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Entanglement dynamics between an isolated atom and a moving atom in the cavity*

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The entanglement dynamics between an isolated atom and a moving atom interacting with a cavity field is investigated. The results show that there appears sudden death of entanglement between the isolated atom and the moving atom and that the time of entanglement sudden death (ESD) is independent of the initial state of the system. It is interesting that the isolated atom can also entangle with a cavity field, though they do not interact with each other originally, which stems from the fact that the entanglement between the isolated atom and the moving atom may turn into the entanglement between the isolated atom and the cavity.

Keywords: entanglement, sudden death, transfer
PACC: 0365, 4250

1. Introduction

Entanglement is a remarkable feature of quantum mechanics, and it is a key resource of quantum computation and quantum information processing.[1] Recently much interest has been aroused in the evolution of the joint entanglement of a pair of qubits exposed to local noisy environments. The reason is related to the discovery by Yu and Eberly.[2] In open systems, entanglement can fall abruptly to zero and will maintain zero for a period of time before entanglement recovers. The occurrence of this phenomenon, termed entanglement sudden death (ESD), has recently received a lot of attention,[3−14] and this phenomenon has been observed in the experiments of entangled photon pairs[15] and atomic ensembles.[16]

The pure quantum nature of entanglement is the property of nonlocal correlation between widely separated particles which have once interacted in the past. Alternatively, much effort has been made to produce the entanglement locally, although there has also been aroused the interest in the construction of entanglement non-locally. One of the schemes to produce entanglement non-locally is entanglement swapping, which enables us to entangle two quantum systems that have never interacted directly with each other.[17] There has recently been another scheme, i.e. entanglement transfer from one quantum system to another. In particular, the entanglement transfer from the field to a qubit system has a primary importance and has been widely studied.[18−21]

In the present paper, we shall investigate the entanglement dynamics of the two space-separated atoms. One of them is moving in the Jaynes–Cummings (J–C) cavity and the other is isolated completely. The evolution of the entanglement between the non-interacting atoms or between the atom and cavity shows some striking features such as ESD and entanglement transfer.

2. Model and its solution

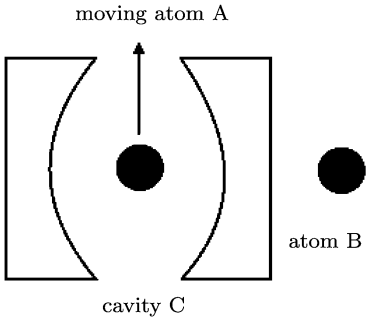


Fig.1. Schematic diagram of the model used in this paper. The atom B is isolated completely.

We now consider a system consisting of a Jaynes–Cummings model with a moving atom A and an iso-

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lated atom B. There is no interaction between atom A and atom B, neither is there an interaction between atom B and cavity C (see Fig.1). Compared with double Jaynes–Cummings model,[22] the model used in the present paper has the advantage, i.e., due to the isolated atom B located outside cavity C, we can conveniently, experimentally manipulate the entanglement between the moving atom A and the isolated atom B.

The effective Hamiltonian for the system in the rotating-wave approximation is ($\hbar = 1$)

$$H = \omega_0 a^\dagger a + \omega \sum_{i=A}^{B} S_z^i + gf(z)(aS_+^A + a^\dagger S_-^A), \tag{1}$$

where S_z^i and $S_\pm^i$ are the pseudo-spin operators of the i-th atom, $a^\dagger(a)$ is the photon creation (annihilation) operator of the coherent mode field, ω_0 and ω are the frequencies of the atomic transition and the mode field, g represents the strength of interaction between the atom A and the cavity C, $f(z)$ is the shape function of the atomic motion along the z-axis so that only the z-dependence of the mode field function needs taking into account. The atomic motion can be incorporated in the usual way

$$f(z) \to f(vt), \tag{2}$$

where v denotes atomic motion speed. To be specific, we define the $\mathrm{TEM}_{\mathrm{mnp}}$ mode as

$$f(z) = \sin\left(\frac{\pi p v t}{L}\right), \tag{3}$$

where p represents the number of half-wave lengths of the field mode inside a cavity of length L.

For simplicity, we consider the resonant case ($\omega = \omega_0$) and assume that the cavity is prepared initially in the Fock state $|n\rangle$ and the two atoms are combined into the Bell state:

$$|\psi_{\mathrm{atom}}(0)\rangle = \cos\alpha\,|+-\rangle + \sin\alpha\,|-+\rangle \tag{4}$$

with the first sign in brackets on the right-hand side denoting the state of atom A and the second denoting the state of atom B (+ represents excited state, - represents ground state). The initial state for the total system is given by

$$|\psi(0)\rangle = \cos\alpha\,|+-n\rangle + \sin\alpha\,|-+n\rangle\,. \tag{5}$$

Using the standard technique,[23] it can be shown that this Hamiltonian gives rise to the following time evolution operator in the interaction picture

$$U_I(t) = \begin{pmatrix} a_{11} & 0 & a_{13} & 0 \\ 0 & a_{22} & 0 & a_{24} \\ a_{31} & 0 & a_{33} & 0 \\ 0 & a_{42} & 0 & a_{44} \end{pmatrix}, \tag{6}$$

where

$$\begin{aligned} a_{11} &= a_{22} = \cos(g\Theta(t)\sqrt{aa^\dagger}), \\ a_{13} &= a_{24} = -\,\mathrm{i}\,\frac{a}{\sqrt{aa^\dagger}}\sin(g\Theta(t)\sqrt{aa^\dagger}), \\ a_{31} &= a_{42} = -\,\mathrm{i}\,\frac{a^\dagger}{\sqrt{a^\dagger a}}\sin(g\Theta(t)\sqrt{a^\dagger a}), \\ a_{33} &= a_{44} = \cos(g\Theta(t)\sqrt{a^\dagger a}) \end{aligned} \tag{7}$$

with

$$\Theta(t) = \int_0^t f(vt')\mathrm{d}t' = \frac{L}{\pi p v}\left[1 - \cos\frac{\pi p v t}{L}\right]. \tag{8}$$

For a particular choice of the atomic motion speed $v = gL/\pi$, $\Theta(t)$ becomes

$$\Theta(t) = \frac{1}{pg}[1 - \cos(pgt)]. \tag{9}$$

At any time $t > 0$, the state vector of the system is given by

$$\begin{aligned} |\psi(t)\rangle &= U_I(t)\,|\psi(0)\rangle \\ &= x_1\,|++n-1\rangle + x_2\,|+-n\rangle \\ &\quad + x_3\,|-+n\rangle + x_4\,|--n+1\rangle\,, \end{aligned} \tag{10}$$

where the coefficients are given as the following time-dependent expressions

$$\begin{aligned} x_1 &= -\,\mathrm{i}\sin\alpha\sin(g\Theta(t)\sqrt{n}), \\ x_2 &= \cos\alpha\cos(g\Theta(t)\sqrt{n+1}), \\ x_3 &= \sin\alpha\cos(g\Theta(t)\sqrt{n}), \\ x_4 &= -\,\mathrm{i}\cos\alpha\sin(g\Theta(t)\sqrt{n+1}). \end{aligned} \tag{11}$$

3. Entanglement between an isolated atom and a moving atom

It has been demonstrated that for the qubit-qubit or qubit-qutrit case, non-positivity of the partial transposition is a necessity and sufficient condition for the entanglement of a mixed state.[24,25] The entanglement of the system can be measured by

$$E = 2\sum_i |\mu_i| \tag{12}$$

where μ_i is the negative eigenvalue of ρ^{T_X}, and T_X denotes the partial transposition with respect to the system X. The value $E = 1$ corresponds to the maximum entanglement while $E = 0$ indicates that the system is in a separable state.[24,25]

The information about the entanglement between two atoms is contained in the reduced density matrix $\rho_{\rm AB}$ for the two atoms which can be obtained from expression (10) by tracing out the photonic parts of the total pure state. The $\rho_{\rm AB}$ can be written in the basis $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$ as

$$\rho_{\rm AB} = \begin{pmatrix} |x_1|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_2 x_3^* & 0 \\ 0 & x_3 x_2^* & |x_3|^2 & 0 \\ 0 & 0 & 0 & |x_4|^2 \end{pmatrix}. \qquad (13)$$

The time-dependent matrix elements are given by expression (11).

The entanglement between an isolated atom B and a moving atom A can be measured by

$$\begin{aligned} E_{\rm AB} = \max(0, &-\cos^2\alpha \sin^2(\sqrt{n+1} g\Theta(t)) \\ &- \sin^2\alpha \sin^2(\sqrt{n} g\Theta(t)) \\ &+ [(\cos^2\alpha \sin^2(\sqrt{n+1} g\Theta(t)) \\ &- \sin^2\alpha \sin^2(\sqrt{n} g\Theta(t)))^2 \\ &+ 4\cos^2\alpha \sin^2\alpha \cos^2(\sqrt{n+1} g\Theta(t)) \\ &\times \cos^2(\sqrt{n} g\Theta(t))]^{1/2}). \end{aligned} \qquad (14)$$

Figure 2 shows the numerical results for the time evolution of the entanglement between atom A and atom B for $n = 3$. In Fig.2(a), the atomic motion in the J-C cavity is not considered for $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line) separately. It is obvious that the entanglement between the two atoms evolves without periodicity, and there appears a distinct phenomenon of ESD. When the motion of the atom in the J-C cavity is taken into account, it is evident that the entanglement between the two atoms evolves periodically with period 2π as shown in Fig.2 (b). This periodical dynamic behaviour of the $E_{\rm AB}$ depends on the mode field parameter p, and it can be understood by the expression (7). From expression (7), we have $g\Theta(t) = 1/p[1 - \cos(pgt)]$. It is observed that $g\Theta(t)$ is a periodical function of scaled time gt with period $2\pi/p$, and when $p = 1$, the atom-atom entanglement evolves in the period 2π. Meanwhile, figure 2(b) also indicates the presence of ESD. The time length of the ESD is independent of the degree of entanglement of the initial state except for the case $\alpha = k\pi$. This feature is different from the result of the double Jaynes-Cummings model where the smaller the initial degree of entanglement of the atoms, the longer the state will stay in the disentangled separable state.[22] From Fig.2(b), we can also see that when the moving atom A and the isolated atom B are initially in a maximum entangled state, they are still in the maximum entangled state at $gt = 2k\pi (k = 0, 1, 2\ldots)$. It is demonstrated that the atomic motion leads to the reducing of ESD time and the changing of the time progress of the ESD between the moving atom and the isolated atom.

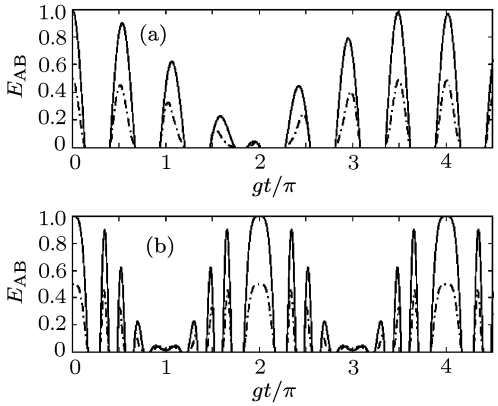


Fig.2. Time evolution of entanglement $E_{\rm AB}$ between moving atom A and isolated atom B for $n = 3$ with different initial states $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line) (a) without taking the motion of atom A into account and (b) with taking the motion of atom A into account for $p = 1$ and $v = gL/\pi$.

4. Entanglement between atom and cavity

When $n \neq 0$, atom A and atom B combine each with cavity C to form a qubit–qutrit system, where the atom system is a qubit, the field is a qutrit with three states: $|n+1\rangle$, $|n\rangle$ and $|n-1\rangle$. The corresponding reduced density matrixes $\rho_{\rm AC}$ and $\rho_{\rm BC}$ are each expressed as an explicit 6×6 matrix in the basis $|+n+1\rangle, |+n\rangle, |+n-1\rangle, |-n+1\rangle, |-n\rangle$ and $|-n-1\rangle$, and they are given as follows:

$$\rho_{\rm AC} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |x_2|^2 & 0 & x_2x_4^* & 0 & 0 \\ 0 & 0 & |x_1|^2 & 0 & x_1x_3^* & 0 \\ 0 & x_4x_2^* & 0 & |x_4|^2 & 0 & 0 \\ 0 & 0 & x_3x_1^* & 0 & |x_3|^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

and

$$\rho_{\rm BC} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |x_3|^2 & 0 & x_3x_4^* & 0 & 0 \\ 0 & 0 & |x_1|^2 & 0 & x_1x_2^* & 0 \\ 0 & x_4x_3^* & 0 & |x_4|^2 & 0 & 0 \\ 0 & 0 & x_2x_1^* & 0 & |x_2|^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{16}$$

According to expression (12), the entanglement between the moving atom A and the cavity C, $E_{\rm AC}$, is obtained as

$$\begin{aligned} E_{\rm AC} = & \sqrt{\cos^4(\sqrt{n}g\Theta(t))\sin^4\alpha + 4\cos^2(\sqrt{n+1}g\Theta(t))\sin^2(\sqrt{n+1}g\Theta(t))\cos^4\alpha} \\ & + \sqrt{\cos^4(\sqrt{n+1}g\Theta(t))\cos^4\alpha + 4\cos^2(\sqrt{n}g\Theta(t))\sin^2(\sqrt{n}g\Theta(t))\sin^4\alpha} \\ & - \cos^2(\sqrt{n}g\Theta(t))\sin^2\alpha - \cos^2(\sqrt{n+1}g\Theta(t))\cos^2\alpha, \end{aligned} \tag{17}$$

while the entanglement between the isolated atom B and cavity C $E_{\rm BC}$ is

$$\begin{aligned} E_{\rm BC} = & \sqrt{\cos^4(\sqrt{n}g\Theta(t))\sin^4\alpha + 4\cos^2(\sqrt{n+1}g\Theta(t))\sin^2(\sqrt{n}g\Theta(t))\cos^2\alpha\sin^2\alpha} \\ & + \sqrt{\cos^4(\sqrt{n+1}g\Theta(t))\cos^4\alpha + 4\cos^2(\sqrt{n}g\Theta(t))\sin^2(\sqrt{n+1}g\Theta(t))\sin^2\alpha\cos^2\alpha} \\ & - \cos^2(\sqrt{n}g\Theta(t))\sin^2\alpha - \cos^2(\sqrt{n+1}g\Theta(t))\cos^2\alpha. \end{aligned} \tag{18}$$

The computation results of $E_{\rm AC}$ and $E_{\rm BC}$ are shown in Fig.3 and Fig.4 respectively. In Fig.3(a), the entanglement dynamics of $E_{\rm AC}$ is presented for the system parameters: $n = 3$, $p = 1$, and two different initial states, i.e. $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line). It is clear that the entanglement $E_{\rm AC}$ between the moving atom A and the cavity C evolves in period 2π. We can also see that with the degree of initial entanglement between the two atoms decreasing, the amplitude of the entanglement $E_{\rm AC}$ between the moving atom A and the cavity C becomes larger (see the curves for the cases of $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line) in Fig.3(a)). In order to show the influence of the parameter p of the mode field on the entanglement $E_{\rm AC}$ between the moving atom A and the cavity C, we take the parameter $p = 2$ in Fig.3(b). A comparison between Fig.3(a) and Fig.3(b) clearly indicates that the period of $E_{\rm AC}$ shortens with the increase of parameter p. The entanglement $E_{\rm BC}$ between the isolated atom B and cavity C also evolves periodically, with the parameters: $n = 3$, $p = 1$, $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line)) adopted, which is shown in Fig.4, though the atom B has no interaction with the cavity C. It implies that entanglement flows also to the noninteracting partitions. This fact can be visualized as follows: initially the entanglement is contained in the partition composed of the moving atom A and the isolated atom B. Then, due to the interaction between the moving atom A and the cavity C, the information about the quantum state of the moving atom A begins to be mapped into the quantum state of the cavity C. Therefore, some of the quantum information contained in the combined system consisting of the moving atom A and the isolated atom B is now present in the combined system composed of the isolated atom B and the cavity C, producing entanglement in this partition.

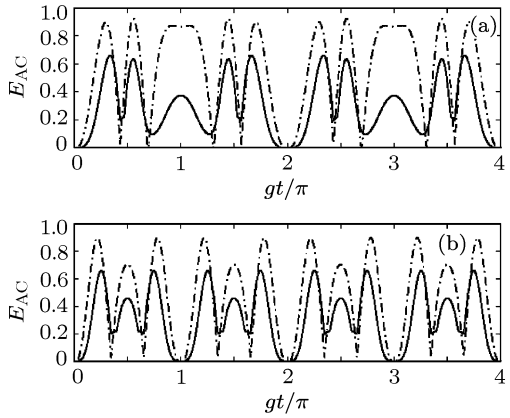


Fig.3. The time evolution of the entanglement E_{AC} between the moving atom A and cavity C for $n = 3$ and two different initial states, i.e. $\alpha = \pi/4$ (solid line) and $\alpha = \pi/12$ (dash line) at (a) $p = 1$ and (b) $p = 2$.

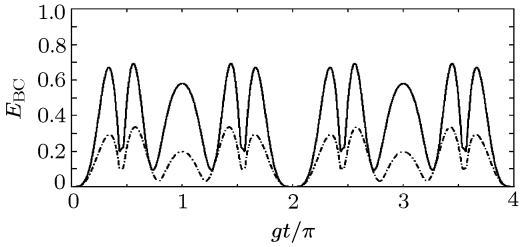


Fig.4. Time evolution of entanglement E_{BC} between isolated atom B and cavity C for $n = 3$, $\alpha = \pi/4$ (solid line), $\alpha = \pi/12$ (dash line) and $p = 1$.

5. Conclusion

In this paper, we study the entanglement dynamics between a moving atom A in the cavity C and an isolated atom B. The results show that there occurs the ESD and the entanglement transfer between the moving atom A and the isolated atom B. This stems from the fact that the entanglement between atoms A and B evolves by the interaction between atom A and cavity C, so the original quantum information contained in the atoms A and B flows to the partition AC composed of atom A and cavity C, and to partition BC consisting of atom B and cavity C. Especially, the entanglement between atoms A and B is transferred completely in a period of time. The motion of the atom in the cavity leads to the periodical evolution and the changes of the time progress of ESD between the moving atom and the isolated atom. We also investigate the entanglement between atom and cavity. The increase of parameter p of the mode field shortens the period of the entanglement between the moving atom and the cavity. It is found that with the decrease of the degree of initial entanglement between the two atoms, the amplitude of the entanglement between the moving atom and the cavity becomes larger, while the amplitude of the entanglement between the isolated atom and the cavity becomes smaller.

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