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Entanglement evolution and transfer in a double Tavis–Cumming model in cavity QED

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We have studied entanglement evolution and transfer in a double Tavis–Cumming model where two pairs of entangled two-level atoms AB and CD interact with two single-mode cavity fields a and b. We show that the Bell-like initial state of atoms AB can exhibit entanglement sudden death which should be independent of the initial entanglement of atoms CD. Also, we show that the initial entanglement of one atomic pair can be transferred into another pair, as well as the possible subsystems, that become entangled during evolution.

Keywords: entanglement evolution, entanglement transfer, Tavis–Cumming model
PACC: 0365, 0367

1. Introduction

Entanglement is not only a fundamental concept in quantum physics, but also identified as a key resource to perform various intriguing global tasks in quantum communication and quantum computation.[1,2] However, in the process of entanglement distribution and particles' manipulation, each particle is unavoidably exposed to its uncontrollable environment. The interaction between system and external environment leads to decoherence which will spoil the necessary entanglement of the shared states. It is therefore of increasing importance to understand entanglement from its role and manifestation in realistic systems. Recently, a peculiar dynamical feature of entanglement has been experimentally confirmed for entangled photon pairs[3] and atomic ensembles[4] that entanglement may disappear in a finite time, in clear contrast with decoherence of an individual qubit that decays asymptotically. Such a surprising phenomenon was termed entanglement sudden death (ESD) and has attracted much attention[5−25] because of its intrinsic and practical interests. It was found that in Refs.[26]–[28] ESD could also appear in multipartite entangled systems besides bipartite entanglement. Here, a natural question in relation to ESD is: where does the lost entanglement finally go? Intuitively, one may think that the lost entanglement has to be transferred to other degrees of freedom of the system considered. It has been shown in Ref.[29] that when ESD occurs for the qubits, there also occurs entanglement sudden birth (ESB) for the corresponding environments. Also, it was shown that ESB can manifest before, simultaneously, or even after ESD.[29] The issues of ESD and ESB are associated to dynamical behaviors of entanglement as well as its transfer among all the possible degrees of freedom of a system, thus the studies of them are necessary and open.

In Refs.[13] and [14], the authors dealt with a double Jaynes–Cummings[30] model (JCM) in which two initially entangled two-level atoms A and B were independently coupled with separate cavity fields a and b, respectively, but there were no interactions at all between the subsystems Aa and Bb. Focusing on the atomic subsystem, they found out that depending on the type of atomic initial states, their entanglement may or may not exhibit ESD.[13,14] Through Ref.[13] and [14], one can see the dynamical features of two entangled qubits interacting locally with their own environments. The authors of Refs.[13] and [14] also formulated distinct forms of entanglement invariant for different initial states, which implies that atomic initial entanglement can be transferred into other degrees of freedom of the whole system. The same double JCM was re-studied in Ref.[27] in which the authors introduced an entanglement invariant under evolution, independent of the initial state of the atoms. In Ref.[27], the authors generalized the pairwise entanglement dynamics into multipartite case and considered a triple JCM in which three entangled atoms were initially prepared in GHZ-type states and locally coupled with three independent cavities. They stud-

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ied the entanglement dynamics for both subsystems of atoms and cavities.[27] The results apparently showed the entanglement transfer between the subsystems of atoms and cavities in multipartite system.[27] In most studies,[13−15,26,27] each particle is assumed to be coupled locally with its own environment. However, in some practical situations, e.g., entanglement swapping and entanglement distillation, particles should be brought into the same environment. In this case, besides the interactions between particles with their environment, there also exist interactions between particles themselves. Therefore, it is necessary and useful in practice to investigate entanglement dynamics for this more complex situation. Motivated by this consideration, in this paper we shall investigate a double Tavis–Cumming (TC) model[31] involving four two-level atoms and two single-mode cavities. The four atoms A, B, C and D are arranged in such a way that A and C are coupled with cavity a via TC interaction at one location, and the same for B and D with cavity b at another location. The two locations ACa and BDb are initially entangled through the entanglement between remote atoms but without any interactions between them. We shall consider not only the entanglement evolution of different atom pairs, but also the entanglement transfer and exchange among them.

2. Entanglement measure and the model

In two-qubit domains, there exist a number of good measures of entanglement such as concurrence[32] and negativity.[33,44] Although the afore-mentioned entanglement measures may be somewhat different quantitatively,[13,14] they are equivalent qualitatively to each other in the sense that all of them are equal to zero for unentangled (i.e., separable with respect to the bipartite partition) states. Here we adopt Wootters' concurrence[32] because of its convenience in definition, normalization and calculation. The concurrence C for the density matrix ρ of a two-qubit system is defined as

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (1)$$

where the quantities λ_i are the eigenvalues of the matrix $\zeta = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, arranged in the decreasing order. Here σ_y are the Pauli matrices and ρ^* is the complex conjugation of ρ in the standard basis: $C(\rho) = 0$ for separate states, whereas $C(\rho) = 1$ for maximally entangled states. For the two-qubit system ij, there are so-called X-form states[16] which are characterized by a density matrix with nonzero elements located only in the main diagonal and antidiagonal as

$$\rho^{ij} = \begin{pmatrix} \varrho_{11}^{ij} & 0 & 0 & \varrho_{14}^{ij} \\ 0 & \varrho_{22}^{ij} & \varrho_{23}^{ij} & 0 \\ 0 & \varrho_{32}^{ij} & \varrho_{33}^{ij} & 0 \\ \varrho_{41}^{ij} & 0 & 0 & \varrho_{44}^{ij} \end{pmatrix}, \quad (2)$$

where ϱ_{kk}^{ij} are real positive and $\varrho_{kl}^{ij} = (\varrho_{lk}^{ij})^*$ are generally complex. The concurrence of the X-form state (2) can be derived easily as

$$\begin{aligned} &C(\rho^{ij}) \\ &= 2\max\{0, |\varrho_{23}^{ij}| - \sqrt{\varrho_{11}^{ij}\varrho_{44}^{ij}}, |\varrho_{14}^{ij}| - \sqrt{\varrho_{22}^{ij}\varrho_{33}^{ij}}\}. \end{aligned} \quad (3)$$

Let us now describe the model we are interested in. The considered system consists of four identical two-level atoms A, B, C, and D with AC and BD being coupled with two independent cavities a and b, respectively, thus the Hamiltonian can be written as a sum of two separate TC interactions as

$$H = H_{AC} + H_{BD}, \quad (4)$$

where

$$\begin{aligned} H_{AC} = &\frac{\omega_0}{2}(\sigma_A^z + \sigma_C^z) + \omega a^\dagger a \\ &+ g\sum_{i=A,C}(a\sigma_i^+ + a^\dagger\sigma_i^-), \end{aligned} \quad (5)$$

and

$$\begin{aligned} H_{BD} = &\frac{\omega_0}{2}(\sigma_B^z + \sigma_D^z) + \omega b^\dagger b \\ &+ g\sum_{i=B,D}(b\sigma_i^+ + b^\dagger\sigma_i^-), \end{aligned} \quad (6)$$

In Eqs.(5) and (6) ω_0 and ω are the frequencies of atoms and cavities, respectively, and g is the coupling constant between the atoms and the corresponding cavities. Here, we assume the resonating case with $\omega_0 = \omega$.

3. Entanglement dynamics

Suppose the initial states of atom pairs AB and CD are prepared in the Bell-like forms as

$$|\psi(0)\rangle_{AB} = \cos\alpha|eg\rangle_{AB} + \sin\alpha|ge\rangle_{AB}, \quad (7)$$

$$|\psi(0)\rangle_{CD} = \cos\beta|eg\rangle_{CD} + \sin\beta|ge\rangle_{CD}. \quad (8)$$

By virtue of states (7) and (8), we can consider not only the case that both atom pairs AB and CD are

entangled, but also the case that one of them is separated by setting, say, $\beta = 0$. We assume the cavities a and b are initially in the vacuum. The total initial state including atoms and cavities at $t = 0$ is thus in the form

$$|\Psi(0)\rangle_{ABCDab} = |\psi(0)\rangle_{AB} \otimes |\psi(0)\rangle_{CD} \otimes |00\rangle_{ab}. \quad (9)$$

Under the governing of Hamiltonian (4), the total initial state after time t will evolve into a six-partite entangled state

$$\begin{aligned}
&|\Psi(t)\rangle_{ABCDab} \\
&= C_1(t)|egeg00\rangle + C_2(t)|ggeg10\rangle + C_3(t)|eggg10\rangle \\
&\quad + C_4(t)|gggg20\rangle + C_5(t)|egge00\rangle + C_6(t)|eggg01\rangle \\
&\quad + C_7(t)|eegg00\rangle + C_8(t)|ggge10\rangle + C_9(t)|gggg11\rangle \\
&\quad + C_{10}(t)|gegg10\rangle + C_{11}(t)|ggee00\rangle \\
&\quad + C_{12}(t)|ggeg01\rangle + C_{13}(t)|geeg00\rangle\rangle \\
&\quad + C_{14}(t)|gege00\rangle + C_{15}(t)|ggge01 \\
&\quad + C_{16}(t)|gegg01\rangle + C_{17}(t)|gggg02\rangle, \qquad (10)
\end{aligned}$$

where

$$\begin{aligned}
C_1(t) &= \frac{1}{3}\cos\alpha\cos\beta(2+\cos\sqrt{6}gt), \\
C_2(t) &= C_3(t) = -\frac{1}{\sqrt{6}}\mathrm{i}\cos\alpha\cos\beta\sin\sqrt{6}gt, \\
C_4(t) &= -\frac{\sqrt{2}}{3}\cos\alpha\cos\beta(1-\cos\sqrt{6}gt), \\
C_5(t) &= \frac{1}{8}\sin(\alpha+\beta)(3+\cos 2\sqrt{2}gt) \\
&\quad - \frac{1}{2}\sin(\alpha-\beta)\cos\sqrt{2}gt, \\
C_6(t) &= C_8(t) = \frac{\sqrt{2}}{4}\mathrm{i}[\sin(\alpha-\beta)\sin\sqrt{2}gt \\
&\quad - \frac{1}{2}\sin(\alpha+\beta)\sin 2\sqrt{2}gt], \\
C_7(t) &= \frac{1}{2}C_9(t) = C_{11}(t) \\
&= -\frac{1}{8}\sin(\alpha+\beta)(1-\cos 2\sqrt{2}gt), \\
C_{10}(t) &= C_{12}(t) = -\frac{\sqrt{2}}{4}\mathrm{i}[\sin(\alpha-\beta)\sin\sqrt{2}gt \\
&\quad + \frac{1}{2}\sin(\alpha+\beta)\sin 2\sqrt{2}gt], \\
C_{13}(t) &= \frac{1}{8}\sin(\alpha+\beta)(3+\cos 2\sqrt{2}gt) \\
&\quad + \frac{1}{2}\sin(\alpha-\beta)\cos\sqrt{2}gt, \\
C_{14}(t) &= \frac{1}{3}\sin\alpha\sin\beta(2+\cos\sqrt{6}gt), \\
C_{15}(t) &= C_{16}(t) = -\frac{1}{\sqrt{6}}\mathrm{i}\sin\alpha\sin\beta\sin\sqrt{6}gt, \\
C_{17}(t) &= -\frac{\sqrt{2}}{3}\sin\alpha\sin\beta(1-\cos\sqrt{6}gt). \qquad (11)
\end{aligned}$$

The reduced density matrix $\rho_{AB}(t)$ ($\rho_{CD}(t)$, $\rho_{AD}(t)$ and $\rho_{BC}(t)$) of atom pair AB (CD, AD and BC) can be obtained by tracing $\rho_{ABCDab} \equiv |\Psi(t)\rangle_{ABCDab}\langle\Psi(t)|$ over the degrees of freedom of atoms CD (AB, BC and AD) and cavities ab. Through straight forward calculations we can obtain $\rho_{AB}(t)$ ($\rho_{CD}(t)$, $\rho_{AD}(t)$ and $\rho_{BC}(t)$) in the X form (2) and the corresponding concurrence C_{AB} (C_{CD}, C_{AD} and C_{BC}) in the form (3).

We first consider the case that atoms AB are initially in the entangled form (7) while CD are in the separate state with $\beta = 0$ in Eq.(8). In Fig.1, we plot concurrences of atoms AB as functions of rescaled time gt and the initial entanglement degree in terms of α. Obviously, in the whole range of α, the entanglement of atoms AB non-smoothly becomes and stays at zero for a finite interval of time before recovering, i.e., it exhibits the so-called ESD behavior. The result is in striking contrast with that in double JC model,[13−15] where atomic state (7) does not undergo ESD for all the possible values of α. Here, a natural question arises; Where does the lost entanglement of atoms AB finally go? This question could be partially answered if we take other degrees of freedom in the total system into account. Therefore, in Fig.2, we plot simultaneously concurrences of atom pairs AB, CD, AD and BC as functions of rescaled time gt for $\alpha = \pi/4$ and $\beta = 0$. From Fig.2 we can see that along with the decrease of entanglement of atoms AB from maximum to zero, the independent atoms AD experience a process from ESB to ESD. During the entanglement vanishing period in atom pairs AB and AD, both of the independent atom pairs CD and BC undergo ESB. In the following evolution, ESD occurs for the newly produced entanglement of CD and BC, while the entanglement of AB and AD arise again during this period. Therefore, the entanglement evolution process is also an entanglement transfer and exchange process among the four atom pairs AB, AD, BC and CD. It is worthwhile noting that there exists a time gap in which the entanglement of all the four remote atom pairs AB, CD, AD and BC can vanish simultaneously. However, we know that the entanglement between the subsystems ACa and BDb could not be separated completely, hence there should exist other forms of entanglement in addition to the bipartite atomic entanglement, such as the bipartite entanglement between two cavities, multipartite entanglement between atoms as well as between atoms and cavities.

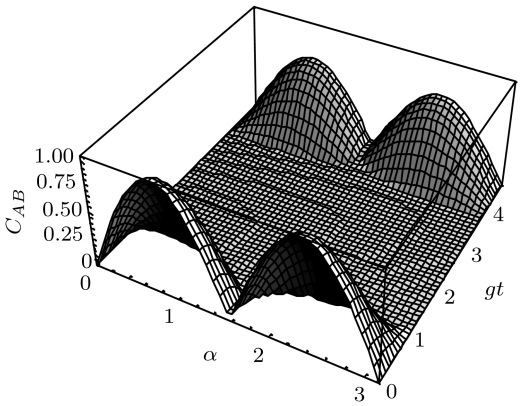


Fig.1. Concurrence of atoms AB as functions of rescaled time gt and the degree of initial entanglement in terms of α for $\beta = 0$.

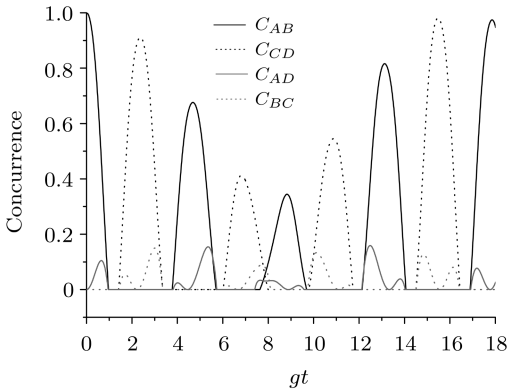


Fig.2. Concurrence of atom pairs AB, CD, AD and BC as a function of rescaled time gt for $\alpha = \pi/4$ and $\beta = 0$.

Next, we consider the case that both atom pairs AB and CD are initially entangled but with different entanglement degrees. Without loss of generality, we choose $\alpha = \pi/4$, $\beta = \pi/12$, and plot in Fig.3 concurrences of atom pairs AB, CD, AD and BC as functions of rescaled time gt. It is obvious that both atom pairs AB and CD suffer from ESD at approximately the same time. Also, we can see that although the initial entanglement of AB is bigger than that of CD, in the later time-evolving process, the entanglement of CD can instead be bigger than that of AB. Furthermore, the maximum entanglement values change between AB and CD during the whole evolution process. This phenomenon again represents the entanglement transfer and exchange among different degrees of freedom of the total system. The simultaneous ESD for the four remote atom pairs AB, AD, CD and BC can also be found in the present case.

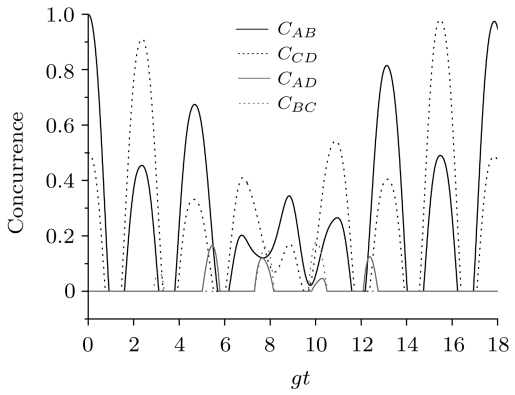


Fig.3. The same as in Fig. 2 but for $\alpha = \pi/4$ and $\beta = \pi/12$.

Though there are no interactions between two subsystems ACa and BDb, as shown by Hamiltonian (4), the local decoherence of atoms would destroy the global initial entanglement. In other words, it is the local photon-exchange between atoms and fields that leads to the decay of atomic entanglement. Therefore, the decay rate of atomic entanglement should be related to the number of excitons in the entire system. Since we assume that there are no photons in the cavities at $t = 0$, unexcited atoms in $|g\rangle$ do not interact with the field, only excited ones in $|e\rangle$ do. Therefore, the larger the number of the initial excited atoms, the sooner the vanishing of atomic entanglement. In the double TC model, there are always two excited atoms among the four. Therefore, for the same two cavity fields, the state (7) undergoes ESD in the double TC model, while it does not in the double JC model[14] where there is only one excited atom.

4. Conclusions

In conclusion, we have investigated entanglement dynamics of several bipartite subsystems in terms of a double TC model, where two atoms A and C are locally coupled with the same cavity a at one location, the same for another two atoms B and D with cavity b at another independent location. The initial states of atoms AB are always prepared in the entangled form (7) while the two cases of initial state of atoms CD are considered. Firstly, we assume that the initial states of atoms CD are separated. In this case we find that ESD can occur for atoms AB for all the possible values of its initial entanglement degree in terms of α in (7). During the ESD period of atoms AB, the entanglements of independent atom pairs CD and BC undergo

an ESB process, which can be thought as an entanglement transfer process from atom pair AB to CD and BC. It is interesting that the ESD can appear simultaneously for the remote atom pairs AB, CD, AD and BC, as shown in Fig.2. We know that the two subsystems ACa and BDb would not be separated completely, therefore beyond the bipartite atomic entanglement there should exist other entanglement forms which properly demonstrate the spreading out of entanglement over all of the system's degrees of freedom that may become entangled with each other in all possible ways through the underlying local interactions. In the second case, the initial state of atoms CD is entangled but with different entanglement degrees from that of atoms AB. Focusing on the atomic pairs AB and CD, we show that both their entanglements suffer from ESD and also the entanglement amount can exchange between them. Since our model is related to realistic quantum information process, such as entanglement swapping and entanglement distillation, the present results would be suitable for implementation into the framework of cavity QED.

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