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Interaction of light filaments in air*

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This paper analytically investigates the interaction of light filaments generated by a femtosecond laser beam in air. It obtains the Hamiltonian of a total laser field and interaction force between two filaments with different phase shifts and crossing angles. The property of the interaction force, which leads the attraction or repulsion of filaments, is basically dependent on the phase shift between filaments. The crossing angle between two filaments can only determine the magnitude of the interaction force, but does not change the property of the force.

Keywords: femtosecond laser beam, air, light filaments, Hamiltonian
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1. Introduction

Filamentation is a well-known phenomenon when an intense femtosecond laser beam propagates in air. If the peak power of a femtosecond laser beam is just higher than the self-focusing threshold, single filament can be formed due to the dynamic balance of Kerr self-focusing and electron de-focusing. If the peak power of femtosecond laser beam is many times higher than the self-focusing threshold, many small-scale filaments will be formed in the spatiotemporal region of the laser beam. The formation and evolution of the multiple filaments have attracted substantial research interests both in theory and experiment[1−13] due to the potential applications in remote sensing and lighting control.[14,15] The formation of multiple filaments was explained by the modulation instability of the laser beam[1] and the optical turbulence model.[2] Recent studies show that for beam power reaching several TW, the stable filaments are initiated by the fluctuations of the laser beam and confined into distinct regions.[3,4] In the stable regions the small-scale filaments are nucleated randomly, in agreement with the optical turbulence model. Furthermore, the influences of initial fluctuation, strong field gradients and phase distortion on the evolution of multiple filaments have been investigated.[5−9] These studies suggest that there should be some mechanism underlined in the interaction of the small-scale filaments.[10] The interaction dynamics of light filaments is critical in understanding and controlling of the stable propagation of femtosecond laser beam. Our previous calculations display the interaction feature of two light filaments, such as attraction, fusion, repulsion and collision, depending on the phase shift and crossing angle between them.[10] Also, the non-planar interaction of two light filaments is investigated and the spiraling propagation is observed. However, the simulation results need a clear analysis to further understand the mechanism of the interaction between light filaments.

In this regard, Bergé *et al*[16−18] developed an analytical method to study the coalescence of two light filaments. A total mean-squared radius of the total beam formed by the two light filaments is defined and the Hamiltonian of the simplified nonlinear Schrödinger (NLS) equation is obtained. The evolution of the total beam can be obtained from the Hamiltonian. They investigated the interactions of two in-phase light filaments and two out-phase light filaments.[16] However, the phase shift and crossing angle between the light filaments, which have significant influences on the interaction of them,[10] have not been discussed yet. In this paper, we analytically derive the Hamiltonian interaction of two filaments with differ-

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ent phase shifts and crossing angles. The property and magnitude of the force between the two filaments can be illustrated directly from the Hamiltonian interaction.

2. Analytical derivation for interaction of two light filaments

To start with the analytical derivation, we introduce the simplified NLS equation which can be written as:[18]

$$2ik_0\frac{\partial E}{\partial z}+\Delta_\perp E+k_0^2\left(n_2\left|E\right|^2-n_{2K}\left|E\right|^{2K}\right)E=0, \tag{1}$$

where E is the electric envelope of the laser beam, z refers the propagation distance and k_0 is the central wave number. The Laplacian operator $\Delta_\perp = \partial^2/\partial x^2+\partial^2/\partial y^2$ describes the beam transverse diffraction. The third and fourth terms describe the instantaneous Kerr effect with the coefficient of n_2 and saturated nonlinearity induced by electron generation, respectively. Here, the electron density is frozen in time and is approached by $\rho=\dfrac{\Delta T\beta^{(K)}}{K\hbar\omega_0}\left|E\right|^{2K}$,[18] where ΔT is the temporal duration of a filament and $\beta^{(K)}$ is the multiphoton ionization coefficient for the number of photon K. Thus, the effective coefficient of saturated nonlinearity is $n_{2K}=\dfrac{\Delta T\beta^{(K)}}{\rho_{\rm c}K\hbar\omega_0}$, where $\rho_{\rm c}$ is the critical plasma density.

The evolution of the electric field of the laser beam is characterized by the total mean-squared radius, which is given by

$$\left\langle r^2\right\rangle=\int r^2\left|E\right|^2{\rm d}\boldsymbol{r}/P, \tag{2}$$

where $r=\sqrt{x^2+y^2}$ is the radial coordinate and $P=\int\left|E\right|^2{\rm d}\boldsymbol{r}$ represents the power of the laser beam. The mean-squared radius of the laser beam obeys the following equation:[17]

$$\begin{aligned}&d_z^2\left\langle r^2\right\rangle\\&=2\left(H+\frac{K-1}{K+1}k_0^2n_{2K}\int\left|E\right|^{2(K+1)}{\rm d}\boldsymbol{r}\right)/k_0^2P,\end{aligned} \tag{3}$$

where H is the total Hamiltonian and invariant for Eq.(1), which is given by

$$\begin{aligned}&H\\&=\int\left(\left|\nabla_\perp E\right|^2-\frac{\pi\left|E\right|^4}{P_{\rm cr}}+\frac{k_0^2n_{2K}}{K+1}\left|E\right|^{2(K+1)}\right){\rm d}\boldsymbol{r},\end{aligned} \tag{4}$$

where $P_{\rm cr}=\lambda_0^2/2\pi n_0n_2$ denotes the critical power for self-focusing.

To simplify the derivation, each light filament is taken as Gaussian profile with the same power and transverse waist. Thus, the electric field envelope formed by the two light filaments is written as:

$$\begin{aligned}&E(x,y,z=0)\\&=\left(\frac{2P_0}{\pi w^2}\right)^{1/2}{\rm e}^{-\frac{(x-a/2)^2+y^2}{w^2}}{\rm e}^{{\rm i}\varphi}\\&\quad\times{\rm e}^{-{\rm i}k_0(x-a/2)\tan\theta}+\left(\frac{2P_0}{\pi w^2}\right)^{1/2}\\&\quad\times{\rm e}^{-\frac{(x+a/2)^2+y^2}{w^2}}{\rm e}^{{\rm i}k_0(x+a/2)\tan\theta},\end{aligned} \tag{5}$$

where P_0 and w are the power and waist of a single light filament, respectively, a is the separation distance of the two light filaments, φ represents the phase shift and 2θ represents the crossing angle between the two light filaments.

First, substituting the electric field envelope of a single light filament $\left(E=\left(\dfrac{2P_0}{\pi w^2}\right)^{1/2}{\rm e}^{-\frac{x^2+y^2}{w^2}}{\rm e}^{{\rm i}\varphi}{\rm e}^{-{\rm i}k_0x\tan\theta}\right)$ into Eq.(2), we can obtain

$$\left\langle r^2\right\rangle=\frac{w^2}{2}. \tag{6}$$

The Hamiltonian contributed by the single light filament is written as

$$\begin{aligned}H_{\rm free}=&\frac{2P_0}{w^2}\left[1+\frac{1}{2}w^2k_0^2\tan^2\theta-\frac{P_0}{2P_{\rm cr}}\right.\\&\left.+\frac{w^2k_0^2n_{2K}}{2(K+1)^2}\left(\frac{2P_0}{\pi w^2}\right)^K\right].\end{aligned} \tag{7}$$

Thus, the evolution of the single light filament is defined by

$$\begin{aligned}d_z^2\left\langle w^2\right\rangle=&4\left[H_{\rm free}+(K-1)\frac{2P_0}{w^2}\right.\\&\left.\times\frac{w^2k_0^2n_{2K}}{2(K+1)^2}\left(\frac{2P_0}{w^2}\right)^K\right]/k_0^2P_0.\end{aligned} \tag{8}$$

Substituting the electric field envelope in Eq.(5), we can obtain the total mean-squared radius of the laser beam formed by the two light filaments. It is given by

$$\begin{aligned}\left\langle r^2\right\rangle=&\frac{w^2}{2}+\frac{a^2}{4}+\frac{w^2}{4}(2-k_0^2w^2\tan^2\theta)\cos\varphi\\&\times\exp\left(-\frac{k_0^2w^2\tan^2\theta}{2}\right)\exp\left(-\frac{a^2}{2w^2}\right).\end{aligned} \tag{9}$$

In the interaction region where we study, $a/w=5$, $\exp(-a^2/2w^2)\ll1$, the third term in Eq.(9) is much

smaller than the first and the second terms, thus, the total mean-squared radius of the laser beam can be written as

$$\langle r^2 \rangle \simeq \frac{a^2}{4} + \frac{w^2}{2}. \tag{10}$$

The total beam Hamiltonian is given by

$$H = 2H_{\text{free}} + H_{\text{int}}, \tag{11}$$

where the first term $2H_{\text{free}}$ (Eq.(7)) refers the contributions of two individual light filaments and H_{int} represents the Hamiltonian interaction which is written as

$$\begin{aligned} H_{\text{int}} = \frac{4P_0}{w^2} \Bigg\{ & \left(1 - \frac{a^2}{2w^2}\right) \cos\varphi \, \mathrm{e}^{-\frac{a^2}{2w^2} - \frac{1}{2}w^2 k_0^2 \tan^2\theta} - \frac{P_0}{2P_{\text{cr}}} \Bigg[(2 + \cos(2\varphi) \, \mathrm{e}^{-w^2 k_0^2 \tan^2\theta}) \, \mathrm{e}^{-\frac{a^2}{w^2}} \\ & + 4\cos\varphi \cos\left(\frac{1}{2} a k_0 \tan\theta\right) \, \mathrm{e}^{-\frac{3a^2}{4w^2} - \frac{1}{4}w^2 k_0^2 \tan^2\theta} \Bigg] + \frac{w^2 k_0^2 n_{2K}}{2(K+1)^2} \left(\frac{2P_0}{\pi w^2}\right)^K \\ & \times \left[(2K+1) \cos\varphi \cos\left(\frac{K}{K+1} a k_0 \tan\theta\right) \mathrm{e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}} \mathrm{e}^{-\frac{k_0^2 w^2 \tan^2\theta}{2(K+1)}} \right] \Bigg\}. \end{aligned} \tag{12}$$

Thus, the evolution of the laser beam formed by the two light filaments is characterized by

$$\begin{aligned} d_z^2 \langle r^2 \rangle = 2 \Bigg\{ & 2H_{\text{free}} + H_{\text{int}} + \frac{4P_0}{w^2} \frac{w^2 k_0^2 n_{2K} (K-1)}{2(K+1)^2} \left(\frac{2P_0}{\pi w^2}\right)^K \\ & \times \left[1 + (2K+1) \cos\varphi \cos\left(\frac{K}{K+1} a k_0 \tan\theta\right) \mathrm{e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}} \mathrm{e}^{-\frac{k_0^2 w^2 \tan^2\theta}{2(K+1)}} \right] \Bigg\} / 2k_0^2 P_0. \end{aligned} \tag{13}$$

Therefore, the evolution of the separation distance between the two light filaments can be deduced from Eqs.(8), (10) and (13), it is written as

$$d_z^2(a^2) = \frac{4H_{\text{int}}}{k_0^2 P_0} + \frac{16 n_{2K}(K-1)}{2(K+1)^2} \left(\frac{2P_0}{\pi w^2}\right)^K (2K+1) \cos\varphi \cos\left(\frac{K}{K+1} a k_0 \tan\theta\right) \mathrm{e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}} \mathrm{e}^{-\frac{k_0^2 w^2 \tan^2\theta}{2(K+1)}}. \tag{14}$$

The second term is much smaller than the first term, thus, the evolution of the separation distance can be written as

$$d_z^2(a^2) = 4H_{\text{int}} / k_0^2 P_0. \tag{15}$$

Here, the Hamiltonian interaction H_{int} gathers all the interaction terms of the two light filaments. If the Hamiltonian interaction is negative ($H_{\text{int}} < 0$), the interaction of two light filaments will lead to the decrease of the separation distance between the two light filaments. The force between them should be attraction. If the Hamiltonian interaction is positive ($H_{\text{int}} > 0$), the interaction of the two light filaments will cause the increase of the separation distance between them. This suggests that the two light filaments repulse each other. The property and magnitude of the interaction force between the two light filaments can be estimated from Eqs.(12) and (15).

3. Results

First, we study the interaction of two parallel filaments with a separation distance $a = 0.5$ mm. In this case, $\theta = 0$, the Hamiltonian interaction is written as

$$\begin{aligned} H_{\text{int}} = \frac{4P_0}{w^2} \Bigg\{ & \left(1 - \frac{a^2}{2w^2}\right) \cos\varphi \, \mathrm{e}^{-\frac{a^2}{2w^2}} - \frac{P_0}{2P_{\text{cr}}} \left[(2 + \cos 2\varphi) \, \mathrm{e}^{-\frac{a^2}{w^2}} + 4\cos\varphi \, \mathrm{e}^{-\frac{3a^2}{4w^2}} \right] \\ & + \frac{w^2 k_0^2 n_{2K}}{2(K+1)^2} \left(\frac{2P_0}{\pi w^2}\right)^K \left[(2K+1) \cos\varphi \, \mathrm{e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}} \right] \Bigg\}. \end{aligned} \tag{16}$$

The power and waist of a single light filament are $P_0 = 9$ GW and $w = 100$ μm respectively, which are taken from Ref.[10]. From Eq.(16), it can be seen that the interacting force between the two parallel light filaments depends on the phase shift between them. Figure 1 shows the Hamiltonian interaction of the two parallel light filaments as a function of their phase shift. For the phase shift varying from 0 to 0.5π, the Hamiltonian interaction is always negative. Moreover, as the increase of the phase shift the absolute value of Hamiltonian interaction decreases. It suggests that the two light filaments with a phase shift less than 0.5π will attract each other. The attraction between the two light filaments becomes weak when the phase shift approaches to 0.5π. For the phase shift $0.5\pi < \varphi \leq \pi$, the Hamiltonian interaction is positive and the two light filaments repulse each other. The results are consistent with those given by numerical simulations.[10]

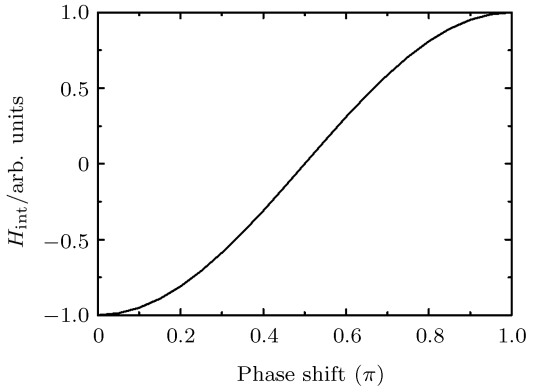


Fig.1. The Hamiltonian interaction of two parallel light filaments as a function of the phase shift between them.

Meanwhile, equation (15) can describe the interaction of two light filaments with incident angles. To simplify the case, we assume that the two light filaments are in phase ($\varphi = 0$). In such a case, the Hamiltonian interaction for the two light filaments with incident angle θ can be written as

$$
\begin{aligned}
H_{\rm int} = \frac{4P_0}{w^2}\Bigg\{ & \left(1-\frac{a^2}{2w^2}\right) \mathrm{e}^{-\frac{a^2}{2w^2}-\frac{1}{2}w^2k_0^2\tan^2\theta} \\
& - \frac{P_0}{2P_{\rm cr}}\left[(2+\mathrm{e}^{-w^2k_0^2\tan^2\theta})\,\mathrm{e}^{-\frac{a^2}{w^2}} + 4\cos\left(\frac{ak_0\tan\theta}{2}\right)\,\mathrm{e}^{-\frac{3a^2}{4w^2}-\frac{1}{4}w^2k_0^2\tan^2\theta}\right] \\
& + \frac{w^2k_0^2n_{2K}}{2(K+1)^2}\left(\frac{2P_0}{\pi w^2}\right)^K\left[(2K+1)\cos\left(\frac{K}{K+1}ak_0\tan\theta\right)\,\mathrm{e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}}\,\mathrm{e}^{-\frac{k_0^2w^2\tan^2\theta}{2(K+1)}}\right]\Bigg\}. \qquad (17)
\end{aligned}
$$

The parameters of filament power and waist are the same as those in Eq.(16). Figure 2(a) shows the Hamiltonian interaction of the two in-phase light filaments as a function of incident angle. For the incident angle from 0° to 0.1° the Hamiltonian interaction is always negative due to the intensity superposition of the two in-phase filaments. It suggests that two in-phase light filaments attract each other no matter how large the incident angle is. But the strength of attraction varies with the incident angle. With the increase of the incident angle, the absolute value of the Hamiltonian interaction decreases. As a result, we can predict that the attraction of the two light filaments decreases with the increase of incident angle.

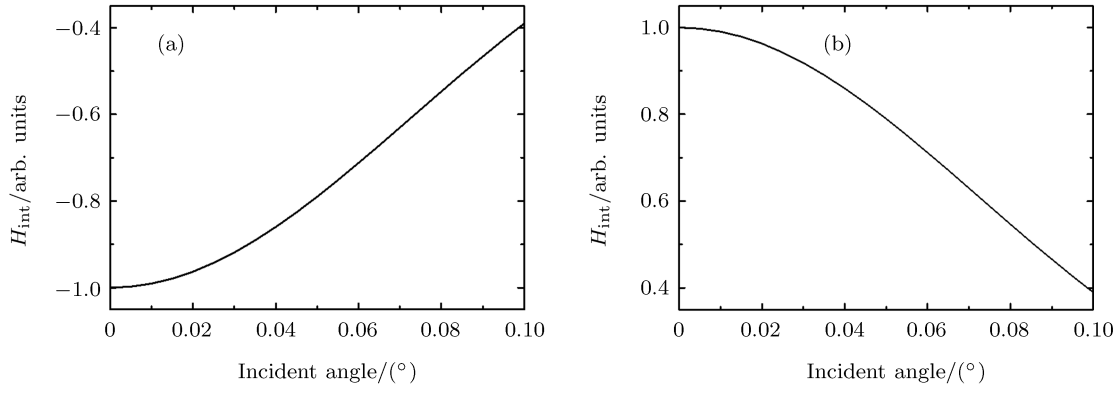


Fig.2. The interaction Hamiltonian of two light filaments as a function of the incident angle for the phase shift being 0 (a) and π (b).

When the two light filaments are out of phase, the Hamiltonian interaction as a function of incident angle is written as

$$H_{\rm int} = \frac{4P_0}{w^2}\left\{-\left(1-\frac{a^2}{2w^2}\right)\ {\rm e}^{-\frac{a^2}{2w^2}-\frac{1}{2}w^2k_0^2\tan^2\theta} - \frac{P_0}{2P_{\rm cr}}\left[\left(2+{\rm e}^{-w^2k_0^2\tan^2\theta}\right)\ {\rm e}^{-\frac{a^2}{w^2}}\right.\right.$$
$$-4\cos\left(\frac{ak_0\tan\theta}{2}\right){\rm e}^{-\frac{3a^2}{4w^2}-\frac{1}{4}w^2k_0^2\tan^2\theta}\Big] - \frac{w^2k_0^2n_{2K}}{2(K+1)^2}\left(\frac{2P_0}{\pi w^2}\right)^K$$
$$\left.\times\left[(2K+1)\cos\left(\frac{K}{K+1}ak_0\tan\theta\right)\ {\rm e}^{-\frac{2K+1}{2(K+1)}\frac{a^2}{w^2}}{\rm e}^{-\frac{k_0^2\tan^2\theta w^2}{2(K+1)}}\right]\right\}. \quad (18)$$

Figure 2 (b) shows the curve of the Hamiltonian interaction of the two out of phase light filaments with different incident angles varying from 0° to 0.1°. It is shown that in such cases, the Hamiltonian interaction is always positive. This suggests that for the two out of phase light filaments, they always repulse each other even if the incident angle tries to move them toward the centers. Furthermore, the absolute value of the Hamiltonian interaction decreases with the increase of the incident angle, like the interaction case of two in-phase filament. It suggests that the crossing angle between filaments destroys the coherence of the two filaments, thus the interaction of them is influenced. The interacting force between two filaments with a large crossing angle is smaller than that in the case of a small crossing angle. The total laser beam cannot propagate at a long distance due to the destructive coherence of the two filaments.

4. Conclusion

In conclusion, the interaction of two light filaments in the transverse plane of the laser beam has been investigated analytically. The interaction force between the two light filaments can be estimated by obtaining the Hamiltonian of them. The property of the interaction force, which leads the attraction or repulsion of filaments, basically depends on the phase shift between them. The two filaments with a phase shift less than 0.5π will attract each other. For the phase shift larger than 0.5π, two filaments repulse each other. With the increase of crossing angle, the interaction force between filaments decreases. The crossing angle only influences the magnitude of the force between the two light filaments, but does not change the property of the force.

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