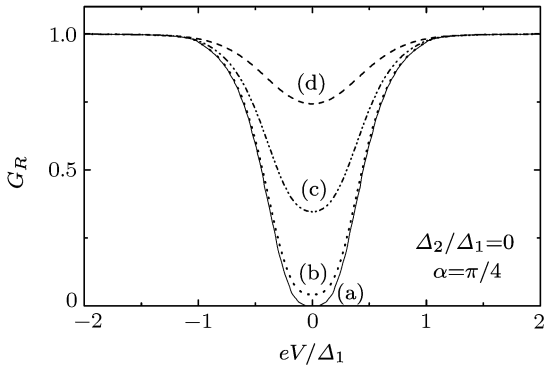


Fig.6. Normalised conductance as a function of the bias voltage in q/I/d-wave junction at $z = 0$ for different quasiparticle lifetime factor $\Gamma/\Delta_1 = 0$ (a), 0.006 (b), 0.06 (c), 0.2 (d). Here $k_\mathrm{F}w/\pi = 1.7$.

In summary, we have studied the tunnelling conductance in q/I/$d_{x^2-y^2}+\mathrm{i}d_{xy}$ junctions. When $\alpha = \pi/4$, above-mentioned results reveal some quite distinctive tunnelling conductance, using those, we can distinguish $d_{x^2-y^2}+\mathrm{i}d_{xy}$ mixed wave superconductor from d-wave and p-wave one.

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