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Einstein–Podolsky–Rosen entanglement in time-dependent periodic pumping non-degenerate optical parametric amplifier*

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The solution of the time-dependent periodic pumping non-degenerate optical parametric amplifier (NOPA) is derived when the pump depletion is considered both above and below thresholds. Based on this solution, the quantum fluctuation calculated shows that a high entanglement and a good squeezing degree of the parametric light beams are achieved near and above thresholds. We adopt two kinds of pump fields: (i) a continuously modulated pump with a sinusoidal envelope; (ii) a sequence of laser pulses with Gaussian envelopes. We analyse the time evolution of continuous variable entanglement by analytical and numerical calculations, and show that the periodic driven pumping also improves the degree of entanglement. The squeezing and Einstein-Podolsky-Rosen (EPR) entanglement by using the two pumping driven functions are investigated from below to above the threshold regions, the tendencies are nearly the same, and the Gaussian driven function is superior to that of the sine driven function, when the maximum squeezing and the minimum variance of quantum fluctuation are considered. In the meantime, the generalization of frequency degenerate OPA to frequency non-degenerated OPA problem is investigated.

Keywords: EPR entanglement, periodic pumping, non-degenerate optical parametric amplifier
PACC: 0367, 4250, 4265K

1. Introduction

Up to now, the interest in continuous variable (CV) entanglement has been aroused by successful experiments on quantum teleportation based on two-mode squeezed state[1] as well as most applications in the field of quantum information. Since then, remarkable theoretical and experimental efforts have been made to generate and quantify CV entangled states.

The CV entanglement states of light fields were studied in Ref.[2] and demonstrated experimentally[3] for a non-degenerate optical parametric amplifier (NOPA). A CV entanglement source was built from two single-mode squeezed states combined on a beam splitter. Obviously, the entanglement states of light field stem from their squeezing. Recently, there has been developed a systematic perturbation procedure to study the quantum property of the time-modulated NOPA system for below- and above-threshold regions. The photon number n_i and phase ϕ_i were calculated.[4] The quantum optical effects have been also demonstrated in a series of experiments.[5]

In the previous paper,[6] we have solved the time-dependent Fokker–Planck (F–P) equation for a degenerate optical parametric amplifier (DOPA), and the squeezing calculated shows a complicated behaviour as a function of time. In summary: (a) in the case of $\mu \leq 1$ (where μ is the pumping power in units of threshold power), the squeezing may be approximately described by the linear theory formula $1/(1+\mu)$; (b) in the case of $\mu > 1$, the squeezing curve is composed of two stages. In the first short time stage, the squeezing is approximately described by the linear theory formula $1/(1+\mu)$, but soon after that it drops to the second long time stationary stage of squeezing 1/2. The more important thing is that a high squeezing cannot be expected to last for a long-time with the help of a constant pump. This fact motivates us to consider that the multi-mode phase-locked periodic pump may be used to achieve a transient high squeezing and a high entanglement result.

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To characterize the CV entanglement, we address the EPR criterion.[2] A sufficient condition for entanglement consists in the negativity of the partial transposition of the quantum state of the two-party system.[7−9] A sufficient condition of inseparability based on the second-order moments of position and momentum operators has been proposed by Simon[10] and Duan.[11] This condition has also been proved to be necessary for bipartite Gaussian state. Recently, the necessary and sufficient conditions for the partial transposition of bipartite quantum states to be non-negative have been provided by Shchukin and Vogel.[12]

In the previous papers, we used the solution of F-P equation for NOPA and Reid's V_1 criterion to deduce the condition for demonstrating the EPR paradox.[13] The model is frequency-degenerated but polarization non-degenerated OPA. According to Ref.[5], if the oscillator is working above threshold, it is also frequency non-degenerated OPA. Considering this circumstance, our model should be modified. In the present paper, we continue to use the minimum uncertain V_1 criterion for investigating the entanglement states of the NOPA system by using the periodic pump. The remainder of the present paper is organized as follows. In Section 2, we formulate the solution of time-dependent F-P equation for the NOPA system by using the periodic pump, and we carry out the quantum fluctuation calculation as well. According to the solution of the time-dependent F-P equation, the squeezing for the time-dependent system is calculated in Section 3. The CV entanglement of DOPA resources by periodic pumping is given in Section 4. In Section 5, the generalization of the frequency-degenerated OPA system to the frequency non-degenerated OPA system is investigated. In Section 6, the results obtained are summarized.

2. Time-dependent F–P equation solution for NOPA and quantum fluctuation calculation

In 1988, Reid pointed out the experimental possibility of demonstrating the original EPR paradox in the NOPA system. Subsequently, a detailed analysis and derivation were performed in Ref.[2]. The argument was based on the Langevin equation, and the pump was assumed to be un-depleted and sufficient in intensity, which could be modelled classically. In fact, near the threshold region, quantum fluctuations are large, the nonlinear effect becomes dominant, and the influence of the pump depletion may not be neglected.[13] In the present paper, we use the solution of the time-dependent F-P equation for NOPA under the consideration of pump depletion. We consider the NOPA system in which there are two modes that have the same frequencies but different polarizations. The total Hamiltonian describing this system may be written as[13,14]

$$H = H_0 + V + W,$$

where

$$\begin{aligned} H_0 = {} & \hbar\omega_0 a_0^\dagger a_0 + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 \\ & + \mathrm{i}\,\hbar(E\mathrm{e}^{-\mathrm{i}\,\omega_L t} a_0^\dagger - E^*\mathrm{e}^{\mathrm{i}\,\omega_L t} a_0) \\ & + \sum_{j,i=1,2} \hbar\omega_j B_{ji}^\dagger B_{ji}, \end{aligned}$$

$$\begin{aligned} V = {} & \hbar\sum k_j (B_{j1} a_1^\dagger + B_{j2} a_2^\dagger) \\ & + \hbar\sum k_j^* (B_{j1}^\dagger a_1 + B_{j2}^\dagger a_2), \end{aligned}$$

$$W = \frac{\mathrm{i}\,\hbar}{2}(2\varepsilon a_1^\dagger a_2^\dagger - 2\varepsilon^* a_1 a_2) \tag{1}$$

with the frequencies ω_0, ω_1 and ω_2 referring to the pump, the signal and the idler field respectively. We assume a phase-matched condition such that $\omega_1+\omega_2 = \omega_0$ and $\hbar\omega_1 = \hbar\omega_2 = \hbar\omega_0/2$ (without loss of generality, here we first adopt the model of frequency-degenerated but polarization non-degenerated OPA, then, we will prove that the solution of the frequency non-degenerated OPA system can be reduced into the solution of the frequency-degenerated OPA system), E is the amplitude of coherent driving field with frequency $\omega_L \approx \omega_0$, α_0 is the pump amplitude in the semi-classical equations given below, included in the gain factor $\varepsilon = \chi\alpha_0$.

Setting

$$\alpha_1 = \frac{b_1 + b_2}{\sqrt{2}}, \quad \text{and} \quad \alpha_2 = \frac{b_1 - b_2}{\sqrt{2}},$$

and considering $b_1 = (a_1 + a_2)/\sqrt{2}$ and $b_2 = (a_1 - a_2)/\sqrt{2}$, we may refer to b_1 and b_2 as the in-phase state operator and the out-phase state operator, respectively.

It is easy to prove that a NOPA system is equivalent to two DOPA systems with different parameters. Furthermore, the F-P equation for NOPA system can be written as a combined system of DOPA 1

and DOPA 2[13]

$$\begin{aligned}\frac{\partial p_1}{\partial t} &= \left[k\left(\frac{\partial}{\partial b_1}b_1+\frac{\partial}{\partial b_1^*}b_1^*\right)\right.\\ &\quad -\left(\varepsilon b_1^*\frac{\partial}{\partial b_1}+\varepsilon^* b_1\frac{\partial}{\partial b_1^*}\right)\\ &\quad \left.+\frac{1}{2}\left(\varepsilon\frac{\partial^2}{\partial b_1^2}+\varepsilon^*\frac{\partial^2}{\partial b_1^{*2}}\right)\right]p_1,\\ \frac{\partial p_2}{\partial t} &= \left[k\left(\frac{\partial}{\partial b_2}b_2+\frac{\partial}{\partial b_2^*}b_2^*\right)\right.\\ &\quad +\left(\varepsilon b_2^*\frac{\partial}{\partial b_2}+\varepsilon^* b_2\frac{\partial}{\partial b_2^*}\right)\\ &\quad \left.-\frac{1}{2}\left(\varepsilon\frac{\partial^2}{\partial b_2^2}+\varepsilon^*\frac{\partial^2}{\partial b_2^{*2}})\right)\right]p_2. \end{aligned}\tag{2}$$

When the pump depletion is neglected, the pump amplitude α_0 and the gain coefficient $\varepsilon=\chi\alpha_0$ are both constant. However, the pump depletion $\alpha_0=\alpha_0(\tau)$ is considered. NOPA and DOPA1(ε,k), DOPA2$(-\varepsilon,k)$ are all time-dependent, so we have $\varepsilon=\chi\alpha_0(\tau)$.

In order to determine $\alpha_0(\tau)$, we need to study the interaction between pump wave and parametric waves. In the positive-P representation, substitute the operators b_1,b_2 by the c numbers α_1,α_2, we derive the semi-classical equations describing the interaction between the pump wave α_0 and two degenerate parametric modes α_1,α_2[6] as follows:

$$\begin{aligned}\frac{\mathrm{d}\alpha_1}{\mathrm{d}\tau} &= -\alpha_1+\frac{\chi}{\gamma_1}\alpha_0\alpha_1^*,\\ \frac{\mathrm{d}\alpha_2}{\mathrm{d}\tau} &= -\alpha_2-\frac{\chi}{\gamma_1}\alpha_0\alpha_2^*,\\ \frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau} &= -\gamma_r\alpha_0+\frac{\bar{\varepsilon}}{\gamma_1}-\frac{\chi}{2\gamma_1}(\alpha_1^2-\alpha_2^2),\end{aligned}\tag{3}$$

where we have used the dimensionless time $\tau=kt=\gamma_1 t$; χ is the coupling constant for the nonlinear coupling between the modes; γ_0 and γ_1 are the damping rates which are related by decay ratio $\gamma_r=\gamma_0/\gamma_1$; $\bar{\varepsilon}$ is the amplitude of external coherent field. Assuming α_0 to be real and writing each of complex numbers α_1 and α_2 into the sum of real and imaginary parts, i.e. $\alpha_1\to\alpha_1+\mathrm{i}\,\tilde{\alpha}_1$ and $\alpha_2\to\alpha_2+\mathrm{i}\,\tilde{\alpha}_2$. The numerical solutions of Eq.(3), with the initial values $\alpha_{10}=\tilde{\alpha}_{10}=\alpha_{20}=\tilde{\alpha}_{20}=1$, yield the time-dependent parametric functions $\alpha_1,\tilde{\alpha}_1,\alpha_2$ and $\tilde{\alpha}_2$ as follows:

$$\begin{aligned}\alpha_1(t) &= \exp\left[-\int_0^t(k-\varepsilon)\mathrm{d}t'\right],\\ \tilde{\alpha}_1(t) &= \exp\left[-\int_0^t(k+\varepsilon)\mathrm{d}t'\right]\\ &= \exp\left[-2\int_0^t k\mathrm{d}t'\right]\alpha_1^{-1}(t),\\ \alpha_2(t) &= \exp\left[-\int_0^t(k+\varepsilon)\mathrm{d}t'\right]=\tilde{\alpha}_1(t),\\ \tilde{\alpha}_2(t) &= \exp\left[-\int_0^t(k-\varepsilon)\mathrm{d}t'\right]=\alpha_1(t).\end{aligned}\tag{4}$$

Write out the variances of the quantum fluctuation of the real part Δx_1 and imaginary part Δy_1 as follows:

$$\begin{aligned}\left\langle(\Delta x_1)^2\right\rangle &= \frac{1}{4}+\frac{\left\langle(\Delta\beta_1)^2\right\rangle}{2}\\ &= \frac{1}{4}+(A_1-A_{10}\alpha_1^2(t)),\\ \left\langle(\Delta y_1)^2\right\rangle &= \frac{1}{4}+\frac{\left\langle(\Delta\,\mathrm{i}\,\tilde{\beta}_1)^2\right\rangle}{2}\\ &= \frac{1}{4}+(\tilde{A}_1-\tilde{A}_{10}\tilde{\alpha}_1^2(t)),\end{aligned}\tag{5}$$

where

$$\begin{aligned}A_1 &= A_{10}\alpha_1^2(t)+\frac{\alpha_1^2(t)}{4\alpha_1^2(t-t_0)}\\ &\quad -\frac{1}{4}+\frac{\alpha_1^2(t)k}{2}\int_{t-t_0}^t\alpha_1^{-2}(t')\mathrm{d}t',\\ \tilde{A}_1 &= \tilde{A}_{10}\tilde{\alpha}_1^2(t)+\frac{\tilde{\alpha}_1^2(t)}{4\tilde{\alpha}_1^2(t-t_0)}\\ &\quad -\frac{1}{4}+\frac{\tilde{\alpha}_1^2(t)k}{2}\int_{t-t_0}^t\tilde{\alpha}_1^{-2}(t')\mathrm{d}t'.\end{aligned}\tag{6}$$

In expression (6), t_0 denotes the time needed for the parametric signal to pass through the amplifier, and the last integral term contains the noise accumulated in the diffusion process.

Similarly, the variances of the quantum fluctuation Δx_2 and Δy_2 read

$$\begin{aligned}\left\langle(\Delta x_2)^2\right\rangle &= \frac{1}{4}+\frac{\left\langle(\Delta\beta_2)^2\right\rangle}{2}\\ &= \frac{1}{4}+(A_2-A_{20}\alpha_2^2(t)),\\ \left\langle(\Delta y_2)^2\right\rangle &= \frac{1}{4}+\frac{\left\langle(\Delta\,\mathrm{i}\,\tilde{\beta}_2)^2\right\rangle}{2}\\ &= \frac{1}{4}+(\tilde{A}_2-\tilde{A}_{20}\tilde{\alpha}_2^2(t)),\end{aligned}\tag{7}$$

where

$$\begin{aligned}A_2 &= A_{20}\alpha_2^2(t)+\frac{\alpha_2^2(t)}{4\alpha_2^2(t-t_0)}\\ &\quad -\frac{1}{4}+\frac{\alpha_2^2(t)k}{2}\int_{t-t_0}^t\alpha_2^{-2}(t')\mathrm{d}t',\\ \tilde{A}_2 &= \tilde{A}_{20}\tilde{\alpha}_2^2(t)+\frac{\tilde{\alpha}_2^2(t)}{4\tilde{\alpha}_2^2(t-t_0)}\\ &\quad -\frac{1}{4}+\frac{\tilde{\alpha}_2^2(t)k}{2}\int_{t-t_0}^t\tilde{\alpha}_2^{-2}(t')\mathrm{d}t'.\end{aligned}\tag{8}$$

Furthermore, the quantum fluctuation of the time-dependent linearly driven NOPA system is

$$\begin{aligned}\left\langle(\Delta X_1)^2\right\rangle &= \frac{1}{2}(4\left\langle(\Delta x_1)^2\right\rangle + 4\left\langle(\Delta x_2)^2\right\rangle) = 1 + 2(A_1 - A_{10}\alpha_1^2(t)) + 2(A_2 - A_{20}\alpha_2^2(t)),\\ \left\langle(\Delta Y_2)^2\right\rangle &= \frac{1}{2}(4\left\langle(\Delta y_1)^2\right\rangle + 4\left\langle(\Delta y_2)^2\right\rangle) = 1 + 2(\tilde{A}_1 - \tilde{A}_{10}\tilde{\alpha}_1^2(t)) + 2(\tilde{A}_2 - \tilde{A}_{20}\tilde{\alpha}_2^2(t)),\end{aligned} \tag{9}$$

where the factor 1/2 introduced manifests the probabilities of the system, half in the 'in phase state' and half in the 'out of phase state'.

Substituting expressions (6) and (8) into expression (9), the quantum fluctuation of the time-dependent linearly driven NOPA system is obtained as

$$\begin{aligned}\left\langle(\Delta X_1)^2\right\rangle &= \frac{\alpha_1^2(t)}{2\alpha_1^2(t-t_0)} + \frac{\alpha_2^2(t)}{2\alpha_2^2(t-t_0)} + k\alpha_1^2(t)\int_{t-t_0}^{t}\alpha_1^{-2}(t')\mathrm{d}t' + k\alpha_2^2(t)\int_{t-t_0}^{t}\alpha_2^{-2}(t')\mathrm{d}t',\\ \left\langle(\Delta Y_2)^2\right\rangle &= \frac{\tilde{\alpha}_1^2(t)}{2\tilde{\alpha}_1^2(t-t_0)} + \frac{\tilde{\alpha}_2^2(t)}{2\tilde{\alpha}_2^2(t-t_0)} + k\tilde{\alpha}_1^2(t)\int_{t-t_0}^{t}\tilde{\alpha}_1^{-2}(t')\mathrm{d}t' + k\tilde{\alpha}_2^2(t)\int_{t-t_0}^{t}\tilde{\alpha}_2^{-2}(t')\mathrm{d}t'.\end{aligned} \tag{10}$$

For simplicity, we adopt the normalized parametric function notations

$$\frac{\alpha_1(t)}{\alpha_1(t-t_0)} \to \alpha_{1n}(t), \quad \frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(t-t_0)} \to \alpha_{1n}(t), \quad \frac{\alpha_2(t)}{\alpha_2(t-t_0)} \to \alpha_{2n}(t), \quad \frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}_2(t-t_0)} \to \alpha_{2n}(t), \tag{11}$$

which can be further rewritten as

$$\begin{aligned}\left\langle(\Delta X_1)^2\right\rangle &= \frac{\alpha_{1n}^2(t)}{2} + \frac{\alpha_{2n}^2(t)}{2} + k\alpha_1^2(t)\int_{t-t_0}^{t}\alpha_1^{-2}(t')\mathrm{d}t' + k\alpha_2^2(t)\int_{t-t_0}^{t}\alpha_2^{-2}(t')\mathrm{d}t',\\ \left\langle(\Delta Y_2)^2\right\rangle &= \frac{\tilde{\alpha}_{1n}^2(t)}{2} + \frac{\tilde{\alpha}_{2n}^2(t)}{2} + k\tilde{\alpha}_1^2(t)\int_{t-t_0}^{t}\tilde{\alpha}_1^{-2}(t')\mathrm{d}t' + k\tilde{\alpha}_2^2(t)\int_{t-t_0}^{t}\tilde{\alpha}_2^{-2}(t')\mathrm{d}t'.\end{aligned} \tag{12}$$

The operator solutions X_1 and Y_2 satisfying expression (13) are

$$\begin{aligned}X_1 &= X_{10}\frac{\alpha_{1n}(t) + \alpha_{2n}(t)}{2} + Y_{20}\frac{\alpha_{1n}(t) - \alpha_{2n}(t)}{2}\\ &\quad + \alpha_1(t)\int_{t-t_0}^{t}\alpha_1^{-1}(t')\xi(t')\mathrm{d}t' - \alpha_2(t')\int_{t-t_0}^{t}\alpha_2^{-1}(t')\eta(t')\mathrm{d}t',\\ Y_2 &= Y_{20}\frac{\tilde{\alpha}_{1n}(t) + \tilde{\alpha}_{2n}(t)}{2} - X_{10}\frac{\tilde{\alpha}_{1n}(t) - \tilde{\alpha}_{2n}(t)}{2}\\ &\quad + \tilde{\alpha}_1(t)\int_{t-t_0}^{t}\tilde{\alpha}_1^{-1}(t')\eta(t')\mathrm{d}t' + \tilde{\alpha}_2(t')\int_{t-t_0}^{t}\tilde{\alpha}_2^{-1}(t')\xi(t')\mathrm{d}t',\end{aligned} \tag{13}$$

where $\xi(t)$ and $\eta(t)$ are the random forces, which satisfy the following relations:

$$\langle\xi(t')\xi(t'')\rangle = \langle\eta(t')\eta(t'')\rangle = k\delta(t'-t'') \quad \text{and} \quad \langle\xi(t')\eta(t'')\rangle = 0;$$

X_{10} and Y_{20} are the vacuum fluctuations with $\langle X_{10}\rangle = \langle Y_{20}\rangle = 0$, $\left\langle X_{10}^2\right\rangle = \left\langle Y_{20}^2\right\rangle = 1$.[2]

Using expression (13), we can calculate the correlation between X_1 and Y_2 as follows:

$$\begin{aligned}\langle X_1 Y_2\rangle &= -\left(\frac{\tilde{\alpha}_{1n}(t) + \tilde{\alpha}_{2n}(t)}{2}\right)\left(\frac{\alpha_{1n}(t) - \alpha_{2n}(t)}{2}\right) + \left(\frac{\tilde{\alpha}_{1n}(t) - \tilde{\alpha}_{2n}(t)}{2}\right)\left(\frac{\alpha_{1n}(t) + \alpha_{2n}(t)}{2}\right)\\ &\quad + \left\langle\alpha_1(t)\int_{t-t_0}^{t}\alpha_1^{-1}(t')\xi(t')\mathrm{d}t' \times \tilde{\alpha}_2(t)\int_{t-t_0}^{t}\tilde{\alpha}_2^{-1}(t'')\xi(t'')\mathrm{d}t''\right\rangle\\ &\quad - \left\langle\alpha_2(t)\int_{t-t_0}^{t}\alpha_2^{-1}(t')\eta(t')\mathrm{d}t' \times \tilde{\alpha}_1(t)\int_{t-t_0}^{t}\tilde{\alpha}_1^{-1}(t'')\eta(t'')\mathrm{d}t''\right\rangle\\ &= \frac{1}{2}(\alpha_{1n}(t)\tilde{\alpha}_{2n}(t) - \tilde{\alpha}_{1n}(t)\alpha_{2n}(t)) + k\alpha_1^2(t)\int_{t-t_0}^{t}\alpha_1^{-2}(t')\mathrm{d}t' - k\tilde{\alpha}_1^2(t)\int_{t-t_0}^{t}\tilde{\alpha}_1^{-2}(t')\mathrm{d}t'.\end{aligned} \tag{14}$$

Substituting the above results into the formula evaluating the variance V_1,[2,15] we have

$$V_1 = \Delta^2 X_1 = \langle X_1^2 \rangle - \frac{\langle X_1 Y_2 \rangle^2}{\langle Y_2^2 \rangle}. \qquad (15)$$

3. Quantum fluctuation in time-dependent periodic pumping driven DOPA system

Using the above solutions, we can obtain the quantum fluctuation of the time-dependent linearly driven DOPA system. The squeezing is calculated from expression (5) to be

$$\langle (\Delta y_1)^2 \rangle = \frac{1}{4} + \frac{\left\langle (\Delta \,\mathrm{i}\, \tilde{\beta}_1)^2 \right\rangle}{2} = \frac{1}{4} + (\tilde{A}_1 - \tilde{A}_{10}\tilde{\alpha}_1^2), \quad (16)$$

where

$$\tilde{A}_1(t) = \tilde{\alpha}_1^2(t)\left(\tilde{A}_{10} + \frac{1}{4}\right) - \frac{1}{4} + \frac{\tilde{\alpha}_1^2(t)k}{2}\int_0^t \tilde{\alpha}_1^{-2}(t')\mathrm{d}t'.$$

The time-dependent parametric light $\tilde{\alpha}_1$ is derived through solving the NOPA system Eq.(3). In the present paper, with the purpose of achieving high squeezing, a periodic function of time, $\bar{\varepsilon}(\tau)$, is substituted for $\bar{\varepsilon}$. In the following, two types of periodic functions are adopted: one is sine function $\bar{\varepsilon}(\tau) = \mu \times (1 + \sin\omega\tau)$, and the other is Gaussian function $\bar{\varepsilon}(\tau) = \mu \times 5.45\exp[-10 \times (\sin\omega\tau)^2]$.

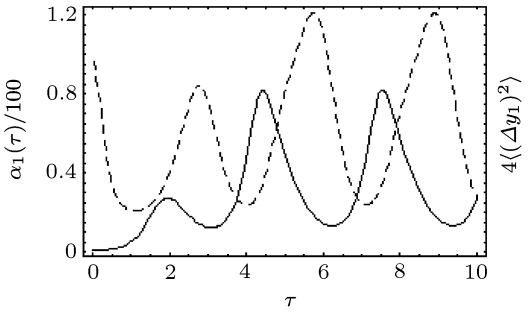


Fig.1. Squeezing for sine function pumping with $\mu = 2$, $\eta = 1/1000$, $\gamma_r = 10$, $\omega = 2$, $\tau_0 = 9$, $\alpha_1(\tau)/100$ vs. τ and $4\langle(\Delta y_1)^2\rangle$ vs. τ combined.

Here, $\eta = \chi^2/2\gamma_1\gamma_0$, $\mu = \bar{\varepsilon}/\bar{\varepsilon}_c$, $\bar{\varepsilon}_c = \gamma_1\gamma_0/\chi$. Through the numerical solution of Eq.(3) and expression (16), the squeezing calculations are shown in Figs.1 and 2 for the sine function and the Gaussian function, respectively. The solid curve represents the field of parametric light $\alpha(\tau)/100$ versus τ, while the dash curve denotes the squeezing $4\langle(\Delta y)^2\rangle$ versus τ. In Fig.1, there is a small deviation between the maximum squeezing (0.2–0.3) and the peak of light field curve, and most of the energy of the light field is squeezed below the vacuum fluctuation 1. While in Fig.2, the maximum squeezing attains 0.2-0.25, the position is nearly coincident with the peak of the light field curve. This shows that the Gaussian function pumping is superior to the sine function pumping. We usually compare the sine function with the Gaussian function by the area under the curve, not by the height of the curve. For both cases, the maximum squeezing attained approaches the value $1/(1+\mu)$ given by the linear theory (the pump depletion is not considered).

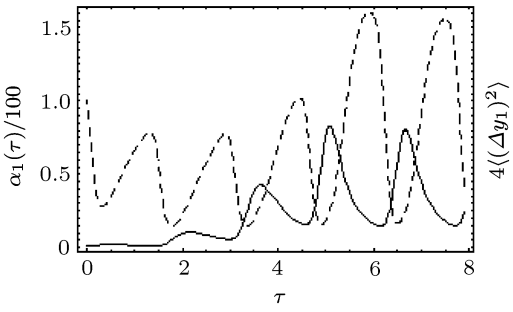


Fig.2. Squeezing for Gaussian function pumping with $\mu = 2$, $\eta = 1/1000$, $\gamma_r = 10$, $\omega = 2$, $\tau_0 = 9$, $\alpha_1(\tau)/100$ vs. τ and $4\langle(\Delta y_1)^2\rangle$ vs. τ combined.

4. Entanglement resources of NOPA system driven by periodic pumping

Through expressions (7)–(19), we calculate the entanglement of NOPA system separately by using

(i) sine function $\bar{\varepsilon} = \mu \times (1 + \sin\omega\tau)$, and

(ii) Gaussian function $\bar{\varepsilon} = \mu \times 5.45\exp[-10 \times \sin^2(\omega\tau)]$.

The pump parameter level $\mu = \bar{\varepsilon}/\bar{\varepsilon}_c$, where $\bar{\varepsilon}_c$ is the threshold. The main results are depicted in Figs.3 and 4 for the sine function and the Gaussian function, respectively.

Figure 3 depicts the variance V_1 as a function of time τ (τ ranges from 25 to 35). (a) $\mu = 0.5$ for below threshold, (b) $\mu = 1.001$ for very near threshold, (c) $\mu = 2$ for above threshold, (d) $\mu = 4$ for far above threshold. From Fig.3, we can see that (i) the minimum variance V_1 attains the values of 0.88, 0.7, 0.5 and 0.4, corresponding to the cases (a), (b), (c) and (d), respectively; (ii) with the value of pump μ increasing, the range of variation of V_1 increases. During $\mu = 4$, the shape of the curve turns asymmetric, the left part of it protrudes slightly.

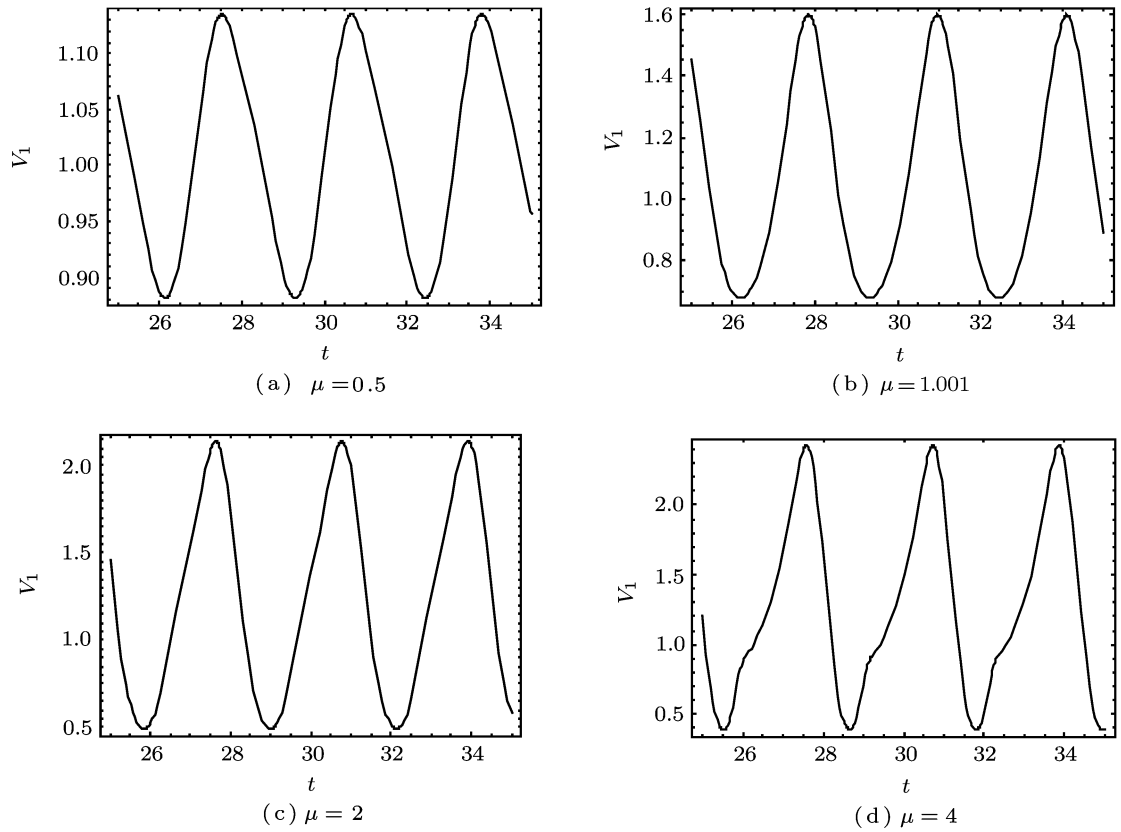


Fig.3. Sine pumping functions with $\eta = 1/1000$, $\gamma_r = 10$, $\omega = 2$, $\tau_0 = 9$, and $\mu = 0.5$ (a), 1.001 (b), 2 (c), and 4 (d).

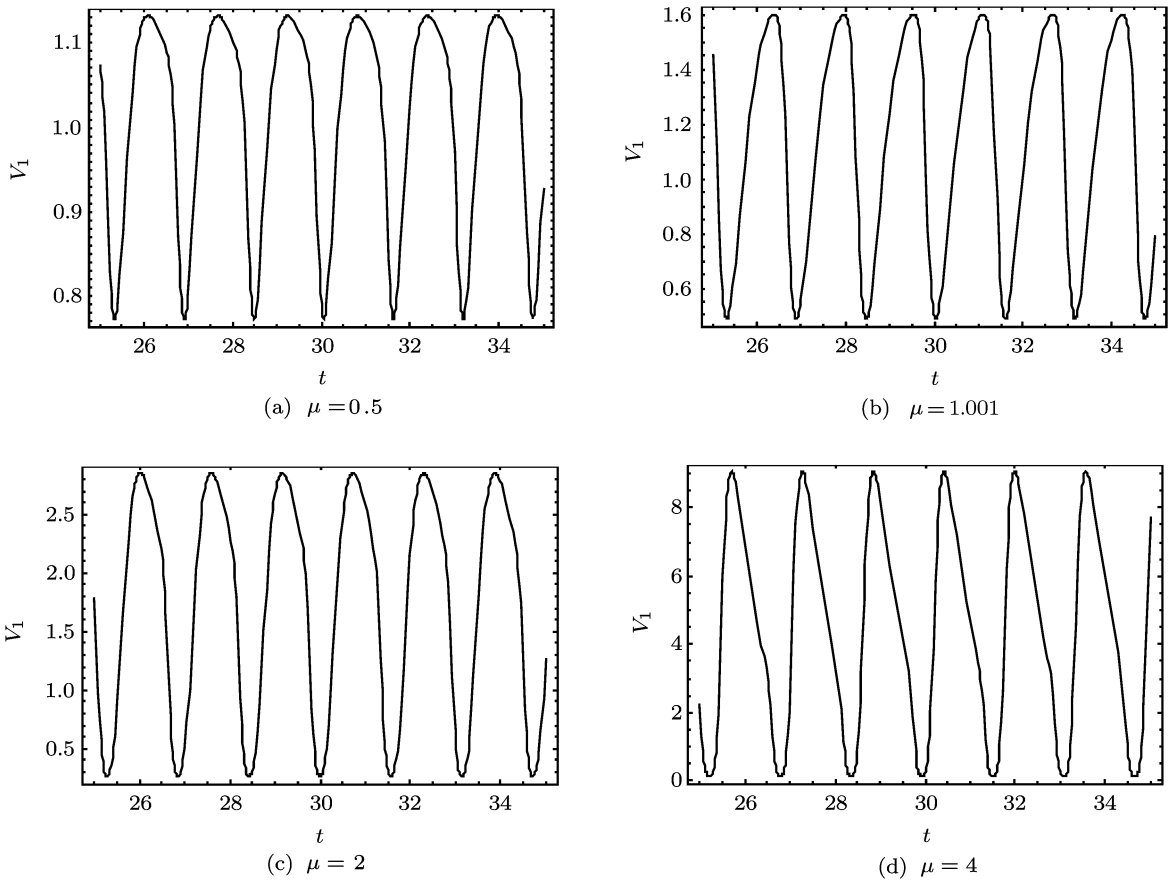


Fig.4. Gaussian pumping function with $\eta = 1/1000$, $\gamma_r = 10$, $\omega = 2$, $\tau_0 = 9$, and $\mu = 0.5$ (a), 1.001 (b), 2 (c), and 4 (d).

Figure 4 depicts the minimum variance V_1 as a function of time τ (τ ranges from 25 to 35). We can see from Fig.4 that (i) from above threshold to below threshold, in the presence of the periodic pumping, the values of variance V_1 attain minimum values of 0.77, 0.48, 0.28 and 0.2 in Figs.4(a), 4(b), 4(c) and 4(d), respectively; (ii) a comparison between Fig.4 and Fig.3 shows that their general features are unchanged but the peaks become denser, the range of variation of V_1 becomes large. However the most interesting thing is that the minimum value (0.2) in Fig.4(d) is much lower than that (0.5) in Fig.3(d), and the minimum values 0.77, 0.48, 0.28 and 0.2 can also be described by the formula $1/(1+\mu)$ for $\mu = 0.5$, 1.001, 2 and 4, respectively.

5. General solution for frequency non-degenerated OPA system

In Section 2, the solution of the NOPA with two modes of the same frequency but different polarizations has been derived. In this Section, the above solution will be generalized to the cases of two-modes with two different frequencies and different polarizations. ω_1 and ω_2 denote the frequencies of two-modes as $\omega_1 = \omega_0/2 - \Delta\omega$ and $\omega_2 = \omega_0/2 + \Delta\omega$; $\omega_1 + \omega_2 = \omega_0$, where ω_0 is the pumping wave frequency. The intracavity fields are described by the boson operators a_0, a_1 and a_2 that satisfy the following equations[16] (omitting the noise term):

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}a_1 &= -\gamma_1 a_1 + \chi a_2^\dagger a_0,\\
\frac{\mathrm{d}}{\mathrm{d}t}a_2 &= -\gamma_2 a_2 + \chi a_1^\dagger a_0,\\
\frac{\mathrm{d}}{\mathrm{d}t}a_0 &= \varepsilon_0 - \gamma_0 a_0 + \chi a_1 a_2. \qquad (17)
\end{aligned}$$

Equation (17) is suitable for the frequency-degenerated OPA system. We denote $\bar{a}_1$ and $\bar{a}_2$ as the parametric wave operators and add terms $\partial\bar{a}_1/\partial t = \mathrm{i}\,\Delta\omega\bar{a}_1$, $\partial\bar{a}_2/\partial t = -\mathrm{i}\,\Delta\omega\bar{a}_2$ to the right hand-side of Eq.(17), and take into account the oscillation frequency detuning of parametric waves relative to the frequency $\omega_0/2$, namely, $\omega_1 = \omega_0/2 + \Delta\omega$ and $\omega_2 = \omega_0/2 - \Delta\omega$. For the frequency non-degenerated OPA system, we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\bar{a}_1 &= \mathrm{i}\,\Delta\omega\bar{a}_1 - \gamma_1\bar{a}_1 + \chi\bar{a}_2^\dagger a_0,\\
\frac{\mathrm{d}}{\mathrm{d}t}\bar{a}_2 &= -\mathrm{i}\,\Delta\omega\bar{a}_2 - \gamma_2\bar{a}_2 + \chi\bar{a}_1^\dagger a_0,\\
\frac{\mathrm{d}}{\mathrm{d}t}a_0 &= \varepsilon_0 - \gamma_0 a_0 + \chi\bar{a}_1\bar{a}_2. \qquad (18)
\end{aligned}$$

The solutions, $\bar{a}_1$ and $\bar{a}_2$, are $\bar{a}_1 = a_1\exp[\mathrm{i}\,\Delta\omega t]$ and $\bar{a}_2 = a_2\exp[-\mathrm{i}\,\Delta\omega t]$, where a_1 and a_2 satisfy Eq.(17) for the frequency-degenerated OPA system; $\gamma_1 = \gamma_2$. These solutions show that $\bar{a}_1$ and $\bar{a}_2$ depend on a_1, a_2 respectively. The quantum fluctuations $\langle(\Delta X_1)^2\rangle$, $\langle(\Delta Y_2)^2\rangle$, $\langle\Delta X_1\Delta Y_2\rangle$ and variance V_1 calculations have been achieved through the solution of degenerated OPA system Eq.(17), which reveals the fact that the solutions of $\langle(\Delta\bar{X}_1)^2\rangle$, $\langle(\Delta\bar{Y}_2)^2\rangle$, $\langle\Delta\bar{X}_1\Delta\bar{Y}_2\rangle$, and $\bar{V}_1$ for the frequency non-degenerated OPA system may be expressed in terms of $\langle(\Delta X_1)^2\rangle$, $\langle(\Delta Y_2)^2\rangle$,$\langle\Delta X_1\Delta Y_2\rangle$ and V_1 for the frequency-degenerated OPA system. Actually, through the derivation given in the appendix, the expressions are obtained as

$$\begin{aligned}
\langle(\Delta\bar{X}_1)^2\rangle &= \cos^2\varphi\,\langle(\Delta X_1)^2\rangle + \sin^2\varphi\,\langle(\Delta Y_2)^2\rangle\\
&\quad - 2\sin\varphi\cos\varphi\,\langle\Delta X_1\Delta Y_2\rangle,\\
\langle(\Delta\bar{Y}_2)^2\rangle &= \sin^2\varphi\,\langle(\Delta X_1)^2\rangle + \cos^2\varphi\,\langle(\Delta Y_2)^2\rangle\\
&\quad + 2\sin\varphi\cos\varphi\,\langle\Delta X_1\Delta Y_2\rangle,\\
\langle\Delta\bar{X}_1\Delta\bar{Y}_2\rangle &= \cos\varphi\sin\varphi(\langle(\Delta X_1)^2\rangle - \langle(\Delta Y_2)^2\rangle)\\
&\quad + (\cos^2\varphi - \sin^2\varphi)\,\langle\Delta X_1\Delta Y_2\rangle, \qquad (19)
\end{aligned}$$

where $\varphi = \Delta\omega t$. Using expression (19), we can evaluate the variance $\bar{V}_1$ for the frequency non-degenerated OPA system as

$$\bar{V}_1 = \langle(\Delta\bar{X}_1)^2\rangle - \frac{\langle\Delta\bar{X}_1\Delta\bar{Y}_2\rangle^2}{\langle(\Delta\bar{Y}_2)^2\rangle}. \qquad (20)$$

Usually the relation $\langle(\Delta X_1)^2\rangle = \langle(\Delta Y_2)^2\rangle$ is established through numerical calculation. In this case, expression (19) reduces into

$$\begin{aligned}
\langle(\Delta\bar{X}_1)^2\rangle &= \langle(\Delta X_1)^2\rangle - \sin 2\varphi\,\langle\Delta X_1\Delta Y_2\rangle,\\
\langle(\Delta\bar{Y}_2)^2\rangle &= \langle(\Delta X_1)^2\rangle + \sin 2\varphi\,\langle\Delta X_1\Delta Y_2\rangle,\\
\langle\Delta\bar{X}_1\Delta\bar{Y}_2\rangle &= \cos 2\varphi\,\langle\Delta X_1\Delta Y_2\rangle. \qquad (21)
\end{aligned}$$

The variance $\bar{V}_1$ becomes

$$\begin{aligned}
\bar{V}_1 &= \frac{\langle(\Delta X_1)^2\rangle^2 - \langle\Delta X_1\Delta Y_2\rangle^2}{\langle(\Delta X_1)^2\rangle + \sin 2\varphi\,\langle\Delta X_1\Delta Y_2\rangle}\\
&= \frac{V_1}{1 + \sin 2\varphi\dfrac{\langle\Delta X_1\Delta Y_2\rangle}{\langle(\Delta X_1)^2\rangle}}. \qquad (22)
\end{aligned}$$

We note that expression (22) is derived under the assumption of the signal parametric wave with a detuning $\Delta\omega$ relative to the frequency $\omega_0/2$, however this detuning may be $-\Delta\omega$. As a result, the sign "+" before $\sin 2\varphi$ in variance $\bar{V}_1$ of expression (22) should be changed into the negative sign, i.e. "–". The key problem is that we cannot definitely say whether the signal detuning is $\Delta\omega$ or $-\Delta\omega$. The reasonable assumption is that each case has an equal probability 1/2, therefore from a physical consideration, expression (22) should be replaced by taking the arithmetic average value as

$$\begin{aligned}\bar{V}_{1A} &= \frac{1}{2}\left(\frac{V_1}{1+\sin 2\varphi\dfrac{\langle \Delta X_1 \Delta Y_2\rangle}{\langle (\Delta X_1)^2\rangle}} + \frac{V_1}{1-\sin 2\varphi\dfrac{\langle \Delta X_1 \Delta Y_2\rangle}{\langle (\Delta X_1)^2\rangle}}\right) \\ &= \frac{V_1}{1-\sin^2 2\varphi\dfrac{\langle \Delta X_1 \Delta Y_2\rangle^2}{\langle (\Delta X_1)^2\rangle^2}}. \end{aligned} \tag{23}$$

Expression (23) suggests that the variance $\bar{V}_{1A}$ for the frequency non-degenerated OPA system is always greater than the variance V_1 for the frequency-degenerated OPA system. The minimum value of $\bar{V}_{1A}$ occurs at $\sin 2\varphi = 0$, and $V_1 = \bar{V}_{1A}$. For the sine function pumping, in the following we study numerically the frequency non-degenerated OPA system, based on the solution shown in Fig.3(c), the results of V_1 and $\bar{V}_{1A}$ versus τ are shown in Figs.5 and 6, respectively.

Figures 5(a)–5(d) show the evolutions of variances V_1 and $\bar{V}_{1A}$ with τ for $V_1(\Delta\omega = 0)$, $\bar{V}_{1A}(\Delta\omega/\gamma_1 = \pi/3)$, $\bar{V}_{1A}(\Delta\omega/\gamma_1 = \pi/9)$, and $\bar{V}_{1A}(\Delta\omega/\gamma_1 = \pi/27)$, respectively. The frequency detuning curves indicated in Figs.5(b) –5(d) display some irregular features in comparison with the curves in Fig.5(a).

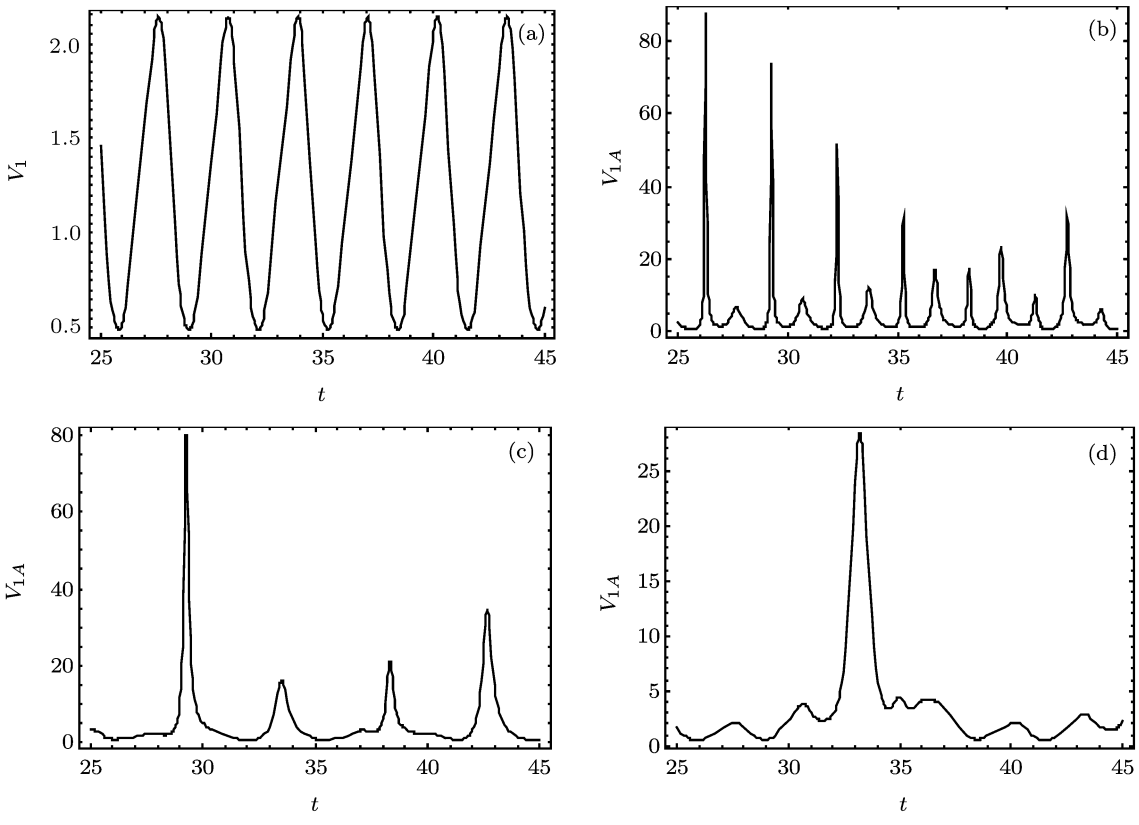


Fig.5. Minimum variances V_1 and $\bar{V}_{1A}$ versus τ with $\mu = 2$, $\eta = 1/1000$, $\gamma_r = 10$.

Figure 6 shows the minimum variances: (a) $V_{1\,\min}(\Delta\omega = 0) = 0.5$, (b) $\bar{V}_{1A\,\min}(\Delta\omega/\gamma_1 = \pi/3) \simeq 0.8$, (c) $\bar{V}_{1A\,\min}(\Delta\omega/\gamma_1 = \pi/9) \simeq 0.7$, and (d) $\bar{V}_{1A\,\min}(\Delta\omega/\gamma_1 = \pi/27) \simeq 0.5$. The most important thing is that the detuning leads to an increase in variance as shown in Fig.6.

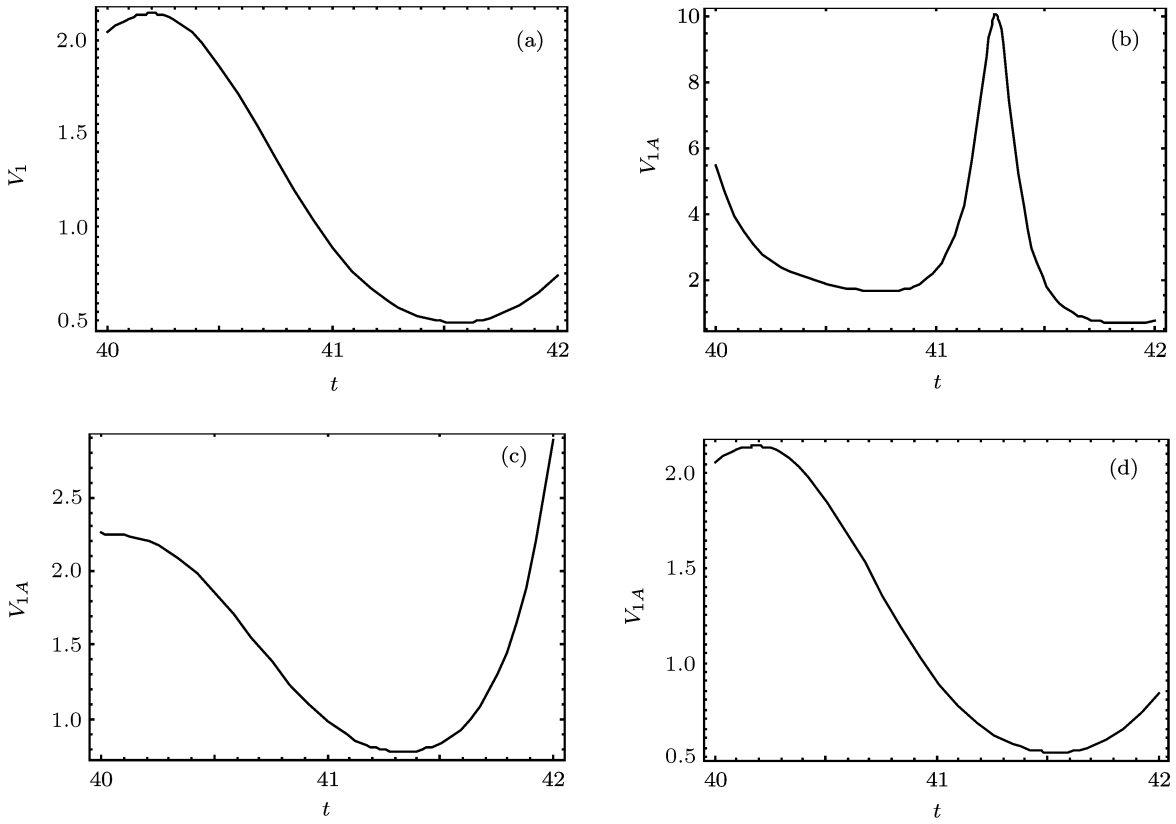


Fig.6. Minimum variances $V_{1\,\min}$ and $\bar{V}_{1A\,\min}$ versus τ with $\mu = 2$, $\eta = 1/1000$, $\gamma_r = 10$.

6. Conclusions

The main results of this paper are summarized as follows.

(i) We investigated theoretically the solution of the time-dependent F-P equation with periodic pumping driven for NOPA system. The quantum fluctuation and entanglement, namely the variance of V_1 for NOPA system, is calculated from expressions (13)-(16).

(ii) In a NOPA system driven by the periodic pump, we adopt two kinds of pump functions:

(a) sine function $\bar{\varepsilon}(\tau) = \mu \times (1 + \sin\omega\tau)$;

(b) Gaussian function $\bar{\varepsilon}(\tau) = \mu \times 5.45\exp[-10 \times \sin^2(\omega\tau)]$.

The squeezings for both cases are shown in Figs.1 and 2. A comparison between the curve for parametric light pulse $\alpha_1/100$ and the curve for squeezing $4\left\langle(\Delta y)^2\right\rangle$ shows that most of the light pulse energy is squeezed in a maximum squeezing state.

(iii) The minimum quantum fluctuations of variance V_1 above threshold and below threshold for the sine function periodic pumping and the Gaussian function periodic pumping are shown in Figs.3 and 4. Due to the accumulation effect of the quantum fluctuations, the envelope of V_1 curve increases linearly with the increase of τ. Both the maximum squeezing and the minimum value of variance V_1 (at least in the case of the Gaussian function periodic pumping) can be described approximately by the linear theory formula $1/(1+\mu)$. The time-modulated pumping produces an improvement in the peak values of the generated EPR-like entanglement. The minimum variance V_1 that determines the degree of entanglement is lower than that for the stationary pump, so that the entanglement at specific, periodic times is stronger than that for the stationary case.

(iv) The generalization of the frequency-degenerated OPA system to the frequency non-degenerated OPA system is investigated. We find out that the detuning of the oscillation frequencies ω_1 and ω_2 from $\omega_0/2$, can lead to an increase of the variance as shown in Figs.5 and 6.

Appendix

Let $\beta_1 = (\alpha_1 + \alpha_2)/\sqrt{2}$, $\tilde{\beta}_1 = (\tilde{\alpha}_1 + \tilde{\alpha}_2)/\sqrt{2}$, and $\beta_2 = (\alpha_1 - \alpha_2)/\sqrt{2}$, $\tilde{\beta}_2 = (\tilde{\alpha}_1 - \tilde{\alpha}_2)/\sqrt{2}$ denote the real parts and the imaginary parts of b_1 and b_2, respectively, where $\alpha_1 = (a_1 + a_1^*)/2$, $\tilde{\alpha}_1 = (a_1 - a_1^*)/2\,\mathrm{i}$, $\alpha_2 = (a_2 + a_2^*)/2$, and $\tilde{\alpha}_2 = (a_2 - a_2^*)/2\,\mathrm{i}$. Similarly, for the modified operators $\bar{b}_1$ and $\bar{b}_2$ and $\varphi = \Delta\omega t$, we have

$$
\begin{aligned}
\bar{\beta}_1 &= \frac{\bar{\alpha}_1 + \bar{\alpha}_2}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\bar{a}_1 + \bar{a}_1^*}{2} + \frac{\bar{a}_2 + \bar{a}_2^*}{2}\right) \\
&= \frac{1}{\sqrt{2}}\left(\frac{a_1 \exp[\mathrm{i}\,\varphi] + a_1^* \exp[-\mathrm{i}\,\varphi]}{2} + \frac{a_2 \exp[-\mathrm{i}\,\varphi] + a_2^* \exp[\mathrm{i}\,\varphi]}{2}\right) \\
&= \frac{1}{\sqrt{2}}\left(\cos\varphi\frac{a_1 + a_1^*}{2} + \mathrm{i}\sin\varphi\frac{a_1 - a_1^*}{2} + \cos\varphi\frac{a_2 + a_2^*}{2} - \mathrm{i}\sin\varphi\frac{a_2 - a_2^*}{2}\right) \\
&= \frac{1}{\sqrt{2}}(\alpha_1\cos\varphi - \tilde{\alpha}_1\sin\varphi + \alpha_2\cos\varphi + \tilde{\alpha}_2\sin\varphi) = \beta_1\cos\varphi - \tilde{\beta}_2\sin\varphi, \\
\tilde{\bar{\beta}}_1 &= \frac{\tilde{\bar{\alpha}}_1 + \tilde{\bar{\alpha}}_2}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\bar{a}_1 - \bar{a}_1^*}{2\,\mathrm{i}} + \frac{\bar{a}_2 - \bar{a}_2^*}{2\,\mathrm{i}}\right) \\
&= \frac{1}{\sqrt{2}}\left(\frac{a_1 \exp[\mathrm{i}\,\varphi] - a_1^* \exp[-\mathrm{i}\,\varphi]}{2\,\mathrm{i}} + \frac{a_2 \exp[-\mathrm{i}\,\varphi] - a_2^* \exp[\mathrm{i}\,\varphi]}{2\,\mathrm{i}}\right) \\
&= \frac{1}{\sqrt{2}}\left(\cos\varphi\frac{a_1 - a_1^*}{2\,\mathrm{i}} + \sin\varphi\frac{a_1 + a_1^*}{2} + \cos\varphi\frac{a_2 - a_2^*}{2\,\mathrm{i}} - \sin\varphi\frac{a_2 + a_2^*}{2}\right) \\
&= \frac{1}{\sqrt{2}}(\tilde{\alpha}_1\cos\varphi + \alpha_1\sin\varphi + \tilde{\alpha}_2\cos\varphi - \alpha_2\sin\varphi) = \tilde{\beta}_1\cos\varphi + \beta_2\sin\varphi, \\
\bar{\beta}_2 &= \frac{\bar{\alpha}_1 - \bar{\alpha}_2}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\bar{a}_1 + \bar{a}_1^*}{2} - \frac{\bar{a}_2 + \bar{a}_2^*}{2}\right) \\
&= \frac{1}{\sqrt{2}}\left(\frac{a_1 \exp[\mathrm{i}\,\varphi] + a_1^* \exp[-\mathrm{i}\,\varphi]}{2} - \frac{a_2 \exp[-\mathrm{i}\,\varphi] + a_2^* \exp[\mathrm{i}\,\varphi]}{2}\right) \\
&= \frac{1}{\sqrt{2}}\left(\cos\varphi\frac{a_1 + a_1^*}{2} + \mathrm{i}\sin\varphi\frac{a_1 - a_1^*}{2} - \cos\varphi\frac{a_2 + a_2^*}{2} + \mathrm{i}\sin\varphi\frac{a_2 - a_2^*}{2}\right) \\
&= \frac{1}{\sqrt{2}}(\alpha_1\cos\varphi - \tilde{\alpha}_1\sin\varphi - \alpha_2\cos\varphi - \tilde{\alpha}_2\sin\varphi) = \beta_2\cos\varphi - \tilde{\beta}_1\sin\varphi, \\
\tilde{\bar{\beta}}_2 &= \frac{\tilde{\bar{\alpha}}_1 - \tilde{\bar{\alpha}}_2}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\bar{a}_1 - \bar{a}_1^*}{2\,\mathrm{i}} - \frac{\bar{a}_2 - \bar{a}_2^*}{2\,\mathrm{i}}\right) \\
&= \frac{1}{\sqrt{2}}\left(\frac{a_1 \exp[\mathrm{i}\,\varphi] - a_1^* \exp[-\mathrm{i}\,\varphi]}{2\,\mathrm{i}} - \frac{a_2 \exp[-\mathrm{i}\,\varphi] - a_2^* \exp[\mathrm{i}\,\varphi]}{2\,\mathrm{i}}\right) \\
&= \frac{1}{\sqrt{2}}\left(\cos\varphi\frac{a_1 - a_1^*}{2\,\mathrm{i}} + \sin\varphi\frac{a_1 + a_1^*}{2} - \cos\varphi\frac{a_2 - a_2^*}{2\,\mathrm{i}} + \sin\varphi\frac{a_2 + a_2^*}{2\,\mathrm{i}}\right) \\
&= \frac{1}{\sqrt{2}}(\tilde{\alpha}_1\cos\varphi + \alpha_1\sin\varphi - \tilde{\alpha}_2\cos\varphi + \alpha_2\sin\varphi) = \tilde{\beta}_2\cos\varphi + \beta_1\sin\varphi. \qquad \text{(A1)}
\end{aligned}
$$

From expression (A1), we can calculate the sum of the variations of the real parts and the sum of the imaginary parts of $\bar{b}_1$ and $\bar{b}_2$, and denote them by $\Delta\bar{X}_1$ and $\Delta\bar{Y}_2$, respectively, as follows:

$$
\begin{aligned}
\Delta\bar{X}_1 &= \Delta\bar{\beta}_1 + \Delta\bar{\beta}_2 = \Delta\beta_1\cos\varphi - \Delta\tilde{\beta}_2\sin\varphi + \Delta\beta_2\cos\varphi - \Delta\tilde{\beta}_1\sin\varphi \\
&= (\Delta\beta_1 + \Delta\beta_2)\cos\varphi - (\Delta\tilde{\beta}_1 + \Delta\tilde{\beta}_2)\sin\varphi = \Delta X_1\cos\varphi - \Delta Y_2\sin\varphi, \\
\Delta\bar{Y}_2 &= \Delta\tilde{\bar{\beta}}_1 + \Delta\tilde{\bar{\beta}}_2 = \Delta\tilde{\beta}_1\cos\varphi + \Delta\beta_2\sin\varphi + \Delta\tilde{\beta}_2\cos\varphi + \Delta\beta_1\sin\varphi \\
&= (\Delta\tilde{\beta}_1 + \Delta\tilde{\beta}_2)\cos\varphi + (\Delta\beta_1 + \Delta\beta_2)\sin\varphi = \Delta Y_2\cos\varphi + \Delta X_1\sin\varphi. \qquad \text{(A2)}
\end{aligned}
$$

Expression (A2) and its inverse can be expressed in the matrix form as follows:

$$\begin{pmatrix} \Delta\bar{X}_1 \\ \Delta\bar{Y}_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \Delta X_1 \\ \Delta Y_2 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} \Delta X_1 \\ \Delta Y_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \Delta\bar{X}_1 \\ \Delta\bar{Y}_2 \end{pmatrix}. \tag{A3}$$

Equation (A3) yields the variations

$$\begin{aligned} \left\langle (\Delta\bar{X}_1)^2 \right\rangle &= \cos^2\varphi \left\langle (\Delta X_1)^2 \right\rangle + \sin^2\varphi \left\langle (\Delta Y_2)^2 \right\rangle - 2\sin\varphi\cos\varphi \left\langle \Delta X_1 \Delta Y_2 \right\rangle, \\ \left\langle (\Delta\bar{Y}_2)^2 \right\rangle &= \sin^2\varphi \left\langle (\Delta X_1)^2 \right\rangle + \cos^2\varphi \left\langle (\Delta Y_2)^2 \right\rangle + 2\sin\varphi\cos\varphi \left\langle \Delta X_1 \Delta Y_2 \right\rangle, \\ \left\langle \Delta\bar{X}_1 \Delta\bar{Y}_2 \right\rangle &= \cos\varphi\sin\varphi(\left\langle (\Delta X_1)^2 \right\rangle - \left\langle (\Delta Y_2)^2 \right\rangle) + (\cos^2\varphi - \sin^2\varphi) \left\langle \Delta X_1 \Delta Y_2 \right\rangle. \end{aligned} \tag{A4}$$

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