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Effect of road structure on the capacity of a signalized road intersection*

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In this paper, we use the stochastic Nagel–Schreckenberg (NaSch) model to investigate the influence of a special right-turning lane connecting two main roads on the capacity of a signalized road intersection. It is found that the magnitude of right-turning traffic flow and the linking position of the special right-turning lane can greatly influence the capacity of the signalized road intersection. The relation between traffic flow and entering probability for different distances between the entrance (exit) of the special right-turning lane and the road intersection is simulated and analysed. The corresponding spatiotemporal pattern and phase diagram on different sections of the main road are given under the condition of close proximity to the signalized road intersection, stop-and-go traffic occur and obstruct the intersection. On the contrary, unchanged flux is maintained as the distance exceeds a critical values. All the studies indicate that setting a special right-turning lane by choosing a suitable location near a signalized road intersection can relieve the load of current traffic on the main road and maintain traffic flow.

Keywords: traffic flow, cellular automaton, phase diagram, road intersection
PACC: 0500, 0520, 0550, 6470

1. Introduction

In recent years there has been considerable interest in traffic flow because of traffic congestions, accidents and environmental impacts occur gradually with increasing demands for movement. The observed complicated nonlinear dynamic phenomena have received much attention from scientists.[1,2] Therefore, for a better understanding of such complex phenomena in traffic, various traffic models such as cellular automation (CA), car-following, gas-kinetic, hydrodynamic models, etc., have been developed[2,3] and many empirical observations have been continuously reported.[4,5] Particularly the CA models have been extensively studied to understand traffic behaviours, due to its simplicity and flexibility in modeling different aspects of real traffic. The typical CA model is the NaSch model proposed by Nagel and Schreckenberg in 1992.[6]

The road intersection is a key ingredient in the management of traffic systems. The study of the road intersection is of significance for making a scientific suggestion of real traffic management and developing traffic theory. Brockfeld *et al* have studied optimizing traffic lights for city traffic.[7] Huang *et al* adopted the NaSch model to study the traffic flow controlled by traffic lights.[8] The effects of traffic flow condition of road on capacity of T-shaped the crossroad system were analysed by Wu *et al.*[9] Jiang and Wu developed a stopping time-dependent randomization the CA model to explain the dependence of saturated current on cycle time.[10] The phase transition of traffic flow on the main road was analysed by Guo *et al.*[11] Recently, Zhao *et al*[12] have investigated the combined effects of the bus stop upstream and downstream and the signal-controlled crossroad. A traffic flow model with considerable signal light influence was proposed by Huang *et al.*[13] Many studies of the traffic behaviours controlled by traffic lights mainly focus on the effects of different traffic policies on the traffic capacity, few studies consider their interference in the

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structure of the road, like a special right-turning lane as shown in Fig.1. More recently, Liu and Chen separately discussed this problem[14−16] by using a hydrodynamic traffic model. The special right-turning lane can play an important part in separating flow to avoid the conflict between the right-turning flow and one on the main road, as compared with the conventional right-turning flow at a signalized road intersection. That is to say, the previous conflict point at the road intersection between the main flow and the right-turning current will be shifted to the road intersection downstream, where there exists an interaction between the signalized road intersection and the special right-turning lane downstream (see Fig.1). Obviously, this separated flow is beneficial to relieving the current load on the main road.

In the present paper, we will discuss the effects of the road structure on the capacity of the road intersection using a CA traffic model from the microscopic point of view. In Section 2, a traffic model for simulation is constructed via the layout of the structure of the road intersection. In Section 3, we carry out the simulation and analysis. Finally, the conclusion is given in Section 4.

2. Structure of the road intersection and model

There widely exist road intersections in city traffic systems over the world. In urban the traffic of China, some road intersections are set up to link several special right-turning lanes to relieve load of current on main road, like Fig.1. Generally speaking, a road intersection is set to link four special right-turning lanes. For simplicity and symmetric consideration, we assume that a signalized road intersection is linked with only one special right-turning lane. Moreover, the road intersection on one cell C_0 is composed of two crossed single lanes including roads A and B. The road intersection consists of five sections A_1, A_2, B_1, B_2 and C in Fig.1. Section A_1 (A_2) is the upstream (downstream) of road A. Section B_1 (B_2) is the upstream (downstream) of road B. Section C is the special right-turning lane. To simplify the problem, the entrance A_0 and the exit B_0 of section C occupy only one cell, the distance between A_0 (B_0) and C_0 is denoted as L_a (L_b).

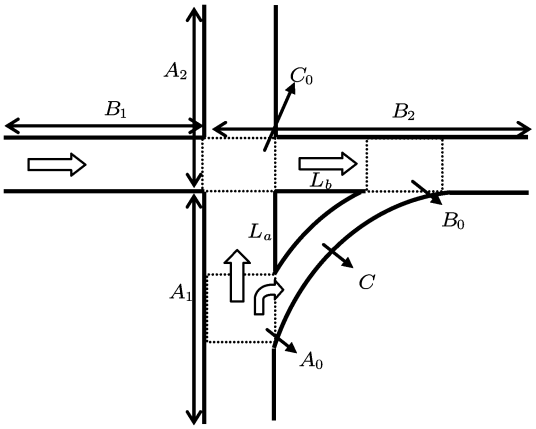


Fig.1. Structure of the road intersection and main road in the simulation.

There are two types of motions of vehicles, one is to move straight ahead and the other is to turn right. The right-turning vehicles enter road A in proportion to $P_{\rm r}$, which is the probability of right-turning vehicles on road A. The cycle time of the traffic light is denoted as T and the green-light time approximately equals the red-light time. We use the NaSch model to simulate traffic on the urban road intersection with a special right-turning lane. The update rules in the NaSch model are as follows:

(i) acceleration: $v_n \to \min(v_n + 1, v_{\max})$,

(ii) deceleration: $v_n \to \min(v_n, {\rm gap}_n)$,

(iii) randomization with probability p : $v_n \to \max(v_n - 1, 0)$,

(iv) motion: $x_n \to x_n + v_n$.

Here, x_n and v_n are the position and the speed of vehicle n, ${\rm gap}_n = x_{n+1} - x_n - 1$ denotes the number of empty cells in front of vehicle n, and x_{n+1} is the position of vehicle $n + 1$. When the traffic light is red, the gap to the nearest vehicle behind the intersection should be calculated as ${\rm gap}_n = x_{\rm light} - x_n$, where $x_{\rm light}$ is the position of the intersection. The maximum speed and the randomization probability are denoted as $v_{\max}$ and p respectively.

The nearest vehicle on section A_1 behind the cell A_0, the nearest vehicle on sections B_2 and C behind the cell B_0 are denoted as A_{latest}, B_{latest} and C_{latest}, respectively. The nearest vehicle on section B_2 before the cell B_0 is B_{before}. Thus, we first examine whether the vehicle A_{latest} is the right-turning vehicle or not in a time step. If it is, then its gap should be calculated as $\text{gap}_n = x_{C_{\text{last}}} - x_n - 1$, where $x_{C_{\text{last}}}$ is the position of last vehicle on section C. Next, we examine whether the vehicle B_{latest} can arrive at cell B_0 or not. If it cannot, then it is obvious that the update of vehicles on section B_2 is not affected by those vehicles on section C, and also the update of vehicles on section C is not affected by those vehicles on section B_2 in this time step. Similarly, if the vehicle C_{latest} cannot arrive at cell B_0 in one time step, the updates of vehicles on both sections B_2 and C are not affected by each other in this time step.

Thus, when both vehicles B_{latest} and C_{latest} can simultaneously arrive at B_0 in one time step, we can calculate the times t_b and t_c needed for vehicles B_{latest} and C_{latest} to arrive at B_0 in the light of the Rui's method:[17]

$$t_b = \frac{x_{B_0} - x_{B_{\text{latest}}}}{\min(v_{\max}, x_{B_{\text{before}}} - x_{B_{\text{latest}}} - 1, v_{B_{\text{latest}}} + 1)}, \tag{1}$$

$$t_c = \frac{L_C - x_{C_{\text{latest}}} + 1}{\min(v_{\max}, L_C - x_{C_{\text{latest}}} + x_{B_{\text{before}}} - x_{B_0}, v_{C_{\text{latest}}} + 1)}, \tag{2}$$

where x denotes the position of the vehicle or the cell, v denotes the speed of the vehicle and L_C denotes the length of section C.

If $t_b < t_c$, it is assumed that vehicle B_{latest} has the priority to occupy cell B_0. For this case, the update of vehicles on section B_2 will not be affected by the vehicles on section C in this time step. However, the update of vehicles on section C will be affected by vehicles on section B_2. To implement the update, two substeps are classified: i) the update of vehicles on sections A_1, A_2, B_1 and B_2 is as usual and the vehicle B_{latest} becomes the new vehicle B_{before}; ii) the update of vehicles takes place on section C. For $t_b > t_c$, the situation is similar to that for $t_b < t_c$, just the roles of sections B and C interchange.

As for $t_b = t_c$, it is reasonable to endow the vehicle nearer to the cell B_0 (provided that the distances to B_0 are different for vehicles B_{latest} and C_{latest}) with top-priority, and the problem becomes the same as that in the case $t_b > t_c$ or $t_b < t_c$. Finally, if $t_b = t_c$ and the distances to B_0 are the same for vehicles B_{latest} and C_{latest}, a reasonable solution is to endow the vehicle B_{latest} with priority because it is straightforward vehicle. The situation turns out to be the same as that for $t_b < t_c$.

The boundary conditions are adopted as follows. With top-priority sections A_1 and B_1 corresponding to x=1 and the entrance of sections A_1 and B_1 include $v_{\max}$ cells, i.e. the vehicles can enter into sections A_1 and B_1 from cell 1. In one time step, when the update of vehicles on the road is finished, we check the positions of the last vehicles on sections A_1 and B_1 and that of the leading vehicles on sections A_2 and B_2, which are denoted as $x_{A_{\text{last}}}$, $x_{B_{\text{last}}}$, $x_{A_{\text{lead}}}$ and $x_{B_{\text{lead}}}$, respectively. If $x_{A_{\text{last}}}$ ($x_{B_{\text{last}}}$) $> v_{\max}$, a vehicle with speed $v_{\max}$ is injected with probability $p_a(p_b)$ at cell 1. Near the exit of the section A_2 and B_2, the leading vehicles are removed if $x_{A_{\text{lead}}}$ ($x_{B_{\text{lead}}}$) $> L$, where L denotes the position of the right cells on sections A_2 and B_2, and the following vehicles become the new leading vehicles and they move without any hindrance.

3. Simulation results and analysis

In simulation, the sections A_1, A_2, B_1 and B_2 are divided into L=1000 cells each. Section C contains the integer of $L_C = \sqrt{L_a^2 + L_b^2}$ cells. Each cell corresponding to an actual road length of 7.5 m is either empty or occupied by one vehicle and each time step corresponds to 1 s. The model parameters are set as follows: $v_{\max}$=5, p=0.25, and T=120 s. The removal rates on the right boundary of sections A_2 and B_2 are 1 and the injection rates on the left boundary of sections A_1 and B_1 are p_a and p_b respectively. In simulation, the first 4×10^4 time steps are discarded to let the transient time die out. The current is obtained by counting the vehicles that pass through a

virtual detector in 1×10^5 time steps. The traffic flow diagrams are obtained by averaging over 100 runs of simulations.

First, we study the effect of different right-turning probabilities for small L_a and L_b near the road intersection. The right-turning probability is $P_{\rm r}$ and other parameters are chosen as follows: $L_a = L_b$=5, p_a=0.8, $P_{\rm r}$=0, 0.2, 0.5, 1.0. The flows of the upstream road B (section B_1) versus the entering probability p_b for different values of $P_{\rm r}$ are shown in Fig.2. It can be seen that the flow increases with entering probability p_b until it reaches an unchanged saturated value like a platform. The saturated flow of section B_1 declines with the increase of the right-turning probability $P_{\rm r}$ and approaches a minimum at a maximum value $P_{\rm r}$ =1. That means the right-turning vehicles can exert a hindrance effect on the traffic of the intersection. The larger the value of $P_{\rm r}$ is, the stronger the hindrance effect will be. From the spatiotemporal pattern in Fig.3, it is obvious that the waiting queue on section B_1 grows, stretching to the farther position due to the right-turning flow. The queue of vehicles on the road upstream of the intersection in the red-light period will mix with the coming vehicles and cannot completely be emptied in the next green-light period. Therefore, the capacity of section B_1 descends with the increase of the right-turning probability $P_{\rm r}$.

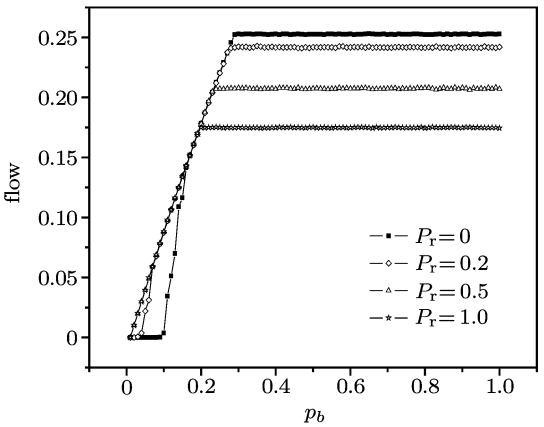


Fig.2. Relationship between flow of section B_1 and entering probability p_b for different values of right-turning probability $P_{\rm r}$ of vehicles.

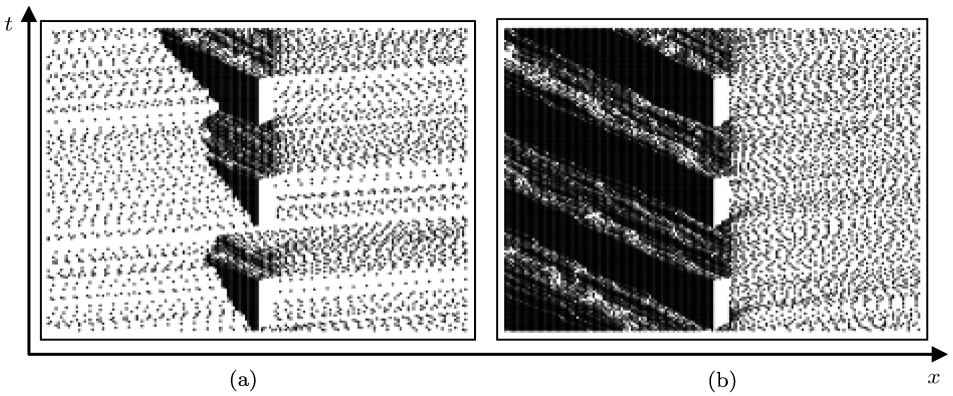


Fig.3. Spatiotemporal patterns for road B (include upstream and downstream road B) where black points denote occupied cells and white points denote empty cells (a) $P_{\rm r}$=0.5 and (b) $P_{\rm r}$=1.0.

In order to solve the above problem, the special right-turning lane C is built to avoid the conflict with one on the main road at the road intersection in Fig.1. We investigate how the special right-turning lane affects the road intersection traffic by simulation. This simulation is divided into three parts as follows:

The first part is to investigate the capacity of section B_1 again by choosing different locations for the special right-turning lane C and parameters: $P_{\rm r}$=0.5, p_a=0.8, L_a=50, L_b=5, 15, 25, 35, 50, 80. The relationship between the traffic flow for section B_1 and the entering probability p_b is shown in Fig.4. From it we can see that the traffic flow of section B_1 increases with the entering probability p_b until the flow reaches an unchanged saturated value. For different values of distance L_b, the saturated value increases and ap-

proaches an unchanged maximum value. That means if L_b is large enough, the special right-turning lane would not exert the hindrance effect on the road intersection, which is coincident with the result of Ref.[14]. It can be demonstrated from the corresponding spatiotemporal pattern and the phase diagram (p_b, L_b) in Figs.5 and 6, respectively. From the spatiotemporal pattern in Fig.5, it can be found that the vehicles waiting for the green light to turn on will be evacuated quickly for a large enough distance L_b. The phase diagram in Fig.6 can be divided into two regions. Region I denotes traffic free while region II is traffic jamming.

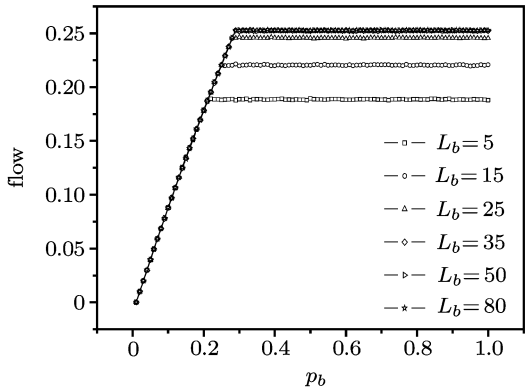


Fig.4. Relationship between flow of section B_1 and entering probability p_b for different values of L_b.

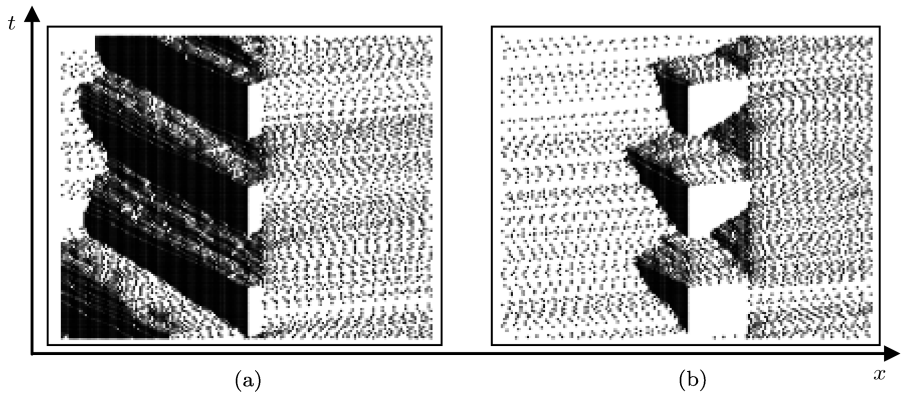


Fig.5. Spatiotemporal patterns for road B upstream and downstream, where black points denote the occupied cells and white points denote empty cells (a) L_b=5 and (b) L_b=25.

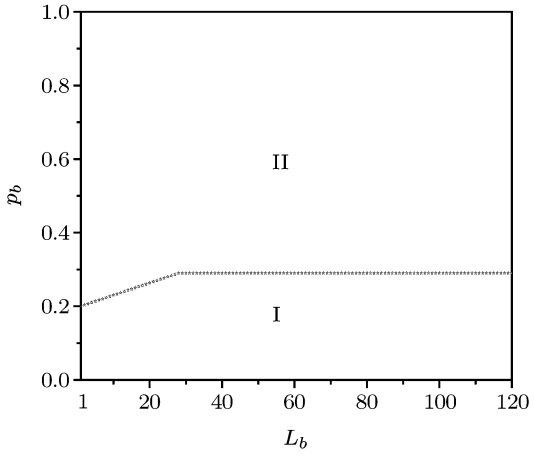


Fig.6. Phase diagram in (p_b, L_b) space for section B_1.

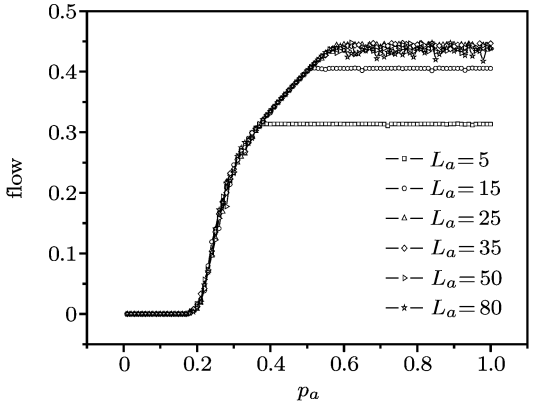


Fig.7. Relationship between flow of section A_1 and entering probability p_a for different values of L_a.

The second one is to investigate the traffic state of section A_1 and its parameters are P_r=0.5, p_b=0.8, L_b=50, L_a=5, 15, 25, 35, 50, 80. The relationship between the traffic flow for section A_1 and the entering probability p_a shown in Fig.7 is similar to that between the flow for section B_1 and the entering probability p_b. The corresponding spatiotemporal pattern and the phase diagram (p_a, L_a) are shown in Figs.8 and 9,

respectively. It is very obvious to show the stop-and-go traffic in Fig.8. The stop-and-go traffic lasts for a long time due to the small distance L_a=5. Conversely, the stop-and-go traffic will fast disperse in the case of distance L_a=80. The waiting queue on section A_1 in Fig.8 grows, stretching to the farther position due to the entry of the special right-turning lane close to the intersection. The length of the queue is longer than the distance L_a in the red-light period and the right-turning vehicles cannot move in this period, thereby the flow for small L_a is lower than that for larger L_a. From the phase diagram in Fig. 9, we can see that there are four regions in the plot. The traffic state on the road is free flowing in regions I, II, III while it is a traffic jam in region IV. The flow in region I approximately equals 0 due to the small entering probability p_a. The flow in region II increases more tempestuous than that in region III with the entering probability.

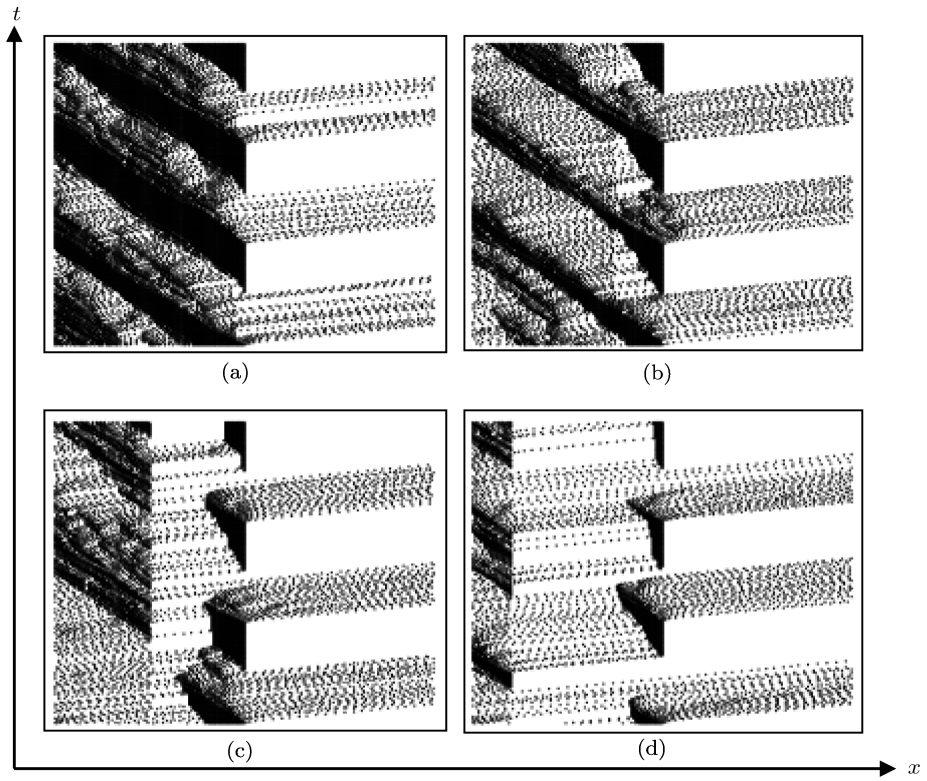


Fig.8. Spatiotemporal patterns for road A upstream and downstream, where black points denote occupied cells and white points denote the empty cells for L_a=5 (a), 25 (b), 50 (c) and 80 (d).

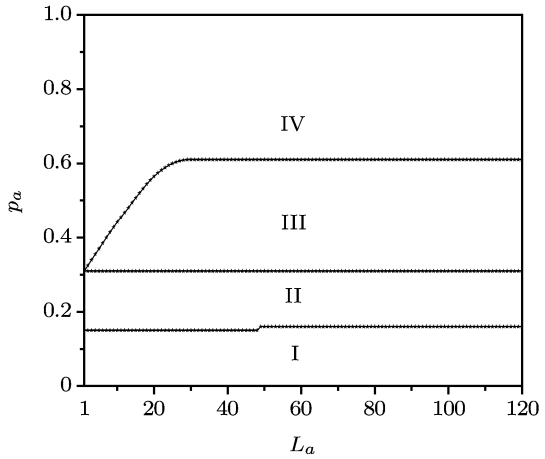


Fig.9. Phase diagram in (p_a, L_a) space for section A_1.

The final section B_2 is quite complicated because it is the location where the flow of vehicles on the main road intersects with the right-turning flow downstream. Considering the positional effect of the special right-turning lane C on traffic on the main road, the parameters are chosen as 1) P_r=0.5, p_a=0.8, L_b=50, L_a=5, 15, 25, 35, 50, 80; 2) P_r=0.5, p_a=0.8, L_a=50,L_b=5, 15, 25, 35, 50, 80 and the simulation results are shown in Figs.10 and 11, respectively. The traffic flow in Fig.10 shows that the relationships between the flow for section B_2 and the entering probability p_b for different values of L_a or L_b are similar to those of the sections A_1 and B_1 in Fig.4 and Fig.7 respectively. These simulation results can also indicate that the increase of distance L_a or L_b can reduce the hindrance effects when L_a or L_b is a small value and

even eliminate the hindrance effects when L_a or L_b is a large value. The phase diagrams for section B_2 in (p_b, L_a) and (p_b, L_b) space in Fig.11 can also illustrate these results. Both spaces can be divided into two regions. The region I shows a free flow while region II indicates a congested state in each phase diagram.

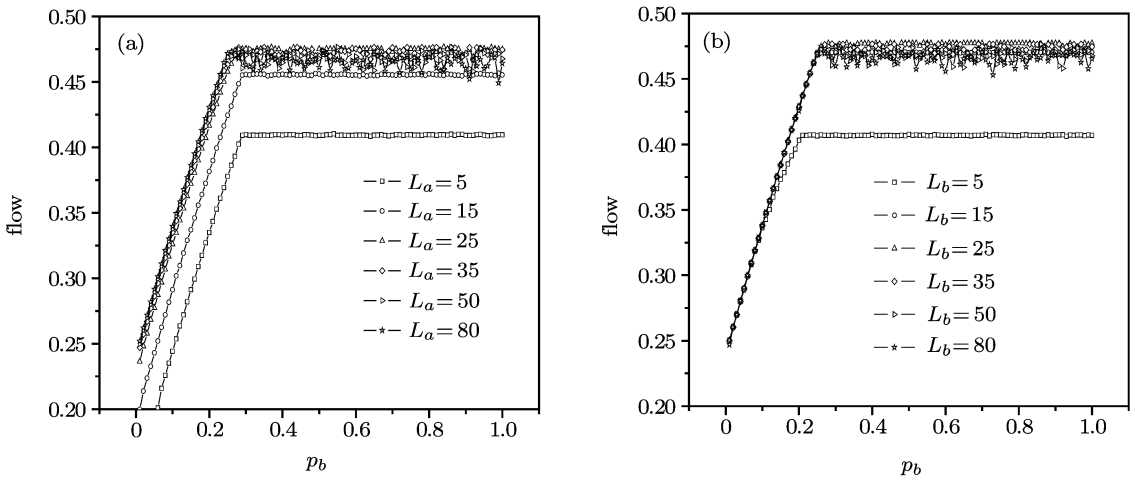


Fig.10. Relationship between flow of section B_2 and entering probability p_b for different values of (a) L_a and (b) L_b.

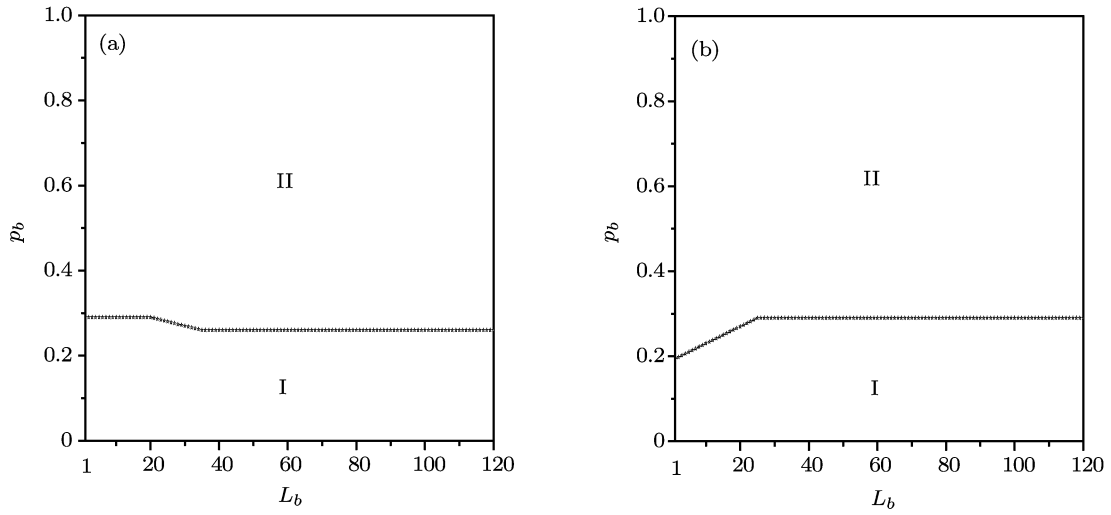


Fig.11. Phase diagrams in (p_b, L_a) space (a) and in (p_b, L_b) space (b) for section B_2.

4. Conclusions

In traffic flow researches, the urban road intersection with special right-turning lanes is of particular interest and significance. In this paper, we study how the structure of the road, such as a special right-turning lane connecting two main roads, affects traffic near a signalized road intersection by using a CA model.

The magnitude of right-turning traffic flow and the position of the special right-turning lane can affect the capacity of signalized road intersection. The traffic flow versus the entering probability p_a (p_b) for different values of distance L_a (L_b) is obtained. The corresponding spatiotemporal pattern and the phase

diagram on different sections of the main road are also yielded. The simulation results and the simulation analyses indicate that the special right-turning lane can relieve the load at the road intersection by separating the flow on the main road. Moreover, the capacities of different regions on the main road are affected by the positions of the special right-turning lane upstream and downstream. Increasing the distance L_a or L_b near the road intersection will induce the gradual growths of the flow on sections A_1, B_1 and B_2. When the distance L_a (L_b) exceeds a critical value, the flows on different sections reach a saturated value and then remain unchanged, which is confirmed by the simulations of the hydrodynamic traffic model and empirical data on field.[14−16] Conversely, the smaller distance L_a (L_b) is not enough to keep traffic free and induce the stop-and-go traffic. Certainly, the larger distance L_a (L_b) can play a part in maintaining the free-flowing of vehicles, but greatly occupy land resource. It is very obvious that setting a reasonable right-turning lane near the road intersection is superior to the conventional right-turning flow at the road intersection. Therefore, these results may be useful for setting the appropriate structure for the road intersection and offering scientific guidance for the design of traffic roads.

But, it must be mentioned that in the present work only the simplest situation is considered, in which only one special right-turning lane connects to the intersection downstream. In reality, each road intersection downstream often links four special right-turning lanes and the main road is multi-lane. The simulation and the analysis of this real traffic is our next study.

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