Vol 18 No 10, October 2009
1674-1056/2009/18(10)/4177–05

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Modulational instability for a self-attractive two-component Bose–Einstein condensate*

Li Sheng-Chang(栗生长) and Duan Wen-Shan(段文山)†

College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China

(Received 11 October 2008; revised manuscript received 5 March 2009)

By means of the multiple-scale expansion method, the coupled nonlinear Schrödinger equations without an explicit external potential are obtained in two-dimensional geometry for a self-attractive Bose-Einstein condensate composed of different hyperfine states. The modulational instability of two-component condensate is investigated by using a simple technique. Based on the discussion about two typical cases, the explicit expression of the growth rate for a purely growing modulational instability and the optimum stable conditions are given and analysed analytically. The results show that the modulational instability of this two-dimensional system is quite different from that in a one-dimensional system.

Keywords: modulational instability, Bose-Einstein condensate, multiple-scale expansion method
PACC: 0530J, 1230, 1110L

1. Introduction

Since the realization of Bose-Einstein condensates (BECs) in dilute alkali-metal atomic vapours,[1] rapid development has taken place in this field. Many properties of BECs have been studied experimentally and theoretically.[2−5] Nonlinearity is one of the most important features of BECs. Therefore, there are many interesting nonlinear excitations of nonlinear coherent structures in BECs. Recently, solitary waves and vortices in BECs have been observed in many experiments.[6−8] Experimental investigations on nonlinear properties in BECs have promoted more intensive theoretical studies.[9−12]

So far, most of studies have focused on a single component BEC with repulsive atomic interaction.[13,14] Furthermore, more and more scientists have paid much attention to the study of the nonlinear phenomena in two-component BECs (TBECs).[15−17] The study of multi-component BECs is particularly interesting, TBECs of different hyperfine states of rubidium atoms in a magnetic trap[18] and of sodium atoms in an optical trap[19] have been realized in experiment. Compared with one-component BEC, two-component condensates have rich collective dynamics and coherent structures due to the interaction between different components. The experimental observations in TBECs have lead to many fundamental questions concerning the dynamic problems, stability and dissipation of nonlinear excitations.

Generally, the dynamic behaviours of TBECs composed of different hyperfine states or different species can be described by the coupled Gross-Pitaevskii (CGP) equations.[20,21] Indeed, the coupled one-dimensional (1D) nonlinear Schrödinger equations[22] and the coupled Korteweg-de Vries (KdV) equations[23] have been used to study the dark solitions in TBECs. On the other hand, the modulational instability (MI) is also an interesting issue in BECs.[24,25] To our knowledge, most of investigations on dynamics and MI in BECs are limited to 1D systems and repulsive atomic interactions.

In the present paper we study the modulational instability for a self-attractive TBEC. It is well-known that the stability issues of the single component BEC can be determined solely by the type of atomic interactions. For an attractive interaction,[26−28] the condensate is stable for a maximum critical number of atoms. In the presence of an external source of atoms when the number increases beyond a critical number, the condensate collapses emitting atoms until the number of atoms is reduced below the critical number and a stable configuration is reached. Then

*Project supported by the National Natural Science Foundation of China (Grant No 10875098) and the Science Foundation of the Northwest Normal University (Grant Nos NWNU-KJCXGC-03-17 and NWNU-KJCXGC-03-48).
†Corresponding author. E-mail: duanws@nwnu.edu.cn

the condensate can grow again and a series of collapses can take place, which was observed experimentally in the BEC of ^{7}Li with attractive interaction.[29,30] However, a study on TBEC has shown numerically[31] that the type of atomic interaction does not determine the stability of the system.

Presently, the stability of a coupled BEC and two colliding BECs are studied by Adhikari[32] and Timmermans[33] separately. The MI of TBEC in a quasi-one-dimensional geometry has also been investigated by Raju *et al.*[25] Motivated by these studies, we choose the coupled nonlinear Schrödinger (CNLS) equations to investigate a self-attractive two-dimensional (2D) TBEC composed of different hyperfine states with a disk-shaped trap. With the help of the linear stability analysis theory, we discuss the modulational instability of the system. Furthermore, we obtain the explicit expression of growth rate and the MI conditions of the TBEC for two typical cases. Finally, we compare our results with those in 1D situation, and find that there exists considerable difference between them.

2. Derivation of CNLS equations

The dynamic behaviour of two-component interacting BEC composed of two different hyperfine states at low temperature can be described by the CGP equations[20,21]

$$\mathrm{i}\hbar\frac{\partial\Psi_j}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_j(\boldsymbol{r})+\sum_{l=1,2}U_{jl}|\Psi_l|^2\right]\Psi_j, \quad (1)$$

where $j=1,2$, $V_j(\boldsymbol{r})=(m/2)\omega_j^2[\lambda(x^2+y^2)+z^2]$ is the external potential, ω_j is the frequency of the trap in the z-direction, λ denotes the aspect ratio of the condensate, which is assumed to be identical for each component. Here we consider the disk-shaped condensate with $\lambda\ll 1$. Ψ_j is the macroscopic wave function and $\int \mathrm{d}\boldsymbol{r}|\Psi_j|^2=N_j$ is the number of atoms for the j-th component, $N=\sum_j N_j$ is the total number of atoms. $U_{jl}=4\pi\hbar^2 a_{jl}/m$ is the interaction constant with m, the mass of atom in TBEC, a_{jl} $(j=l)$ and a_{jl} $(j\neq l)$ are negative scattering lengths corresponding to identical and different hyperfine state atomic interactions, respectively.

In order to consider the two-component condensate, generally one must distinguish the linear oscillator frequencies ω_1 and ω_2 corresponding to their different magnetic moments. This fact implies that two components are confined in two different traps, and their distributions are very different. We are currently interested in the TBEC with a thin disk-shaped trap, which means that the conditions $a_j\ll l_j$ can be satisfied, where $l_j=(4\pi n_j a_j)^{-1/2}$ is the healing length and $a_j=(\hbar/m\omega_j)^{1/2}$ is the linear oscillator length of the j-th component with $n_j=N_j/a_j^3$, the density of each component. It is clear that the dimensionless ratio λ is small enough and the densities of both components can be assumed to be low enough. In this situation one can use the multiple-scale expansion method to reduce the original 3D CGP equations into CNLS equations.[22] To do this, we first introduce the dimensionless variables $(X,Y,Z)=(x,y,z)a_1^{-1}$, $T=\omega_1 t/2$, and $\psi_j=(2a_1^3/N)^{1/2}\Psi_j$. By substituting these new variables into Eq.(1), we have

$$\begin{aligned}&\mathrm{i}\frac{\partial\psi_1}{\partial T}\\&=\left[-\nabla^2+V_1'(\boldsymbol{r}')+U_{11}'|\psi_1|^2+U_{12}'|\psi_2|^2\right]\psi_1, \quad (2)\\&\mathrm{i}\frac{\partial\psi_2}{\partial T}\\&=\left[-\nabla^2+V_2'(\boldsymbol{r}')+U_{21}'|\psi_1|^2+U_{22}'|\psi_2|^2\right]\psi_2, \quad (3)\end{aligned}$$

where $V_1'(\boldsymbol{r}')=[\lambda(X^2+Y^2)+Z^2]$ and $V_2'(\boldsymbol{r}')=\omega^2 V_1'(\boldsymbol{r}')$ with $\omega=\omega_2/\omega_1=(a_1/a_2)^2$, $U_{jj}'=4\pi N a_{jj}/a_1$ and $U_{ij}'=4\pi N a_{ij}/a_1$ $(i\neq j)$ are the dimensionless effective interaction constants of identical and different hyperfine state atomic collisions, respectively, which are both negative due to the attractive interaction.

As is well known, the intrinsic nonlinearity of a BEC can lead to mode interaction. Otherwise, because the small parameter for the system can be defined as $\epsilon=8\pi N_1\sqrt{\lambda}a_{11}/a_1\ll 1$, we can consider the nonlinearity to be weak enough, and the main state of the BEC as a weakly modulated ground state. In this situation, based on the multiple scale method, an effective 2D system of CNLS equations can be derived from the original 3D CGP equations. Before the multiple-scale expansion, we must solve the following two equations:

$$-\nabla^2 G_1+[\lambda^2(X^2+Y^2)+Z^2]G_1=\nu_1 G_1, \quad (4)$$

$$-\nabla^2 G_2+\omega^2[\lambda^2(X^2+Y^2)+Z^2]G_2=\nu_2 G_2. \quad (5)$$

Equations (4) and (5) are well-known eigenvalue problems of 3D quantum harmonic oscillator. Their normalized ground state solutions take the forms of

$$G_1=[\lambda^{1/4}/\pi^{3/4}]\exp\{-[\lambda(X^2+Y^2)+Z^2]/2\}, \quad (6)$$

$$G_2=[\lambda^{1/4}(\omega/\pi)^{3/4}]\exp\{-\omega[\lambda(X^2+Y^2)+Z^2]/2\} \quad (7)$$

with the eigenvalues $\nu_1 = \lambda + 2$ and $\nu_2 = \omega(\lambda + 2)$ respectively. To obtain useful CNLS equations from systems (2) and (3) by multiple-scale expansion, as the first step, we introduce the scaled variables $(\xi_i, \eta_i, \zeta_i) = \epsilon^i(X, Y, Z)$, and $\tau_i = \epsilon^i T$ $(i = 0, 1, 2, \ldots)$, which are taken to be independent of each other. Hence one has the following expansions:

$$\psi_j = \left[\frac{1}{|U_{11}'|\lambda^{1/2}}\right]^{1/2} \sum_{i=1}^{\infty} \epsilon^i \psi_{ji} \tag{8}$$

with $\psi_{j1} = P_j(\xi_1, \eta_1, \tau_2) G_j(\xi_0, \eta_0, \zeta_0) \exp(-\mathrm{i}\nu_j \tau_0)$, for $j = 1, 2$. It should be mentioned here that P_j represents the slow modulation of the ground state G_j due to the weak nonlinearity in the system. Inserting the above expansions into Eqs.(2) and (3), equating all terms at each order of ϵ, eliminating all secular terms, and letting $a_{12} = a_{21}$, one can obtain the following CNLS equations at the order of ϵ^3:

$$\begin{aligned} &\mathrm{i}\frac{\partial P_1}{\partial \tau_2} + \frac{\partial^2 P_1}{\partial \xi_1^2} + \frac{\partial^2 P_1}{\partial \eta_1^2} - \alpha_1 |P_1|^2 P_1 \\ &- \beta |P_2|^2 P_1 = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} &\mathrm{i}\frac{\partial P_2}{\partial \tau_2} + \frac{\partial^2 P_2}{\partial \xi_1^2} + \frac{\partial^2 P_2}{\partial \eta_1^2} - \beta |P_1|^2 P_2 \\ &- \alpha_2 |P_2|^2 P_2 = 0, \end{aligned} \tag{10}$$

where $\alpha_1 = -(2\pi)^{-3/2}$, $\alpha_2 = (\omega/2\pi)^{3/2}\, a_{22}/|a_{11}|$, and $\beta = [\omega/(\pi(1+\omega))]^{3/2}\, a_{21}/|a_{11}|$. It is noted that even for initially identical scattering lengths, the two components have different nonlinearities[22] because of different atomic densities of two components. Here we must know that in the case when the healing length becomes comparable with the latitude size of the BEC the previously developed method does not hold true any more.

3. Modulational instability analysis for TBEC

We now restrict our attention to the modulational instability of two constant-amplitude components of the TBEC against quasi-stationary density perturbations. Instead of a stationary solution, here we consider the dynamic solution of CNLS equations derived in the previous section. Accordingly, one can separate the amplitude into two parts

$$P_j = [P_{j0} + \varepsilon_j(\theta)] \exp(-\mathrm{i}\delta_j \tau_2), \tag{11}$$

where $j = 1, 2$, $\varepsilon_j \ll P_{j0}$, $\theta = K_\xi \xi_1 + K_\eta \eta_1 - \Omega \tau_2$ is the modulation phase with $\boldsymbol{K} = \boldsymbol{i}K_\xi + \boldsymbol{j}K_\eta$ $(K^2 = K_\xi^2 + K_\eta^2)$ and Ω being respectively, the wavenumber and the frequency of modulation, P_{j0} is the constant (real) amplitude, ε_j is the small amplitude perturbation, and δ_j is a nonlinear frequency shift. For convenience, we change the notations (ξ_1, η_1, τ_2) into (ξ, η, τ). By substituting expression (11) into Eqs.(9) and (10), we obtain

$$\delta_1 = \alpha_1 |P_{10}|^2 + \beta |P_{20}|^2, \tag{12}$$

$$\delta_2 = \alpha_2 |P_{20}|^2 + \beta |P_{10}|^2, \tag{13}$$

in the zeroth-order approximation of ε_j. At the next order of ε_j approximation, we obtain the following coupled equations:

$$\begin{aligned} &\mathrm{i}\frac{\partial \varepsilon_1}{\partial \tau} + \frac{\partial^2 \varepsilon_1}{\partial \xi^2} + \frac{\partial^2 \varepsilon_1}{\partial \eta^2} - \alpha_1 |P_{10}|^2 \tilde{\varepsilon}_1 \\ &- \gamma_c \beta |P_{20}|^2 \tilde{\varepsilon}_2 = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} &\mathrm{i}\frac{\partial \varepsilon_2}{\partial \tau} + \frac{\partial^2 \varepsilon_2}{\partial \xi^2} + \frac{\partial^2 \varepsilon_2}{\partial \eta^2} - \alpha_2 |P_{20}|^2 \tilde{\varepsilon}_2 \\ &- \gamma_c \beta |P_{10}|^2 \tilde{\varepsilon}_1 = 0, \end{aligned} \tag{15}$$

where $\tilde{\varepsilon}_j = \varepsilon_j + \varepsilon_j^*$ with ε_j^* being the conjugate complex of ε_j, and $\gamma_c = P_{10} P_{20}$ is the effective coupled constant. Introducing $\varepsilon_j = A_j + \mathrm{i}B_j$, substituting it into Eqs.(14) and (15), and separating the real and imaginary parts, yield

$$\frac{\partial A_j}{\partial \tau} + \frac{\partial^2 B_j}{\partial \xi^2} + \frac{\partial^2 B_j}{\partial \eta^2} = 0, \tag{16}$$

$$\begin{aligned} &\frac{\partial B_1}{\partial \tau} - \frac{\partial^2 A_1}{\partial \xi^2} - \frac{\partial^2 A_1}{\partial \eta^2} + 2\alpha_1 |P_{10}|^2 A_1 \\ &+ 2\gamma_c \beta A_2 = 0, \end{aligned} \tag{17}$$

$$\begin{aligned} &\frac{\partial B_2}{\partial \tau} - \frac{\partial^2 A_2}{\partial \xi^2} - \frac{\partial^2 A_2}{\partial \eta^2} + 2\alpha_2 |P_{20}|^2 A_2 \\ &+ 2\gamma_c \beta A_1 = 0. \end{aligned} \tag{18}$$

To solve the system of Eqs.(17) and (18), we assume that the amplitude perturbations $\varepsilon_j = A_j + \mathrm{i}B_j$ vary as $A_j = A_{j0} \exp[\mathrm{i}(K_\xi \xi + K_\eta \eta - \Omega\tau)]$ and $B_j = B_{j0} \exp[\mathrm{i}(K_\xi \xi + K_\eta \eta - \Omega\tau)]$. By substituting these expressions into Eqs.(17) and (18), and using the relations $B_{j0} = -\mathrm{i}\Omega A_{j0}/K^2$ derived from Eq.(16), we have

$$(\Omega^2 + K^4 + 2\alpha_1 K^2 |P_{10}|^2) A_1 + 2\gamma_c \beta K^2 A_2 = 0, \tag{19}$$

$$(\Omega^2 + K^4 + 2\alpha_2 K^2 |P_{20}|^2) A_2 + 2\gamma_c \beta K^2 A_1 = 0. \tag{20}$$

We can easily obtain the nonlinear dispersion relation for the amplitude modulation of the TBEC. For simplicity, we introduce the notations $\gamma_j = K^4 + 2\alpha_j K^2 |P_{j0}|^2$ with $j = 1, 2$ and $\gamma_c' = 2K^2 \beta \gamma_c$, then the dispersion relation can be written in the following form

$$(\Omega^2 + \gamma_1)(\Omega^2 + \gamma_2) + \gamma_c'^2 = 0. \tag{21}$$

4. Examples

In order to understand the details of modulational instability of the TBEC, we discuss and analyse two typical cases.

4.1. Case 1. $\gamma_c' = 0$

In the case $\gamma_c' = 0$, the dispersion relation has no coupled term. In other words, two components in the system are decoupling. This means that one can rewrite Eq.(21) as

$$(\Omega^2 + \gamma_1)(\Omega^2 + \gamma_2) = 0. \tag{22}$$

Indeed, two uncoupled components admit respective dispersion relation ($\Omega^2 = -\gamma_j^2$). It is noted from Eq.(22) that when $\gamma_j > 0$, then $\Omega^2 < 0$ for a small value of modulation wave number K. It suggests, for this case, that both uncoupled components of the TBEC are unstable for $|P_{j0}|^2 > -K^2/2\alpha_j$. Furthermore, the growth rate of each component is $\gamma_j^{1/2}$. In Figs.1 and 2, we choose the typical parameters of ^{23}Na to show the growth rate of components 1 and 2 for various parameter values. As anticipated, there appears sideband because of the modulational instability of both components in the TBEC.

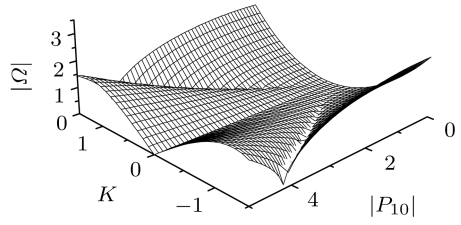


Fig.1. The modulational instability (MI) in component 1 of the TBEC for the zero-coupling case. The parameter values are $\omega = 1.4$, $N = 10^6$, $a_1 = 0.74$ μm, and $(a_{11}, a_{12}, a_{22}) = (-2.70, -2.71, -2.67)$ nm.

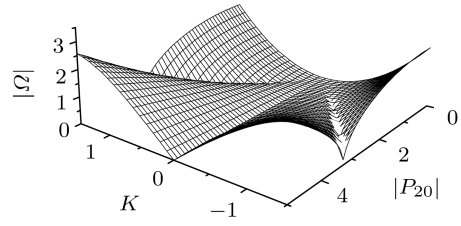


Fig.2. The MI in component 2 of the TBEC for the zero-coupling case. Parameter values are the same as those in Fig.1.

4.2. Case 2. $\gamma_c' \neq 0$

The zero-coupling is a simple and special case in our system, and in general, $\gamma_c' \neq 0$. This situation is our main problem of interest, for studying the coupling effects of two-component TBEC. We first assume $\Omega^2 = -\gamma$, and by inserting it into Eq.(21), one has

$$\gamma^2 - (\gamma_1 + \gamma_2)\gamma + \gamma_1\gamma_2 + \gamma_c' = 0. \tag{23}$$

Equation (23) is a well-known quadratic equation, and we can easily obtain its solution by using the concrete expressions of γ_j and γ_c', which are

$$\gamma = K^4 + K^2(\alpha_1|P_{10}|^2 + \alpha_2|P_{20}|^2) \pm C \tag{24}$$

with $C = K[K^2 - 4\beta K^2 P_{10}^2 P_{20}^2 + \alpha_1|P_{10}|^2 + \alpha_2|P_{20}|^2 - (K^2 + 2\alpha_1|P_{10}|^2)(K^2 + 2\alpha_2|P_{20}|^2)]^{1/2}$. It is concluded from expression (24) that for $K^4 + K^2(\alpha_1|P_{10}|^2 + \alpha_2|P_{20}|^2) \pm C > 0$ both coupled components of the TBEC are unstable. Under this condition, the growth rate of each component is $\gamma^{1/2}$. We find from both Figs.3 and 4 that there is an obvious sideband to compartmentalize the stable regions and unstable regions of the TBEC. To this end we can obtain the stability conditions for the above two cases, which can be written as follows:

$$|P_{j0}|^2 \leq -K^2/2\alpha_j, \tag{25}$$

$$K^4 + K^2(\alpha_1|P_{10}|^2 + \alpha_2|P_{20}|^2) \pm C \leq 0. \tag{26}$$

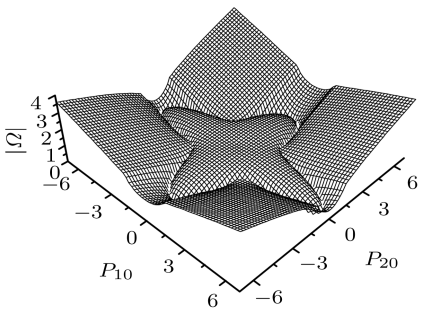


Fig.3. The MI of the TBEC in the (P_{10}, P_{20}) space for the nonzero-coupling case with $+C$. The parameter values are $\omega = 1.4$, $N = 10^6$, $a_1 = 0.74$ μm, $(a_{11}, a_{12}, a_{22}) = (-2.70, -2.71, -2.67)$ nm, and $K = 1$.

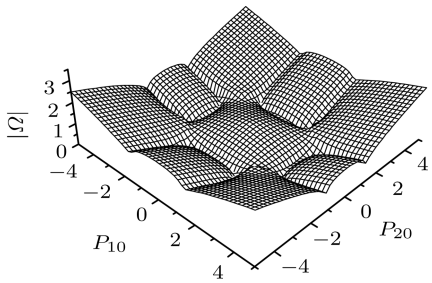


Fig.4. The MI of the TBEC for the nonzero-coupling case with $-C$, in the (P_{10}, P_{20}) space. Parameter values are the same as those in Fig.3.

One important issue that should be emphasized here is that in this letter, we investigate the self-attractive two-component condensate, thus in our models a_{11}, a_{22} and a_{12} are assumed to be negative. We compare our results with those in the 1D case[25] and find that for the zero-coupling case, the MI of the 1D TBEC and self-attractive 2D TBEC are very similar to each other. However, for the coupled case, due to the differences in atomic interaction and geometry, the MI of the TBEC becomes more complex in the 2D system, which is quite different from the 1D case. It means that the structure of the sideband becomes a little bit complicated (see in Figs.3 and 4).

5. Conclusion

We have studied the modulational instability of a self-attractive two-component BEC. The 2D CNLS equations have derived from original 3D CGP equations by using the multiple-scale method. Based on the CNLS equations, we discussed the zero-coupling and coupled cases and found distinct sideband region due to the modulation. For the first case, it is noted that if the components of the TBEC lie in the inside region of the curve $|\Omega| = 0$, they are both stable. Otherwise, they are unstable. For the latter, we predict that the modulationally unstable TBEC may generate periodic or localized solitions, and this is our future work.

References

[1] Davis K B, Mewes M O, Andrews M R, Van Druten N J, Durfee D S, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969
Bradley C C, Sackett C A, Tollett J J and Hulet R G 1995 *Phys. Rev. Lett.* **75** 1687

[2] Fu L B, Liu J and Ma Y 2006 *Acta Phys. Sin.* **55** 5623 (in Chinese)

[3] Li H M 2006 *Chin. Phys.* **15** 2216

[4] Huang G X 2004 *Chin. Phys.* **13** 1866

[5] Li S C and Duan W S 2008 *Euro. Phys. J.* B **62** 485

[6] Dutton Z, Budde M, Slowe C and Hau L V 2001 *Science* **293** 663

[7] Khaykovich L, Schreck F, Ferrari G and Bourdel T 2002 *Science* **296** 1290

[8] Strecker K E, Partridge G B, Truscott A G and Hulet Randall G 2002 *Nature* (London) **417** 150

[9] Liang Z X, Zhang Z D and Liu W M 2005 *Phys. Rev. Lett.* **94** 050402

[10] Liu J, Zhang C, Raizen M G and Niu Q 2006 *Phys. Rev.* A **73** 013601

[11] Huang G X, Makarov V A and Velarde M G 2003 *Phys. Rev.* A **67** 023604

[12] Duan W S, Chen J H and Yang H J 2006 *Chin. Phys. Lett.* **23** 2000

[13] Reinhardt W P and Clark C W 1997 *J. Phys.* B **30** L785

[14] Scott T F, Ballagh R J and Burnett K 1998 *J. Phys.* B **31** L329

[15] Wei W 2005 *Chin. Phys.* **14** 2407

[16] Zhang G F, Yin W, Liang J Q and Yan Q W 2004 *Chin. Phys.* **13** 988

[17] Xie B H and Jing H 2002 *Chin. Phys.* **11** 115

[18] Myatt C J, Burt E A, Ghrist R W, Comell E A and Wieman C E 1997 *Phys. Rev. Lett.* **78** 586

[19] Stamper-Kurn D M, Andrews M R, Chikkatur A P, Inouye S, Miesner H J, Stenger J and Ketterle W 1998 *Phys. Rev. Lett.* **80** 2027

[20] Modugno M, Dalfovo F, Fort C, Maddaloni P and Minardi F 2000 *Phys. Rev.* A **62** 063607

[21] Busch T and Anglin J R 2001 *Phys. Rev. Lett.* **87** 010401

[22] Kostov N A, Enol'skii V Z, Gerdjikov V S, Konotop V V and Salerno M 2004 *Phys. Rev.* E **70** 056617

[23] Brazhnyi N A and Konotop V V 2005 *Phys. Rev.* E **72** 026616

[24] Chen H F, Huang J S and Xie Z W 2008 *Acta Phys. Sin.* **57** 3435 (in Chinese)

[25] Raju T S, Panigrahi P K and Porsezian K 2005 *Phys. Rev.* A **71** 035601

[26] Li L, Malomed B A, Mihalache D and Liu W M 2006 *Phys. Rev.* E **73** 066610

[27] Kagan Y, Muryshev A E and Shlyapnikov G V 1998 *Phys. Rev. Lett.* **81** 933

[28] Gammal A, Frederico T, Tomio L and Chomaz Ph 2000 *Phys. Rev.* A **61** 051602

[29] Bradley C C, Sackett C A, Tolett J J and Hulet R G 1995 *Phys. Rev. Lett.* **75** 1687

[30] Sackett C, Stoof H T C and Hulet R G 1998 *Phys. Rev. Lett.* **80** 2031

[31] Goldstein E V and Meystre P 1997 *Phys. Rev.* A **55** 2935

[32] Adhikari S K 2001 *Phys. Lett.* A **281** 265

[33] Timmermans E 1998 *Phys. Rev. Lett.* **81** 5718