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Surface defect gap solitons in one-dimensional dual-frequency lattices*

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We study the surface defect gap solitons in an interface between a defect of one-dimensional dual-frequency lattices and the uniform media. Some unique properties are revealed that such lattices can broaden the region of semi-finite gap, and the semi-finite gap exists not only in the positive and zero defects but also in the negative defect; unlike in the regular lattices, the semi-finite gap exists in the positive and zero defects but does not exist in the negative defect. In particular, stable solitons exist almost in the whole semi-finite gap for the positive and zero defects. These properties are different from other lattices with defects. In addition, it is found that the existence of surface dual-frequency lattice solitons does not need a threshold power.

Keywords: dual-frequency lattices, surface defect gap solitons
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1. Introduction

Surface waves are a special type of wave, which may exist at the interface between two different media, and appear in diverse areas of physics, chemistry, biology, display properties that have no counterpart in the bulk.[1] In history, investigation of surface modes initiated by Tamm in 1932 in the Kronig-Penney model found electronic states localized at the edge of the solid,[2] and subsequently studied by Shockley in 1939.[3] In linear optics, Kossel suggested that localized states could exist near the boundary between a homogeneous and a layered medium in 1966,[4] and successfully observed in AlGaAs multilayer structures.[5,6] Also, such waves were shown to exist at metal-dielectric interfaces[7] as well as at the interfaces of anisotropic materials.[8] In nonlinear optics, surface waves including TE, TM and mixed-polarization modes in the interface between homogeneous dielectric media have been theoretically predicted.[9−11]

The formation of surface solitons was predicted and almost directly observed for the cases of surface gap solitons.[12−14] Moreover, surface solitons have been theoretically predicted for the case of the hetero-interface between two different semi-infinite waveguide arrays,[15] as well as at the boundaries of two-dimensional nonlinear lattices.[15,16−19] It has been shown that surface solitons in the form of vector,[20,21] kink[22] and vortex[23] can exist. Polychromatic surface modes have been studied and experimentally observed.[24−26] Recently, the existence of discrete surface solitons has been predicted in one-dimensional (1D) arrays of nonlinear optical waveguides.[27]

These states were predicted to be located at the edge of a semi-infinite array and exhibited a power threshold necessary for their existences.[12,13,27−29] The formation and properties of discrete surface solitons have been explored theoretically in detail,[27,28,30−34] and these solitons were quickly created in experiments performed in arrays of optical waveguides.[29] Higher dimensional surface discrete solitons have been reported theoretically and experimentally.[18,19,35,36]

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Defect solitons have been investigated in the waveguide arrays with defocusing cubic nonlinearity.[37] In particular, in one-dimensional optical lattice, low-power defect solitons are linearly stable in lower bandgaps, whereas at higher powers defect solitons can remain stable in negative defects.[38] Recently, surface solitons formed in the defect of one-dimensional regular optical lattices, superlattices, and Bessel lattices have been studied.[39−41]

Linear modes occur in the linear periodic system and have been examined in photonic lattices.[42−44] Their transmission spectra are divided into bands and gaps. In the gaps, the propagation of linear modes in the periodic direction cannot be carried out. When nonlinearities are introduced to the linear media, the transmission spectra of modes are modified, which causes the eigenenergy (or propagation constant) of modes to get into the gaps, thus the modes become localized nonlinear modes (solitons) in periodic system. So it can be concluded that the nonlinear modes (solitons) in optical lattices would bifurcate from the linear Bloch modes. For the different nonlinearities, such nonlinear modes have new properties, e.g., lattice solitons are supported in the periodic system with cubic and quintic nonlinearities.[45]

However, from the above researches on surface solitons, one easily finds that the lowest gap (semi-finite gap), which is more important than other gaps, is usually narrow, and most of results have shown that surface solitons have a threshold power. An essential question is whether the semi-finite gap can be further broadened and whether or not the surface solitons need a threshold power.

In this work, we exploit a new kind of lattices that are constructed by spacing modulation, called as dual-frequency optical lattices. We find that stable surface defect gap solitons (SDGSs) can be well confined at the interface between a defect of dual-frequency optical lattices and uniform saturable nonlinear media. In this system, we obtain a broad semi-finite gap, whose region is larger than that in usual regular lattices and the superlattices. Moreover, stable surface solitons can be formed in the positive, zero, and negative defects, and the existence of surface solitons do not need a threshold power.

2. The model

We consider the beam propagation at the interface of uniform and one-dimensional optical lattice in the focusing saturable nonlinear media described by the nonlinear Schrödinger equation. The nondimensionalized equation for the light field q is[13,39,40]

$$\mathrm{i}\frac{\partial q}{\partial z} + q_{xx} - \frac{E_0}{1 + I_L(x) + |q|^2}q = 0, \tag{1}$$

where I_L is the intensity profile of the optical lattice when $x > \pi/2$,

$$I_L = I_0 \sin^2[\Omega_1(x + \pi/2)] \times \sin^2[\Omega_2(x + \pi/2)], \tag{2}$$

for $-\pi/2 < x < \pi/2$,

$$I_L = I_0(0.6 + \varepsilon) \sin^2[\Omega_1(x + \pi/2)], \tag{3}$$

and $I_L = 0$ for $x < -\pi/2$. In Eq.(1), the diffraction length is $L_\mathrm{d} = 2k_1D^2/\pi^2$, x is the transverse scale whose real unit is D/π, E_0 is the applied dc field whose real unit is $\pi^2/(k_0^2 n_\mathrm{e}^4 D^2 \gamma_{33})$, I_0 is the peak intensity of optical lattice indicated by Eq.(2), the modulation parameters Ω_1 and Ω_2 describe lattices' period and asymmetry (in this paper we take $\Omega_1 = 1$ and $\Omega_2 = 2$), D is the lattice spacing with $\Omega_1 = D/\pi$ and $\Omega_2 = 2D/\pi$, $k_0 = 2\pi/\lambda_0$ is the wave number (λ_0 is the wavelength), $k_1 = k_0 n_\mathrm{e}$, n_e is the unperturbed refractive index, and γ_{33} is the electro-optic coefficient of the crystal. ε indicates the modulation deep of defect intensity (the defect is shown by Eq.(3). In a photorefractive crystal, the effective lattice potential given by Eqs.(2) and (3) can be induced optically, when illuminating the sample with a broad laser beam passed through a properly designed amplitude mask.[18] The mask can control the distribution of optical intensity that forms a defect between the optical lattices and the uniform media. Such a lattice has been created successfully in experiment.[46]

The lattices described by Eqs.(2) and (3) are very different from the superlattices as used in Ref.[13]. Here, the spacing between single lattices is modulated as dual frequency. While the superlattices are produced by modulating the amplitude of lattices, but the spacing between lattices is symmetrical. This difference will significantly influence the properties of solitons since the high sensitivity of the bandgap on the refractive index modulation occurs in various systems.[13−19]

The intensity distributions of semi-infinite dual-frequency lattices expressed by Eqs.(2) and (3) are

displayed in Figs.1(a)–1(c), composed of dense and nondense lattices alternately carrying various surface defects. To show the existent condition for SDGSs, we search for the Floquet–Bloch spectrum by substituting a solution $q(x,z) = f(x)\exp(\mathrm{i}kx - \mathrm{i}\mu z)$ into the linear version of Eq.(1), where μ is the propagation constant, k is the Bloch wave number and $f(x)$ is periodic function [here $f(x) = f(x+\pi)$]. Figure 1(d) shows bandgap structure of the lattice at $I_0 = 3$, which is typical in experimental conditions. Compared with the regular lattices and the superlattices, the semi-finite gap is larger. For the fixed parameter $E_0 = 6$, we find that the border of semi-finite gap is about 3.75; while for the regular lattices and the superlattices, it is about 2.6.[13,39,40] Therefore, such new dual-frequency lattices can extend the region of the semi-finite gap up to 31% approximately, as compared with that in the above-motioned lattices.

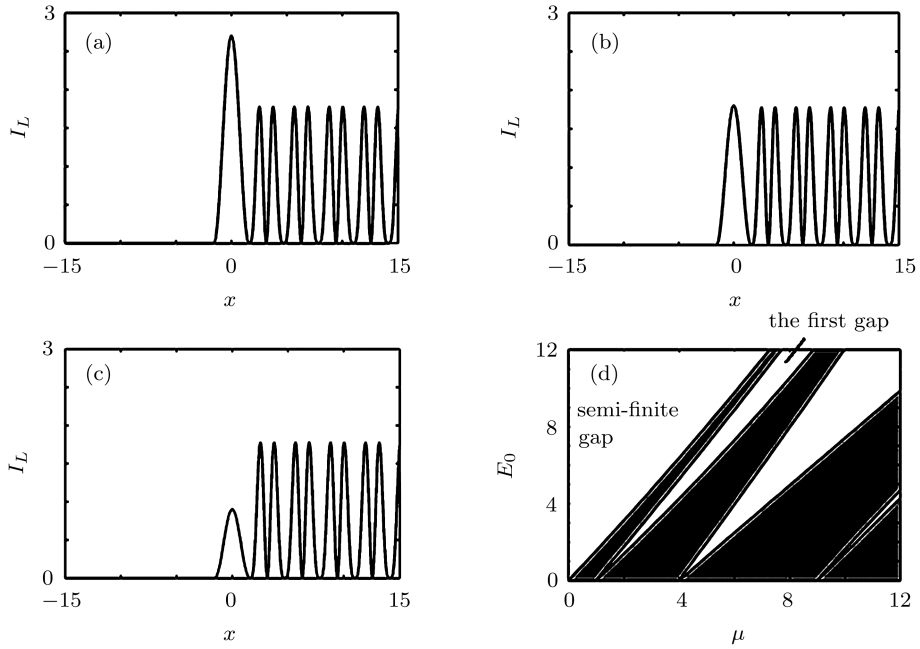


Fig.1. Intensity distribution of one-dimensional dual-frequency optical lattices (arranged by dense and nondense lattices alternatively) with $I_0 = 3$, $\Omega_1 = 1$ and $\Omega_2 = 2$, thus, the maximum amplitude of I_L is about 1.8, with a defect at the surface (a) positive defect $\varepsilon = 0.3$, (b) zero defect $\varepsilon = 0$ (the maximum intensity of defect is equal to the maximum intensity of the lattices), and (c) negative defect $\varepsilon = -0.3$. (d) Linear spectrum in the plane of (E_0, μ); the gray areas are Bloch bands.

3. Numerical results

We search for the stationary soliton profiles in the form of $q(x,z) = f(x)\exp(-\mathrm{i}\mu z)$, where $f(x)$ is the real function satisfying the following equation

$$\frac{\partial^2 f}{\partial x^2} - \frac{E_0}{1 + I_L(x) + |f|^2} f + f\mu = 0. \qquad (4)$$

The energy P of a soliton is defined as $P = \int_{-\infty}^{\infty} f^2(x)\mathrm{d}x$. By numerically solving Eq.(4) using the shooting method, we obtain the soliton profiles. The soliton families are mainly defined by propagation constant μ, lattice modulation parameters Ω_1 and Ω_2, and lattice modulation depth I_0. For the positive defect with $\varepsilon = 0.3$, figure 2(a) plots the power P versus the propagation constant μ. We find that SDGSs exist only in the semi-infinite gap, but do not exist in the first gap. Solitons are stable in the entire region of the semi-infinite gap. Stable examples are shown in Figs.2(b) and 2(d) and Figs.2(c) and 2(e). For zero defect with $\varepsilon = 0$, SDGSs exist only in the semi-infinite gap, but do not exist in the first gap, see Fig.3(a). Solitons can be stabilized in large domain, but unstable in a narrow region (corresponding to the part of low power). For the negative defect with $\varepsilon = -0.3$, SDGSs exist not only in the semi-infinite gap but also in the first gap, see Fig.4(a). From Fig.4(a), only a narrow region is unstable. Figures 4(i) and 4(j) give

a stable soliton existing in the first gap (as shown at point D in Fig.4(a)). We find that when the defect $\varepsilon < -0.29$ solitons may exist in both the semi-finite gap and the first gap. Note that although the soliton profiles look like to be symmetric, however, in fact, they are slightly asymmetric.

To further examine the stability of SGSs, we search for the perturbed solution of Eq.(1) in the form $q(x,z) = [f(x) + h(x,z) + \mathrm{i}e(x,z)]\exp(-\mathrm{i}\mu z)$, where $h(x,z)$ and $e(x,z)$ are the real and imaginary parts of a perturbation eigenmode, respectively, with a complex instability growth rate δ. By linearizing Eq.(1) around f and keeping only the first-order terms in $h(x,z)$ and $e(x,z)$, we arrive at coupled equations for the eigenmode:

$$\delta h = -\partial^2 e/\partial x^2 - \mu e + E_0 e(1 + I_L + f^2)$$

and

$$\delta e = \partial^2 h/\partial x^2 + \mu h - [1/(1 + I_L + f^2) - 2f^2/(1 + I_L + f^2)^2]E_0 h.$$

These equations are solved numerically to get the perturbation growth rate $\mathrm{Re}(\delta)$ as shown in Figs.3(b) and 4(b) [$\mathrm{Re}(\delta) = 0$ indicates the stability domains], which is in agreement with those shown in Figs.3(a) and 4(a), respectively.

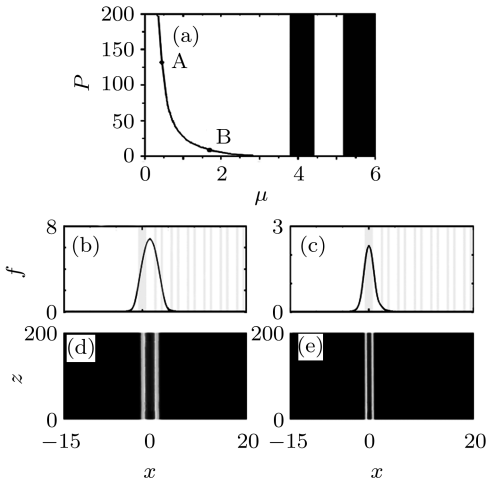


Fig.2. For positive defect with $\varepsilon = 0.3$. (a) The power P versus the propagation constant μ (solid line shows stable region). Solitons are stable in the entire region. Stable examples: (b) and (d) $\mu = 0.45$ (point A); (c) and (e) $\mu = 1.70$ (point B).

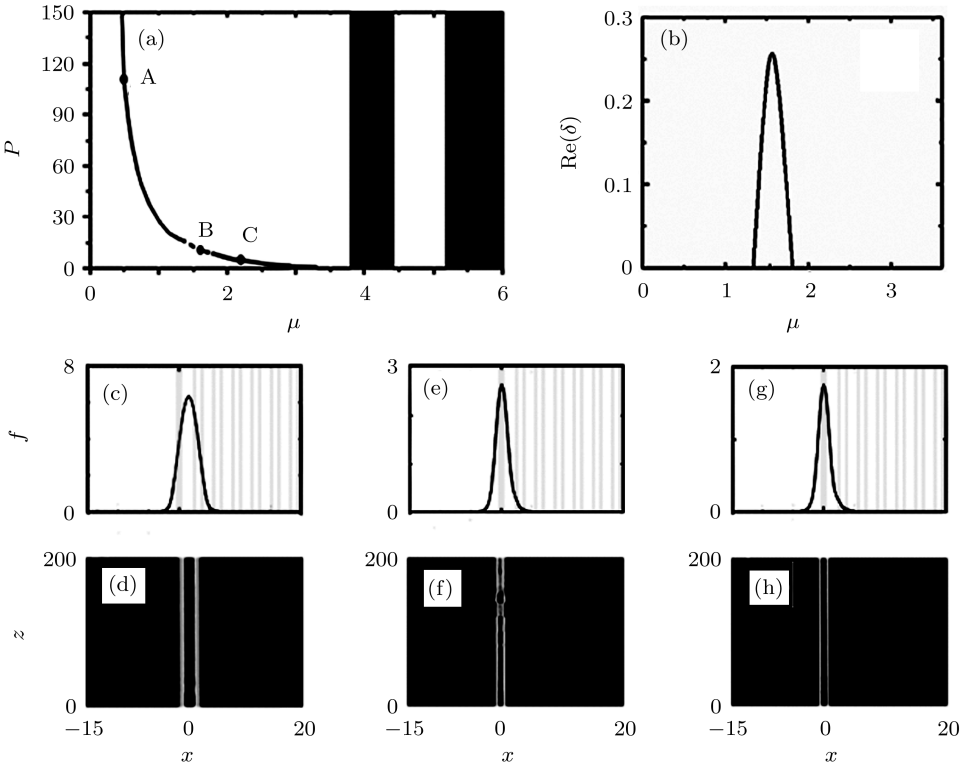


Fig.3. For zero defect with $\varepsilon = 0$. (a) The power P versus the propagation constant μ (solid lines show stable regions and the dashed line shows the unstable region). (b) Perturbation growth rate versus the propagation constant μ. Stable examples: (c) and (d) $\mu = 0.5$ (point A) and (g) and (h) $\mu = 1.60$ (point C); unstable example (e) and (f) $\mu = 2.20$ (point B).

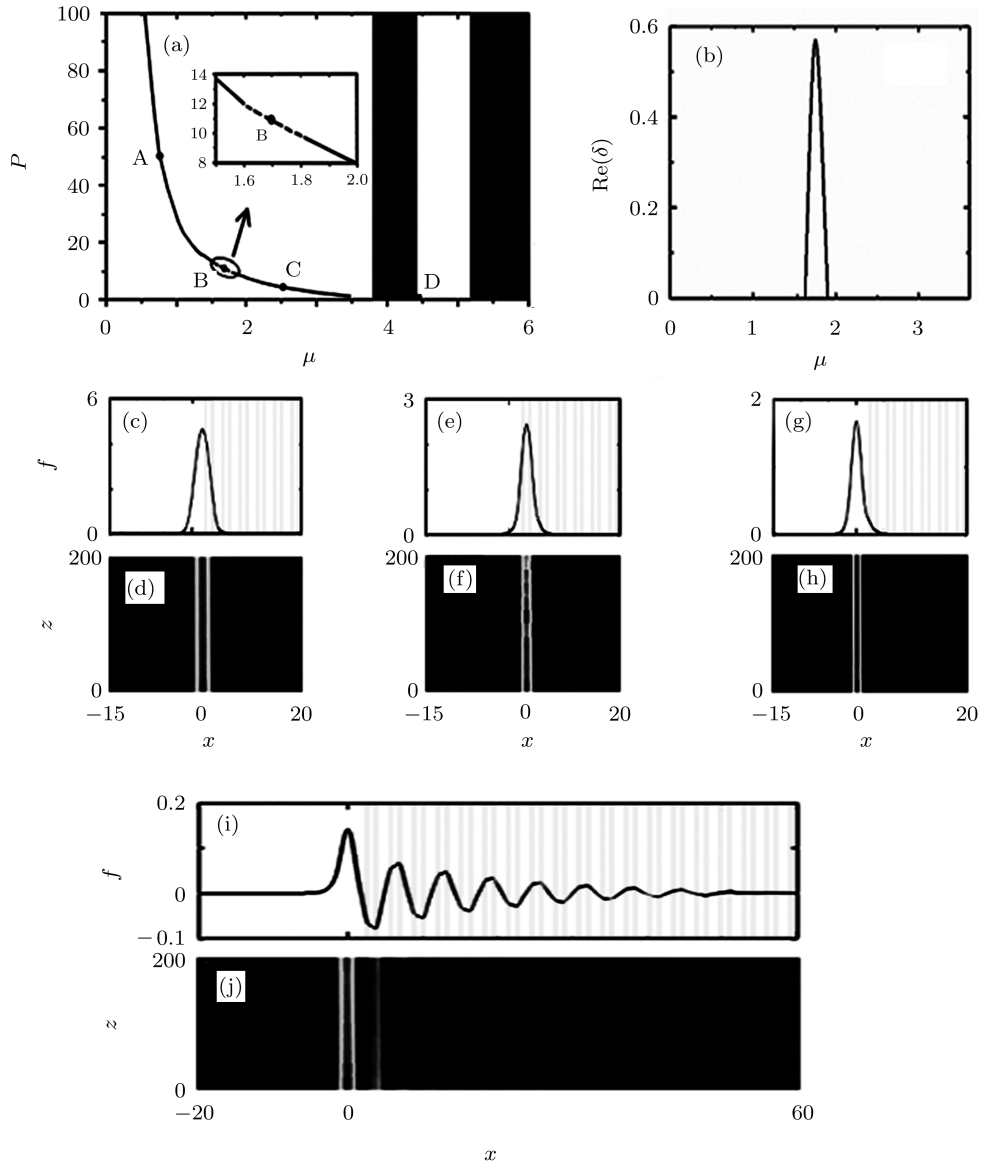


Fig.4. For negative defect with $\varepsilon = -0.3$. (a) The power P versus the propagation constant μ. (b) Perturbation growth rate versus the propagation constant μ. Stable examples: (c) and (d) $\mu = 0.75$ (point A) and (g) and (h) $\mu = 2.5$ (point C); unstable example (e) and (f) $\mu = 1.70$ (point B). Only a narrow region in place of low power is unstable (dashed line). (i) and (j) Stable soliton exists in the first gap (as shown at point D in (a)).

The robustness of perturbed SDGSs is tested in direct simulations of the evolution of the nonlinear modes with added random perturbations, whose relative amplitude is set at the 10% level. Note that the instability of solitons is different from that obeying the Vakhitov–Kolokolov (VK) criterion with the sign of differential $\mathrm{d}P/\mathrm{d}\mu$ change in the slope of the power diagram within the unstable region.[37,38]

Next, we also give the distance of soliton peak position away from the surface as the change of soliton power, see Fig.5. One of the important results is that solitons will get close to the surface of lattices as the power in various defects decreases, as clearly indicated in Figs.5(a)–5(d). Note that the surface of lattices is located at $r_0 = 0$. From the above results, it is obvious that SDGSs can be formed under very low power, i.e., they do not need a threshold power with various defects. This is different from other previous studies in other optical lattices without defect, surface solitons must possess larger power than a threshold power, otherwise, they will be unstable,[12,27–29] or cannot be localized at the surface of lattices (i.e., solitons move inside lattices under low power, thus, they do not belong to surface states).[13,40] This may be the consequence that the dual-frequency lattices produce larger semi-finite gaps than those of regular lattice and superlattice,[39,44] which

results in solitons stabilized in the edge of gaps (corresponding to the low power).

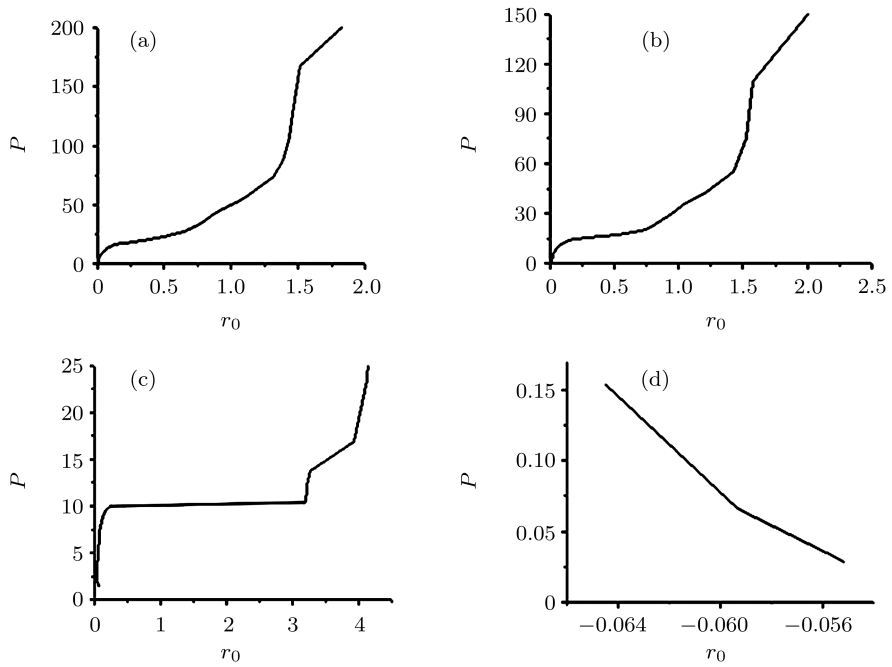


Fig.5. Soliton power P versus soliton peak position r_0 for (a) $\varepsilon = 0.3$, (b) $\varepsilon = 0$, (c) and (d) $\varepsilon = -0.3$; soliton exists in the semi-finite gap (a)–(c) and in the first gap (d). The surface of lattices is located at $r_0 = 0$. These results show that solitons are close towards the surface of lattices with the decrease of power.

Finally, we give the summary of the stability and instability regions of SDGSs for the dependences of defect parameters ε on the propagation constant μ, as shown in Fig.6. In fact, the stabilities and instabilities

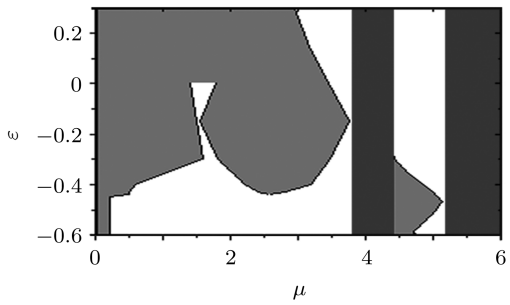


Fig.6. Stability (gray shade) and instability (white) regions based on the dependence of the defect parameter ε on the propagation constant μ. When $-0.6 \leq \varepsilon < -0.29$, solitons exist in both the semi-finite gap and the first gap; otherwise solitons exist in the semi-finite gap.

of SDGSs are complex due to the complicated refractive modulation. For $\varepsilon \geq -0.29$, solitons exist only in the semi-finite gap; while for $-0.6 \leq \varepsilon < -0.29$, solitons exist in both the semi-finite gap and the first gap. In addition, from Fig.6, stable solitons exist in most of the semi-finite gap for positive and zero defects. These properties are different from the surface defect gap solitons in the regular lattices with defect,[39] where solitons do not exist in the semi-finite gap for the negative defect and the stable region of solitons occupies a small part in the semi-finite gap for the positive and zero defects.

4. Conclusions

We have studied the solitons localized at the interface between the dual-frequency lattices with a defect and the focusing saturable nonlinear media. Such lattices with the positive, zero, and negative defects can lead solitons to exist in the semi-finite gap. This is in contrast to the regular lattices, in which solitons exist only in the semi-finite gap for the positive and zero defects, but they do not exist in the semi-finite gap for the negative defect. More important thing is that the semi-finite gap is broadened by about 30% more than that in regular lattices and the superlat-

tices. In particular, stable surface solitons exist almost in the whole semi-finite gap for the positive and zero defects. In addition, it is found that the existence of surface solitons does not need a threshold power. Whether or not such new lattices can induce some remarkable phenomena in other systems, such as Kerr media and Bose–Einstein condensates, is a topic deserving a study in the future.[47]

References

[1] Zangwill A 1988 *Physics at Surfaces* (Cambridge: Cambridge University Press)
[2] Tamm I 1932 *Phys. Z. Sowjetunion* **1** 733
[3] Shockley W 1939 *Phys. Rev.* **56** 317
[4] Kossel D 1966 *J. Opt. Soc. Am.* **56** 1434
[5] Yeh P, Yariv A and Cho A Y 1978 *Appl. Phys. Lett.* **32** 104
[6] Ng W, Yeh P, Chen P C and Yariv A 1978 *Appl. Phys. Lett.* **32** 370
[7] Barnes W L, Dereux A and Ebbesen T W 2003 *Nature* **424** 824
[8] Artigas D and Torner L 2005 *Phys. Rev. Lett.* **94** 013901
[9] Seaton C T, Valera J D, Shoemaker R L, Stegeman G I, Chilwell J T and Smith S D 1985 *IEEE J. Quantum Electron.* **21** 774
[10] Mihalache D, Stegeman G I, Seaton C T, Wright E M, Zanoni R, Boardman A D and Twardowski T 1987 *Opt. Lett.* **12** 187
[11] Mihalache D, Bertolotti M and Sibilia C 1989 *Prog. Opt.* **27** 229
[12] Kartashov Y V, Vysloukh V A and Torner L 2006 *Phys. Rev. Lett.* **96** 073901
[13] He Y J, Chen W H, Wang H Z and Malomed B A 2007 *Opt. Lett.* **32** 1390
[14] Dong L and Wang H 2007 *Opt. Express* **15** 1706
Rosberg C R, Neshev D N, Krolikowski W, Mitchell A, Vicencio R A, Molina M I and Kivshar Y S 2006 *Phys. Rev. Lett.* **97** 083901
[15] Makris K G, Hudock J, Christodoulides D N, Stegeman G I, Manela O and Segev M 2006 *Opt. Lett.* **31** 2774
[16] Kartashov Y V, Vysloukh V A, Mihalache D and Torner L 2006 *Opt. Lett.* **31** 2329
[17] Kartashov Y V and Torner L 2006 *Opt. Lett.* **31** 2172
[18] Wang X, Bezryadina A, Chen Z, Makris K G, Christodoulides D N and Stegeman G I 2007 *Phys. Rev. Lett.* **98** 123903
[19] Szameit A, Kartashov Y V, Dreisow F, Pertsch T, Nolte S, Tunnermann A and Torner L 2007 *Phys. Rev. Lett.* **98** 173903
[20] Kartashov Y V, Ye F and Torner L 2006 *Opt. Express* **14** 4808
[21] Garanovich I L, Sukhorukov A A, Kivshar Y S and Molina M 2006 *Opt. Express* **14** 4780
[22] Kartashov Y V, Vysloukh V A and Torner L 2006 *Opt. Express* **14** 12365
[23] Kartashov Y V, Vysloukh V A, Egorov A A and Torner L 2006 *Opt. Express* **14** 4049
[24] Motzek K, Sukhorukov A A and Kivshar Y S 2006 *Opt. Express* **14** 9873
[25] Motzek K, Sukhorukov A A and Kivshar Y S 2006 *Opt. Lett.* **31** 3125
[26] Sukhorukov A A, Neshev D N, Dreischuh A, Fischer R, Ha S, Krolikowski W, Bolger J, Mitchell A, Eggleton B J and Kivshar Y S 2006 *Opt. Express* **14** 11265
[27] Makris K G, Suntsov S, Christodoulides D N, Stegeman G I and Hache A 2005 *Opt. Lett.* **30** 2466
[28] Suntsov S, Makris K G, Christodoulides D N, Stegeman G I, Morandotti R, Yang H, Salamo G and Sorel M 2007 *Opt. Lett.* **32** 3098
[29] Suntsov S, Makris K G, Christodoulides D N, Stegeman G I, Hache A, Morandotti R, Yang H, Salamo G and Sorel M 2006 *Phys. Rev. Lett.* **96** 063901
[30] Molina M I, Garanovich I L, Sukhorukov A A and Kivshar Y S 2006 *Opt. Lett.* **31** 2332
[31] Molina M I, Vicencio R A and Kivshar Y S 2006 *Opt. Lett.* **31** 1693
[32] Mihalache D, Mazilu D, Lederer F and Kivshar Y S 2007 *Opt. Express* **15** 589
[33] Stepić M, Smirnov E, Rüter C E, Kip D, Maluckov A and Hadžievski L 2007 *Opt. Lett.* **32** 823
[34] Smirnov E, Stepić M, Rüter C E, Kip D and Shandarov V 2006 *Opt. Lett.* **31** 2338
[35] Susanto H, Kevrekidis P G, Malomed B A, Carretero-González R and Frantzeskakis D J 2007 *Phys. Rev.* E **75** 056605
[36] Mihalache D, Mazilu D, Lederer F and Kivshar Y S 2007 *Opt. Lett.* **32** 3173
[37] Sukhorukov A A and Kivshar Y S 2001 *Phys. Rev. Lett.* **87** 083901
[38] Yang J and Chen Z 2006 *Phys. Rev.* E **73** 026609
[39] Chen W H, He Y J and Wang H Z 2006 *Opt. Express* **14** 11271
[40] Chen W H, He Y J and Wang H Z 2007 *J. Opt. Soc. Am.* B **24** 2584
[41] Dong L W, Yang X Y and Chen H Y 2008 *Chin. Phys.* B **17** 988
[42] Fedele F, Yang J and Chen Z 2005 *Opt. Lett.* **30** 1506
[43] Fedele F, Yang J and Chen Z 2005 *Stud. Appl. Math.* **115** 279
[44] Chen W H, He Y J and Wang H Z 2008 *Phys. Lett.* A **372** 3525
[45] Wang J, Ye F, Dong L, Cai T and Li Y P 2005 *Phys. Lett.* A **339** 74
[46] Makasyuk I, Chen Z and Yang J 2005 *Technical Digest Series, Conference on Nonlinear Guided Waves and Their Applications* (Dresden: Optical Society of America, Germany), p.TuC8.
[47] Yang R S and Yang J H 2008 *Chin. Phys.* B **17** 1189