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Asymptotic quasinormal modes of scalar field in a gravity's rainbow*

Liu Cheng-Zhou(刘成周)[a] and Zhu Jian-Yang(朱建阳)[b]†

[a] *Department of Physics, Shaoxing College of Art and Science, Shaoxing* 312000, *China*

[b] *Department of Physics, Beijing Normal University, Beijing* 100875, *China*

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In the framework of the gravity's rainbow, the asymptotic quasinormal modes of the modified Schwarzschild black holes undergoing a scalar perturbation are investigated. By using the monodromy method, we analytically calculated the asymptotic quasinormal frequencies, which depend on not only the mass parameter of the black hole, but also the particle's energy of the perturbation field. Meanwhile, the real parts of the asymptotic quasinormal modes can be expressed as $T_{\rm H}\ln 3$, which is consistent with Hod's conjecture. In addition, for the quantum corrected black hole, the area spacing is independent of the particle's energy, even though the area itself depends on the particle's energy. And that, by relating the area spectrum to loop quantum gravity, the Barbero-Immirzi parameter is given and it remains the same as from the usual black hole.

Keywords: gravity's rainbow, quasinormal modes, monodromy method, quantum effect
PACC: 0470, 0430

1. Introduction

As is well known, the response of the black hole background to the initial configuration is dominated by the quasinormal modes (QNMs) - a set of complex frequencies which only depend on the parameters of the black hole rather than the initial perturbation (see Refs.[1] and [2] for reviews). The characterization of QNMs could play an important role in the astronomic observations of the black hole in the universe. Meanwhile, the properties of QNMs can be used to investigate the black hole stability. In addition, QNMs have got an interpretation in conformal field theory.[3−8] And, the asymptotic QNMs can be related to the quantum theory of spacetime.[9,10] Motivated by Hod's conjecture,[9] it is suggested that the value of asymptotic QNMs may carry important information about the quantization of the black hole area and help to fix certain parameters in a quantum gravity theory.[10] Thus, QNMs for different black holes have been studied extensively (see, for example, Refs.[11]–[16] and their references).

Based on the numerically results of QNMs,[17] Hod suggested that the real parts of highly damped QNMs in the Schwarzschild black hole can be expressed as $\omega_{\rm R} = T_{\rm H}\ln 3$.[9] And that, by using the Bohr's correspondence principle, the real part of the asymptotic QNMs could be identified as the characteristic transition frequency of black holes and the fundamental quanta of the black hole area as $l_{\rm p}^2 4\ln 3$ can be obtained.[9,10] Alternatively, as a main candidate theory of quantum gravity, loop quantum gravity (LQG) (see Refs.[18] and [19] for reviews) has a remarkable result: the discrete area spectrum. However, there is an unknown natural constant called the Barbero-Immirzi parameter and the area spectrum has an ambiguity. By using the area quanta of black holes, Dreyer presented the free parameter and proposed that LQG should be based on SO(3) rather than SU(2).[10] Thus, one hopes that the asymptotic QNMs will have a role in the quest for quantum geometry, especially for the quantum properties of black holes.

For analytically computing the asymptotic value of QNMs and confirming Hod's conjecture, the monodromy method of investigating quasinormal modes was proposed.[20,21] In the method, the radial coordinate is analytically continued into the complex plane and the properties of the potential and tortoise coor-

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†Corresponding author. E-mail: zhujy@bnu.edu.cn

dinate around the origin, the horizons and infinity are used.[17] The obtained asymptotic QNMs present an analytical proof on Hod's conjecture for the scalar field and some gravitational perturbations in Schwarzschild black hole. In the Refs.[22]–[30], the monodromy technique has been extended to many cases such as different perturbations like the Dirac field and the electromagnetic field in more general static black holes and the asymptotic QNMs have been obtained. However, the researches imply that whether the argument that $\omega_{\rm R} = T_{\rm H} \ln 3$ applies to universal cases is still an open problem and much less attention has been paid to the case of analytically calculating the asymptotic QNMs in the quantum corrected black hole.[28]

It is believed that, at Planck scale, the quantum effect of spacetime will emerge and a new physics will be necessary. In Refs.[31] and [32], the Planck scale corrected spacetime named as gravity's rainbow has been presented. The main feature of the gravity's rainbow is that the spacetime geometry depends on the energy of particles moving in it. That is to say, for the quantum corrected spacetime, there are different geometries for probe particles with different energies. The modified geometries can be described by one parameter family of metric as a function of the particle's energy observed by an inertial observer. Moreover, the modified Schwarzschild solution in the gravity's rainbow has been given.[32] Its asymptotic flatness and some thermodynamics quantities have been investigated.[33,34]

In this paper, we shall investigate the asymptotic QNMs of a massless scalar field in the modified Schwarzschild spacetime from the gravity's rainbow. Our main aim is to verify the influence of the spacetime's quantum effects on the asymptotic QNMs. Moreover, in the quantum corrected spacetime, we want to test Hod's conjecture and verify its implication for the quantum theory of spacetime. By using the monodromy method, we analytically calculate the asymptotic quasinormal frequencies in the quantum modified black hole. The results show that, when the quantum effect of spacetime is taken into account, the asymptotic QNMs depend on not only the mass parameter of the black hole but also the energy of the perturbation field. And in the gravity's rainbow, the real part of the asymptotic QNMs can be expressed as $T_{\rm H} \ln 3$ and hence Hod's conjecture is valid. In addition, as in usual spactime,[9,10] the area spacing of the modified black hole is calculated and the free parameter of LQG is presented. The obtained results are independent of the particle's energy and remain the same as those from the usual Schwarzschild black hole.[9,10]

The paper is organized as follows. Section 2, the modified Schwarzschild solution from the gravity's rainbow is introduced briefly. Then in Section 3, in the monodromy framework, the asymptotic QNMs of the massless scalar field in the modified spacetime are analytically calculated. In Section 4, for the quantum corrected black hole, Hod's conjecture and its implication for the quantum theory of black holes and LQG are investigated. The last Section is the summary and conclusions.

2. Gravity's rainbow and modified Schwarzschild black holes

Let us firstly briefly introduce the modified Schwarzschild solution from the gravity's rainbow. When we keep Planck energy as an invariant scale, namely a universal constant for all inertial observers, to preserve the relativity of inertial frames, the double special relativity (DSR) as a deformed special relativity has been proposed.[35−39] The starting point of the main result of DSR is the modified dispersion relation (MDR)

$$E^2 f_1^2 (E;\lambda) - p^2 f_2^2 (E;\lambda) = m_0^2, \tag{1}$$

where f_1 and f_2 are two energy functions from which rotational symmetry can be preserved, λ is a parameter of the order of the Planck scale. Equation (1) shows that, MDR is energy-dependent. That is to say, particles with different energies have different energy-momentum relations. However, in the low energy realm, i.e. $E/E_{\rm p} \ll 1$, f_1 and f_2 approach to unit and MDR can return to the usual energy-momentum relation, where $E_{\rm p} \equiv 1/\sqrt{8\pi G}$ is the Planck energy. This is consistent with Bohr's correspondence principle.

In the context of DSR, the deformed spacetime geometry has been investigated and some proposals have been presented.[31,32,40−43] In Refs.[31] and [32], the authors put forward that the flat spacetime with Planck scale corrections has the invariant as shown below

$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{f_1^2} + \frac{\mathrm{d}r^2}{f_2^2} + \frac{r^2}{f_2^2}\mathrm{d}\Omega^2. \tag{2}$$

This indicates that, the DSR spacetime depends on the energy of particles moving in it. That is to

say, particles with different energies will probe different DSR spacetimes. Thus, the DSR spacetimes are endowed with an energy-dependent quadratic invariant, i.e., an energy-dependent metric, namely rainbow metric. Just like DSR, when particle's energy is a little quantity in comparison with the Planck energy, the rainbow metric will turn to the usual flat spacetimes. And it has been pointed out that the DSR spacetimes have equality with the usual flat spacetimes.[34]

By extending Eq.(2) to incorporate curvature, in Ref.[32], a gravity's rainbow has been presented and a few quantum corrected spacetimes from the gravity's rainbow have been given. Thereamong, the modified Schwarzschild solution can be expressed in terms of energy-independent coordinates and the energy-independent mass parameter as

$$\mathrm{d}S^2 = -\frac{\left(1-\frac{2GM}{r}\right)}{f_1^2}\mathrm{d}t^2 + \frac{1}{f_2^2\left(1-\frac{2GM}{r}\right)}\mathrm{d}r^2 + \frac{r^2}{f_2^2}\mathrm{d}\Omega^2. \tag{3}$$

Obviously, the metric of the modified Schwarzschild black hole is also energy-dependent. That is, if a given observer probes the spacetime using the quanta with different energies, he will conclude that spacetime geometries have different effective descriptions. Here, the particle's energy E denotes its total energy measured at infinity from the black hole. In addition, the energy dependence of the gravity's rainbow probably may have some relation to the background-independent property of LQG.[18,19] In fact, as a semi-classical limit, the MDR has been suggested by LQG.[44−47] In a word, as a gravity's rainbow, the modified Schwarzschild spacetime is endowed with Planck scale modification, i.e., quantum correction.

3. Asymptotic quasinormal modes

By using the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x_\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x_\nu}\right)\phi = 0, \tag{4}$$

and the ansatz of separation of variable $\phi(t,r,\theta,\varphi) = (1/r)\mathrm{e}^{\mathrm{i}\,\omega t}\psi(r)\mathrm{Y}(\theta,\varphi)$, the radial perturbation equation for the scalar field ϕ in the quantum corrected spacetime (3) can be given as

$$\left(r^2-2GMr\right)\frac{\partial^2\psi(r)}{\partial r^2} + 2GM\frac{\partial\psi(r)}{\partial r} + \left[r^2\frac{f_2^2}{f_1^2}\left(1-\frac{2GM}{r}\right)^{-1}\omega^2 - \frac{2GM}{r^3} - \frac{l\,(l+1)}{r^2}\right]\psi(r) = 0, \tag{5}$$

where l is the orbital angular momentum.

For convenience, we define the tortoise coordinate for the quantum modified black hole as

$$x = \int\frac{\mathrm{d}r}{h\,(r)} = \frac{f_1}{f_2}\left[r + 2GM\ln\left(\frac{r}{2GM}-1\right)\right] \tag{6}$$

with

$$h\,(r) = \frac{f_2}{f_1}\left(1-\frac{2GM}{r}\right). \tag{7}$$

Obviously, at the infinity, we have $x \to +\infty$. And that, in the vicinity of $r = 0$, we have $h\,(r) \sim -(f_2/f_1)(2GM/r)$, and the tortoise coordinate takes the form

$$x \sim -\int\frac{f_1}{f_2}\frac{r}{2GM}\mathrm{d}r = -\frac{f_1}{f_2}\frac{r^2}{4GM}. \tag{8}$$

In addition, from Eq.(3), it is seen that the event horizon of the modified black hole is at

$$r_+ = 2GM. \tag{9}$$

Then, near the event horizon, we have $h\,(r) \sim (r-r_+)\,h^{'}\,(r_+) = \frac{f_2}{f_1}\frac{r-2GM}{2GM}$ and

$$x \sim \int\frac{f_1}{f_2}\frac{2GM}{r-2GM}\mathrm{d}r = \frac{f_1}{f_2}2GM\ln\left(r-2GM\right). \tag{10}$$

This shows that, at the horizon, we have $x \to -\infty$.

Thus, by substituting Eq.(6) into Eq.(5), the radial wave equation is translated into

$$\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}x^2} + \left[\omega^2 - V\,(r)\right]\psi(r) = 0, \tag{11}$$

where $V(r)$ is the effective potential with

$$V(r)=\frac{f_2^2}{f_1^2}\left(1-\frac{2GM}{r}\right)\left(\frac{l(l+1)}{r^2}+\frac{2GM}{r^3}\right). \quad (12)$$

We see that, in the quantum corrected black hole, the effective potential depends on not only the radial coordinate and the angular quantum number but also the particle's energy. This shows that the present potential has some Planck scale modifications. But, as in the usual black hole,[48] while $x\to\pm\infty$, we have $V(r)\to 0$, then equation (11) can be translated into the standard wave equation in terms of the tortoise coordinate Eq.(6). That is, the solutions of Eq.(11) behave as $\psi(x)\sim \mathrm{e}^{\pm \mathrm{i}\omega x}$ at the horizon and infinity.

As the same in the usual black hole, the QNMs in the modified black hole can be supposed to be the solutions of perturbation equation of Eq.(11) with the boundary conditions of purely ingoing waves at event horizon and purely outgoing waves at infinity, namely

$$\psi(x)\sim \mathrm{e}^{+\mathrm{i}\omega x}, \quad x\to-\infty, \quad (13)$$

$$\psi(x)\sim \mathrm{e}^{-\mathrm{i}\omega x}, \quad x\to+\infty. \quad (14)$$

Then, to analytically obtain the asymptotic QNMs for the scalar field in the modified Schwarzschild black hole, we calculate the differential equation (11) by using the monodromy method. In the technology, it is essential to analytically extend Eq.(11) from its physical region $r_+<r<\infty$ to the whole complex r-plane. By extension, it is found that the solutions $\psi(r)$ behave multivaluedness around the singular points $r=0$ and $r=r_+$ and that the asymptotic QNMs will be obtained by computing the monodromy around a chosen contour in the complex plane. As in Ref.[21], we put a branch cut from $r=0$ to $r=r_+$ and the monodromy of $\psi(r)$ can be defined by the discontinuity across the cut. Moreover, from Eq.(6), the Stokes line $\mathrm{Re}(x)=0$ in the modified spacetime can be obtained and the contour L can be chosen as shown in Fig.1.

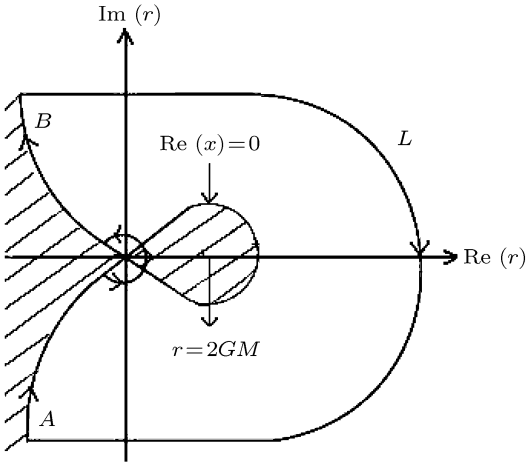


Fig.1. The Stokes line $\mathrm{Re}(x)=0$ and the contour L for the modified Schwarzschild black hole from the gravity's rainbow. The area of $\mathrm{Re}(x)<0$ is denoted by the regions with the hachures.

We see that, inside the contour, the only singularity of $\psi(r)$ is the event horizon. According to the boundary condition Eq.(13) and the tortoise coordinate Eq.(10), we can obtain that the monodromy of $\psi(r)$ around the contour L is the same as the monodromy of $\mathrm{e}^{\mathrm{i}\omega x}$ and must be[21]

$$\mathrm{e}^{\pi\omega/k}, \quad (15)$$

where κ is the surface gravity on the event horizon. For the quantum modified black hole as shown in Eq.(3), κ can be obtained by

$$\kappa=-\frac{1}{2}\lim_{r\to r_+}\sqrt{\frac{-g^{rr}}{g^{tt}}}\frac{1}{g^{tt}}\frac{\partial g^{tt}}{\partial r}=\frac{f_2}{f_1}\frac{1}{4GM}. \quad (16)$$

Then, the Hawking temperature of the black hole can be given as

$$T_{\mathrm{H}}=\frac{\kappa}{2\pi}=\frac{f_2}{f_1}\frac{1}{8\pi GM}. \quad (17)$$

We see that the surface gravity and the temperature for the modified Schwarzschild black hole all depend on the energy of the probe particle. That is, using the quanta with different energies, an observer at infinity will probe different effective temperatures and surface gravities for the quantum corrected black hole. It is easy to verify that, in the context of the gravity's rainbow, the energy dependence of surface gravity and Hawking temperature should arise from the spacetime quantum effect. And that, this should have some modification effects on the black hole physics, for example the QNMs.

Next, we compute the local monodromy of $\psi(x)$ around the same contour. For $r\sim 0$, from Eqs.(8) and (12), we have

$$V(r)\sim-\frac{1}{4x^2}. \quad (18)$$

Here, we could consider that the energy dependence of the effective potential is involved in the tortoise coordinate.

Similar to Ref.[21] by using Bessel's equation, we first rewrite the potential in the form

$$V(r)\sim-\frac{1-j^2}{4x^2}, \quad (19)$$

where j is the spin of the perturbation field. For our information, the results for the scalar perturbation could be obtained from the case of $j = 0$. Thus, near $r = 0$, we obtain the perturbation equation

$$\left(\frac{\partial^2}{\partial x^2} + \omega^2 + \frac{1-j^2}{4x^2}\right)\psi(x) = 0, \tag{20}$$

and its solutions can be expressed as[22]

$$\psi(x) \sim B_+\sqrt{2\pi\omega x}\mathrm{J}_{j/2}(\omega x) + B_-\sqrt{2\pi\omega x}\mathrm{J}_{-j/2}(\omega x), \tag{21}$$

where $B_\pm$ are integral constants and J_ν represents the Bessel function of order ν.

For the asymptotic QNMs, ω is approximately is purely imaginary and hence the line $\mathrm{Im}(\omega x) = 0$ is almost the same as the line $\mathrm{Re}(x) = 0$. Then, on the contour L, $x \to +\infty$ can actually be rotated to $\omega x \to +\infty$ and the boundary condition Eq.(14) can be expressed as

$$\psi \sim \mathrm{e}^{-\mathrm{i}\omega x}, \quad \omega x \to +\infty. \tag{22}$$

And that, along the contour L, it is convenient to obtain the asymptotic forms of $\psi(x)$ away from the origin $r = 0$ to $\omega x \to +\infty$. For $z \gg 1$, considering the asymptotic behavior

$$\mathrm{J}_\nu(z) = \sqrt{\frac{2}{\pi z}}\cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right), \tag{23}$$

we obtain

$$\begin{aligned}\psi(x) &\sim 2B_+\cos(\omega x - \alpha_+) + 2B_-\cos(\omega x - \alpha_-)\\ &= \left(B_+\mathrm{e}^{-\mathrm{i}\alpha_+} + B_-\mathrm{e}^{-\mathrm{i}\alpha_-}\right)\mathrm{e}^{\mathrm{i}\omega x}\\ &\quad + \left(B_+\mathrm{e}^{\mathrm{i}\alpha_+} + B_-\mathrm{e}^{\mathrm{i}\alpha_-}\right)\mathrm{e}^{-\mathrm{i}\omega x},\end{aligned} \tag{24}$$

where the phase shifts $\alpha_\pm = \frac{\pi}{4}(1 \pm j)$. Thus, from the boundary condition of Eq.(22), it is seen that, as $\omega x \to +\infty$, the $B_\pm$ must satisfy

$$B_+\mathrm{e}^{-\mathrm{i}\alpha_+} + B_-\mathrm{e}^{-\mathrm{i}\alpha_-} = 0, \tag{25}$$

and the solution $\psi(x)$ takes the form

$$\psi(x) \sim \left(B_+\mathrm{e}^{\mathrm{i}\alpha_+} + B_-\mathrm{e}^{\mathrm{i}\alpha_-}\right)\mathrm{e}^{-\mathrm{i}\omega x}. \tag{26}$$

Now, along the contour L, let us from point A approach to point B. In the process, we must turn an angle $3\pi/2$ around the origin, i.e., 3π around $x = 0$. As is well known, for $z \sim 0$, the Bessel functions satisfy

$$\mathrm{J}_\nu(z) = z^\nu w(z), \tag{27}$$

where $w(z)$ is an even holomorphic function. Then, after the rotation, we have

$$\sqrt{2\pi\mathrm{e}^{\mathrm{i}3\pi}\omega x}\mathrm{J}_{\pm\frac{j}{2}}\left(\mathrm{e}^{\mathrm{i}3\pi}\omega x\right) \sim 2\mathrm{e}^{6\mathrm{i}\alpha_\pm}\cos(-\omega x - \alpha_\pm), \tag{28}$$

and the solutions for $\omega x \to -\infty$ can be obtained as

$$\begin{aligned}\psi(x) &\sim 2B_+\mathrm{e}^{6\mathrm{i}\alpha_+}\cos(-\omega x - \alpha_+)\\ &\quad + 2B_-\mathrm{e}^{6\mathrm{i}\alpha_-}\cos(-\omega x - \alpha_-)\\ &= \left(B_+\mathrm{e}^{7\mathrm{i}\alpha_+} + B_-\mathrm{e}^{7\mathrm{i}\alpha_-}\right)\mathrm{e}^{\mathrm{i}\omega x}\\ &\quad + \left(B_+\mathrm{e}^{5\mathrm{i}\alpha_+} + B_-\mathrm{e}^{5\mathrm{i}\alpha_-}\right)\mathrm{e}^{-\mathrm{i}\omega x}.\end{aligned} \tag{29}$$

Next, we continue along the contour L in the right half-plane. In the large semicircle of L, the term ω^2 dominates the potential V and hence the solutions $\psi(x)$ of the perturbation equation of Eq.(20) can be approximated as plane waves. In this way, when we complete the contour, the coefficient of $\mathrm{e}^{-\mathrm{i}\omega x}$ in the $\psi(x)$ can remain unchanged and the coefficient of $\mathrm{e}^{\mathrm{i}\omega x}$ makes only an exponentially small contribution to the solutions. Thus, when we return to the points A from B, the coefficient of $\mathrm{e}^{-\mathrm{i}\omega x}$ gets multiplied by

$$\frac{B_+\mathrm{e}^{5\mathrm{i}\alpha_+} + B_-\mathrm{e}^{5\mathrm{i}\alpha_-}}{B_+\mathrm{e}^{\mathrm{i}\alpha_+} + B_-\mathrm{e}^{\mathrm{i}\alpha_-}}. \tag{30}$$

In addition, similar to Eq.(15), the monodromy of $\mathrm{e}^{-\mathrm{i}\omega x}$ around the contour is $\mathrm{e}^{-\pi\omega/k}$. Thus, for the solutions $\psi(x)$, to have the required monodromy of Eq.(15), we get

$$\frac{B_+\mathrm{e}^{5\mathrm{i}\alpha_+} + B_-\mathrm{e}^{5\mathrm{i}\alpha_-}}{B_+\mathrm{e}^{\mathrm{i}\alpha_+} + B_-\mathrm{e}^{\mathrm{i}\alpha_-}}\mathrm{e}^{-\pi\omega/k} = \mathrm{e}^{\pi\omega/k}. \tag{31}$$

Considering Eq.(25), the above equation can be translated into

$$\mathrm{e}^{2\pi\omega/k} = \frac{\mathrm{e}^{6\mathrm{i}\alpha_+} - \mathrm{e}^{6\mathrm{i}\alpha_-}}{\mathrm{e}^{2\mathrm{i}\alpha_+} - \mathrm{e}^{2\mathrm{i}\alpha_-}} = -(1 + 2\cos\pi j). \tag{32}$$

Therefore, letting $j = 0$ and considering Eq.(16), for scalar perturbation in the modified Schwarzschild black hole from the gravity's rainbow, the asymptotic QNMs are obtained as

$$\begin{aligned}\omega &= \frac{k}{2\pi}\ln 3 + \mathrm{i}\,k\left(n + \frac{1}{2}\right)\\ &= \frac{f_2}{f_1}\frac{\ln 3}{8\pi GM} + \mathrm{i}\,\frac{f_2}{f_1}\frac{1}{4GM}\left(n + \frac{1}{2}\right),\end{aligned} \tag{33}$$

where $n \to \infty$.

It is found that, the present asymptotic quasinormal frequencies depend on not only the mass parameter of the black hole, but also the energy functions f_1 and f_2. This shows that, the QNMs in the gravity's rainbow have got the Planck scale correction behaving as the dependence on the particle's energy.

Therefore, the result obtained here is different with the QNMs from the usual Schwarzschild black hole.[21] Certainly, in low energy, both f_1 and f_2 approach to 1, then equation (33) will return to the case of the usual Schwarzschild spacetime.

Equation (33) shows that, the real part of the asymptotic QNMs is

$$\mathrm{Re}\,(\omega) = \frac{f_2}{f_1}\frac{\ln 3}{8\pi GM}. \tag{34}$$

That is, the real part also has energy dependence and is different with the value from the usual Schwarzschild black hole. Moreover, considering Eq.(17) of the black hole temperature, equation (34) can be expressed as

$$\mathrm{Re}\,(\omega) = T\ln 3. \tag{35}$$

We see that, the asymptotic quasinormal frequencies for the modified black hole are consistent with Hod's conjecture. Thus, Hod's conjecture gets support from the gravity's rainbow as a quantum corrected spacetime. It should be pointed out that, the temperature T in Eq.(35) is energy dependent and hence the present Hod's conjecture should have some Planck scale effects. And that, for the quantum corrected black hole, the relationship of the asymptotic QNMs with the quantum theory of spacetime should be tested and verified.

4. Area spectrum

In this Section, as for the usual black hole,[9,10] we investigate the area spectrum of the gravity's rainbow and its relationship with LQG. Based on Bohr's correspondence, Hod argued that,[9] the highly damped QNMs of black holes should be identical to the quantum transition of the corresponding system and the real part of the asymptotic QNMs should be equal to the quantum transition frequency. Thus, in the modified Schwarzschild black hole, the energy of the radiated quanta corresponding to the radiated wave as shown in Eq.(33) is obtained as follows:

$$E = \hbar\,\mathrm{Re}\,(\omega) = \frac{f_2}{f_1}\frac{\hbar}{8\pi GM}\ln 3. \tag{36}$$

On the other hand, from Eqs.(3) and (2), we can see that the modified Schwarzschild solution from the gravity's rainbow is asymptotically DSR. And, the asymptotically DSR spacetimes have equality with the asymptotically flat spacetimes.[35] Then, using the Komar integrals, we define the Arnowitt-Deser-Misner (ADM) mass for the modified Schwarzschild black hole by

$$M_{\mathrm{ADM}} = -\frac{1}{8\pi G}\int_s \varepsilon_{abcd}\nabla^c\xi^d = \frac{M}{f_1 f_2}. \tag{37}$$

We find that, for the gravity's rainbow, the ADM mass is not equal to the mass parameter M, and the total energy of the spacetimes depends on the energy of the probe particle. That is, for the probe particles with different energies, the ADM mass of the gravity's rainbow has different effective descriptions. It is easy to verify that, the energy property of the gravity's rainbow should have a behavior of the spacetime quantum effects. Obviously, this is different from the usual Schwarzschild black hole, in which $M_{\mathrm{ADM}} = M$.

Then, by applying energy conservation to the quantum corrected spacetime, the changes for the ADM mass and the mass parameter of the black hole before and after the radiation of Eq.(36) are obtained, respectively,

$$\Delta M_{\mathrm{ADM}} = E, \tag{38}$$

$$\Delta M = f_1 f_2 E = f_2^2\frac{\hbar}{8\pi GM}\ln 3. \tag{39}$$

In addition, from Eqs.(3) and (9), the event horizon area of the modified black hole can be obtained as

$$A = \int \left(g_{\theta\theta}g_{\varphi\varphi}\right)_{r_+}\mathrm{d}\theta\mathrm{d}\varphi = \frac{16\pi G^2M^2}{f_2^2}. \tag{40}$$

It is seen that, although the horizon of Eq.(9) is universal for all observers, the horizon area of the gravity's rainbow framework is energy-dependent and different from the value of the usual black hole.

Thus, from Eqs.(39) and (40), the area change of the gravity's rainbow corresponding to the asymptotic QNMs shown in Eq.(33) can be obtained as follows:

$$\Delta A = \frac{1}{f_2^2}32\pi G^2M\Delta M = 4\ln 3 l_p^2 \tag{41}$$

with Planck length $l_p = \sqrt{G\hbar}$.

We find that, despite the fact that the area itself is different from the value of the usual black hole and for different observers, the area spacing of the quantum modified black hole is not only the same as the usual black hole,[9,10] but also is the same for all the observers. This may have some relationship with the quantum theory of spacetime and should be investigated in future.

As mentioned in Section 2, the gravity's rainbow may have some relevancy with LQG. Next, as in Ref.[10], we relate the area spectrum of the quantum corrected black hole to quantum geometry. In LQG, the area of black holes is quantized and the area element is given by[18,19,49]

$$A(j_{\min}) = 8\pi l_p^2 \sqrt{j_{\min}(j_{\min}+1)}, \tag{42}$$

where γ is the so-called Barbero–Immirzi parameter and $j_{\min}$ is the smallest spin of the spin network insetting the horizon. Then, under the argument[10]

$$A(j_{\min}) = \Delta A, \tag{43}$$

the γ can be expressed as

$$\gamma = \frac{\ln 3}{2\pi\sqrt{j_{\min}(j_{\min}+1)}}. \tag{44}$$

In addition, the black hole entropy from LQG is[18,19,50]

$$S = N \ln(2j_{\min}+1), \tag{45}$$

where $N = A/A(j_{\min})$ is the number of area element on the horizon.

Thus, considering Eq.(43), we have

$$S = \frac{A}{4l_p^2}\frac{\ln(2j_{\min}+1)}{\ln 3}. \tag{46}$$

Letting the black hole entropy equal the Bekenstein–Hawking entropy

$$S_{\rm bh} = \frac{A}{4l_p^2}, \tag{47}$$

the smallest spin value is presented as

$$j_{\min} = 1. \tag{48}$$

The result is the same as that obtained from the usual black hole and suggested that LQG should be based on SO(3).[10] By substituting the spin value into Eq.(44), the Barbero-Immirzi parameter can be fixed as

$$\gamma = \frac{\ln 3}{2\pi\sqrt{2}}. \tag{49}$$

It is seen that, as in the case of the smallest spin value, the Barbero-Immirzi parameter is not dependent on the particle's energy and has the same result as from the usual black hole.[10] Obviously, as an elementary parameter of LQG, its independence of observer and spacetime background is satisfactory.

5. Summary and conclusions

In the present work, we investigated the asymptotic QNMs in a quantum corrected spacetime. By using the monodromy method, we analytically calculated the asymptotic QNMs for the massless scalar field in the modified Schwarzschild black holes from a gravity's rainbow. The results show that, the highly damped quasinormal frequencies depend not only on the mass parameter of the modified black hole but also on the particle's energy. Comparing the results from the usual Schwarzschild black hole, in which the asymptotic QNMs only depend on the mass parameter,[21] the energy dependence of the asymptotic QNMs in the gravity's rainbow is a novel property. The property may be seen as a Planck scale effect of the asymptotic QNMs and should arise from the quantum effect of the background spacetimes.

It is also found that, in the context of the gravity's rainbow, the energy dependence of the asymptotic quasinormal frequencies is involved in the Hawking temperature and the real part of the asymptotic QNMs can be expressed as $T_{\rm H}\ln 3$. Thus, in support of Hod's conjecture.[9] the present research provides further evidence from a quantum corrected spacetime. In addition, by relating Hod's conjecture to the quantum theory of black holes,[9,10] the area quantization of the quantum modified black hole is investigated and the area spacing is obtained. The result is energy-independent and is the same as that coming from the usual Schwarzschild black hole,[9,10] despite the fact that the area itself is energy-dependent and different from the value of the usual black hole. In addition, in the present quantum corrected spacetime, by relating the area quantization to LQG, the smallest spin value and the free parameter in LQG are given. Also, the obtained results are energy-independent and the same as those in the usual black holes.[10] It is consistent with expectation that the present area spectrum and the obtained elementary parameter are independent of the observer and background spacetime.

The present results should be proved by other cases, such as different fields in different gravity's rainbows. Further research is in progress.

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