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Nuclear energy generation rates on magnetar surfaces*

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Based on the new screening model, this paper discusses the influence of superstrong magnetic fields on nuclear energy generation rates on the surface of magnetars. The obtained result shows that the superstrong magnetic fields can increase the nuclear energy generation rates by many orders of magnitude. The enhancement may have a significant influence for further study of the magnetars, especially for the cooling, the x-ray luminosity observation and the evolution of the magnetars.

Keywords: superstrong magnetic fields, nuclear energy generation rate, neutron stars
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1. Introduction

It is universally believed that the typical estimated magnitude of the surface magnetic fields for most of the neutron stars are about 10^8–10^9 T.[1] There probably exist magnetars with the magnetic field strength exceeding the quantum critical threshold, $B_{\rm rel} = 4.414 \times 10^9$ T,[2−5] which is much larger than the typical magnetic field of the neutron stars. Soft γ-ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs) are classes of candidates for magnetars.[6] The magnetic field of the magnetars may be so strong that they can reach two orders of magnitude above the quantum critical threshold.[7] The most recent observations[8−10] provide strong evidence for the validity of the old hypothesis that the surface magnetic fields of the magnetars are about 10^{10}–10^{11} T. Moreover, the observed x-ray luminosities of the AXPs may require a magnetic field strength B$\geq 10^{12}$ T.[11] And the population statistics of SGRs suggest that magnetars may comprise a significant fraction ($\geq 10\%$) of the neutron star population.[12] The giant magnetic field of magnetar has a significant and observable effect on quantum electrodynamics processes occurring near the star. Hence the magnetars give rise to great interest, their many properties have been extensively studied (e.g., Refs.[13–21] and references therein).

The commonly accepted explanation for x-ray bursts is that the thermonuclear ignition of the accreted material in the atmosphere of the neutron star leads to a thermonuclear explosion.[12] The thermonuclear reactions are the main source of energy in the outer crust of neutron stars. Previous studies[20,21] show that the superstrong magnetic field has a significant effect on the thermonuclear reaction rates. In this paper we therefore discuss the effect of superstrong magnetic field on nuclear energy generation rates on the magnetar surfaces. We describe the composition of the matter by the hydrogen mass fraction X, helium mass fraction Y, and heavy element fraction $Z = 1-X-Y$. The mass fractions at $\Sigma = 0$, X_0, Y_0, and Z_0, are those of the accreted plasma. In this work, we assume that all heavy elements are CNO and that the composition of the accreted matter is that of the Sun: $X_0 = 0.7$, $Y_0 = 0.28$, $Z_0 = 0.02$. The conclusion derived may be crucial to the study of the cooling, the x-ray luminosity observation and the evolution of magnetars.

2. Nuclear energy generation rates

As discussed in Ref.[19], the interesting nuclear reactions are hydrogen burning by the p–p chain and the CNO cycle as well as helium burning by triple-alpha reaction under their corresponding conditions on magnetar surfaces. Then the total nuclear energy

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generation rates on the surface of magnetars can be expressed as

$$\varepsilon = \varepsilon_{\rm pp} + \varepsilon_{\rm CNO} + \varepsilon_{\rm He}, \tag{1}$$

where $\varepsilon_{\rm pp}$, $\varepsilon_{\rm CNO}$ and $\varepsilon_{\rm He}$ are the energy generation rates by hydrogen burning via the p–p chains, via the CNO cycle and by helium burning via the triple-alpha reaction, respectively. According to previous work,[21] the hydrogen burning by the p–p chains is still negligible for the relevant accretion rates on the magnetar surfaces. This work therefore neglects the contributions from the p–p chains hydrogen burning and only considers the contributions from the CNO cycle hydrogen burning as well as the triple-alpha reaction helium burning.

Helium burns to carbon via the triple-alpha reaction, and the corresponding energy generation rate $\varepsilon_{\rm He}$ is a strong function of temperature, but it depends also on element abundance Y and the density ρ. Hansen *et al*[22] have simplified matters by approximating $\varepsilon_{\rm He}$ to be a function only of temperature, namely

$$\varepsilon_{\rm He} = E_{\rm He}\gamma_{\rm He}\left(T_0/T\right)^3 \exp\left(-T_0/T\right), \tag{2}$$

where $T_0 = 4.4027 \times 10^9\,{\rm K}$ and $E_{\rm He}$ is the energy released for triple-alpha reaction, and this work chooses $\gamma_{\rm He} = 0.1\,{\rm s}^{-1}$ such that $\varepsilon_{\rm He}$ in this model is a reasonable representation of the true $\varepsilon_{\rm He}\,(T, Y, \rho)$ at densities typical of those at which helium burns on the magnetar surfaces.

The nuclear energy generation rate in full CNO cycle can be obtained through the following expression

$$\varepsilon = \frac{\Sigma_{ij} Q_{ij} n_i/t_{ij}}{\rho}, \tag{3}$$

where i refers to the isotope, j differentiates between fusion and β decay reactions for a given isotope, and Q_{ij} is the energy released in the specified reaction. In particular, the CNO cycle has a fork at ^{13}N, when the temperature is cold and/or density is diffuse, ^{13}N nuclei preferentially undergo β decay and become ^{13}C; in contrast, they become ^{14}O by proton capture. Therefore, the nuclear reaction timescales of the full CNO cycle, t_{ij}, are determined by

$$\frac{n_i}{t_{ij}} = \Sigma_k \frac{n_k}{t_k} - \Sigma_j \frac{n_i}{t_{ij}}, \tag{4}$$

where k is the reaction product through radionuclide i nuclides. With this definition of nuclear reaction timescales, the proton capture timescales are given by

$$t_{\rm capture} = \frac{1}{n_{\rm p}\langle\sigma v\rangle} = \frac{m_{\rm p}}{X\rho}\frac{1}{\langle\sigma v\rangle}, \tag{5}$$

where $\langle\sigma v\rangle$ is the reaction rate, which is only a function of temperature and can be obtained from LUNA.[23] If the thermal perturbation timescales are much longer than reaction timescales, the reaction network is always in equilibrium for a given temperature and pressure. The time for a given nucleus to go once around the CNO cycle is

$$t_{\rm cycle} \simeq \frac{Q_{\rm CNO} Z_{\rm CNO}}{14 m_{\rm p} \varepsilon}, \tag{6}$$

where $14\,{\rm m}m_{\rm p}$ is the mean molecular weight in CNO cycle, $Z_{\rm CNO}$ is the mass fraction of CNO isotopes and this work assumes the total metallicity to be $Z = 3Z_{\rm CNO}$. According to Ref.[24], this timescale is always much shorter than the perturbation timescale for all metallicities and thermally stable accretion rates. Thus, ${\rm d}n_i/{\rm d}t = 0$, then we can solve for all abundances in terms of one of the abundances in a given conditions of temperature and pressure. Defining the weighting factor as follows

$$f_{ij}\,(T, \rho) \equiv \frac{n_i/t_{ij}}{n_{12{\rm C}}/t_{12{\rm C}}}, \tag{7}$$

then we have

$$\varepsilon = \frac{X n_{12{\rm C}} \langle\sigma v\rangle_{12{\rm C}}}{m_{\rm p}} \Sigma_{ij} Q_{ij} f_{ij}\,(T, \rho)\,. \tag{8}$$

Defining another weighting factor, $h_i(T, \rho) \equiv n_i/n_{12{\rm C}}$, we get

$$\begin{aligned} \varepsilon &= \frac{X Z_{\rm CNO}}{m_{\rm p}^2} \rho \langle\sigma v\rangle_{12{\rm C}} \frac{\Sigma_{ij} Q_{ij} f_{ij}\,(T, \rho)}{\Sigma_i A_i h_i\,(T, \rho)} \\ &= X Z_{\rm CNO} G\,(T, \rho)\,. \end{aligned} \tag{9}$$

where A_i is the atomic mass of the isotope, and $G(T, \rho)$ is a function that depends only on the temperature and density.

The nuclear energy generation rate in full CNO cycles may approximately separate 'cold' and 'hot' CNO cycle contributions. If the temperature and density at the burning depth are very high, all fusion reactions in the CNO cycle occur more rapidly than the β-decays. Then the nuclear energy generation rate is determined by the β-decays of ^{14}O and ^{15}O, which are independent of temperature and density, and given by $\varepsilon_{\rm hot} = 5.8 \times 10^8 Z_{\rm CNO}{\rm J{\cdot}g^{-1}{\cdot}s^{-1}}$. In contrary, the hydrogen burning proceeds via the CNO cycle at the lower temperatures and density region, the slowest step is $^{14}{\rm N}(p,\gamma)^{15}{\rm O}$ in the CNO cycle, and so nearly all catalytic nuclei are in the form of ^{14}N. In

this "cold" CNO limit, the nuclear energy generation rate is given by[24]

$$\begin{aligned}\varepsilon_{\rm Cold} &= \frac{Q_{\rm CNO}}{\rho}\frac{{\rm d}n_{14}}{{\rm d}t}\\ &= 4.4\times10^{25}\rho X Z_{\rm CNO}\times\left[\frac{\exp\left(-15.231/T_9^{1/3}\right)}{T_9^{2/3}}\right.\\ &\quad\left.+8.3\times10^{-12}\frac{\exp\left(-3.0057/T_9\right)}{T_9^{3/2}}\right]\\ &\quad\times{\rm J}\cdot{\rm g}^{-1}\cdot{\rm s}^{-1},\end{aligned}\tag{10}$$

where $Q_{\rm CNO}=4.0\times10^{-12}$J is the energy released for one loop of the CNO cycle, T_9 is the temperature in units of 10^9K. The reaction rate for $^{14}{\rm N}(p,\gamma)^{15}{\rm O}$ is obtained from Lemut *et al*,[23] the first term in the parentheses is the non-resonant reaction with an S-factor of 1.7 keV-barn, and the second represents the 259 keV resonant reaction, which dominates for $T_9=0.15-1$.

3. Effect of superstrong magnetic fields

The charge particle-induced thermonuclear reaction rates are the sum of the resonant and non-resonant reaction contributions to the nuclear reaction mechanism. Since the thermonuclear reaction rates is governed by the non-resonant reaction in the lower temperature range such as on the surface of magnetars, only the magnetic field effects on the non-resonant reaction rate is considered. Our recent work[20] has defined an enhancement factor for considering the effects of magnetic fields on the thermonuclear reaction rates, which is given by

$$f=\exp\left[\pi\eta(E)\left(U_{\rm s1}-U_{\rm s2}\right)E^{-1}\right],\tag{11}$$

where $\eta(E)=z_1z_2{\rm e}^2\sqrt{\mu/2\hbar^2}E^{-1/2}$ is the Sommerfeld parameter, E is the centre of mass bombarding energy, $U_{\rm s1}$ and $U_{\rm s2}$ represent the screening potentials with magnetic field and without magnetic field respectively.

According to Nomoto *et al*,[25] the $\alpha+\alpha$ and $\alpha+^8$Be reactions are dominated by non-resonant reaction for $T_9<0.028$ and 0.074, respectively. Thus, considering the magnetic fields effect on the triple alpha reaction, we can only take the smaller value of two enhancement factors, namely

$$f'=\min\left(f^{\alpha\alpha}\cdot f^{\alpha^8{\rm Be}}\right),\tag{12}$$

where $f^{\alpha\alpha}$ and $f^{\alpha^8{\rm Be}}$ are the enhancement factors for the $\alpha+\alpha$ and $\alpha+^8$Be reaction, respectively. Then the nuclear energy generation rate for the helium burning via the triple alpha reaction, due to the effect of superstrong magnetic fields, can be expressed as

$$\varepsilon'_{\rm He}=E_{\rm He}\gamma_{\rm He}\left(T_0/T\right)^3f'\exp\left(-T_0/T\right).$$

Similarly, the nuclear energy generation rate for the hydrogen burning by the "cold" CNO cycles on magnetar surfaces is given by

$$\begin{aligned}\varepsilon'_{\rm Cold} &= 4.4\times10^{25}\rho X Z_{\rm CNO}\times\left[\frac{\exp\left(-15.231/T_9^{1/3}\right)}{T_9^{2/3}}\right.\\ &\quad\left.\times f+8.3\times10^{-12}\frac{\exp\left(-3.0057/T_9\right)}{T_9^{3/2}}\right]\\ &\quad\times{\rm J}\cdot{\rm g}^{-1}\cdot{\rm s}^{-1},\end{aligned}\tag{13}$$

where f is the enhancement factor for the $^{14}{\rm N}(p,\gamma)^{15}{\rm O}$ reaction, which is given by Eq.(11).

By above discussion, the nuclear energy generation rates on the magnetar surfaces, due to the effect of superstrong magnetic fields, should be corrected as

$$\varepsilon=\varepsilon'_{\rm Cold}+\varepsilon'_{\rm He}.\tag{14}$$

4. Numerical analysis and discussion

For considering the influence of superstrong magnetic field on nuclear energy generation rates on the magnetar surfaces, we introduce two enhancement factors $f_{\rm Cold}=\varepsilon'_{\rm Cold}/\varepsilon_{\rm Cold}$ and $f_{\rm He}=\varepsilon'_{\rm He}/\varepsilon_{\rm He}$ which stand for the hydrogen burning by the "cold" CNO cycle and the helium burning by triple-alpha reaction, respectively. Figure 1 shows the effect of different magnetic fields on nuclear energy generation rate of the "cold" CNO cycle under the conditions in density $\rho/\mu_{\rm e}=10^5{\rm mol\cdot cm}^{-3}$ and temperatures $T_9=0.01-1$.

We find that the stronger the magnetic fields are, the greater the effect on the nuclear energy generation rate is. The magnetic field effect on nuclear energy generation rate can be reduced with the increasing temperature. In considering temperature range, the magnetic field has a significant effect on the nuclear energy generation rate, especially in the lower temperature range ($T_9<0.1$), which can increase the rates

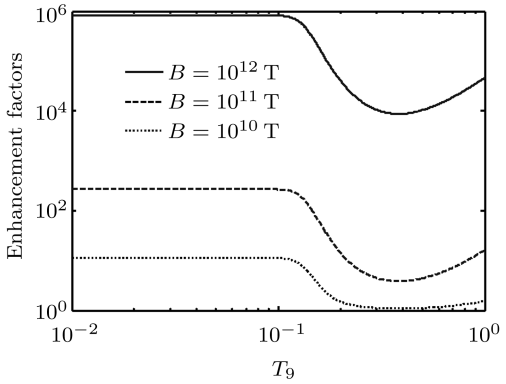


Fig.1. The enhancement factors as a function of temperature for energy generation rate via "cold" CNO cycle under the condition in $\rho/\mu_e = 10^5 \text{mol}\cdot\text{cm}^{-3}$ and at different magnetic fields.

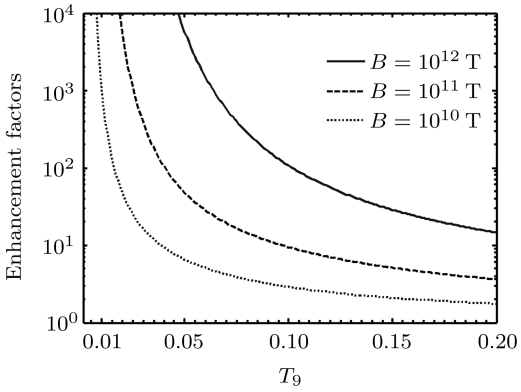


Fig.2. The enhancement factors as a function of temperature for energy generation rate via triple-alpha reaction under the condition in $\rho/\mu_e = 10^6 \text{mol}\cdot\text{cm}^{-3}$ and at different magnetic fields.

by many orders of magnitude. For the temperature $T_9 < 0.1$, the magnetic field can increase the nuclear energy generation rate by a factor of 10, 260 and 8.25×10^5 at $B = 10^{10}$T, 10^{11}T and 10^{12}T respectively. On the magnetar surfaces, the superstrong magnetic field has a significant effect on the nuclear energy generation rate for the hydrogen burning by the 'cold' CNO cycle.

Figure 2 shows the effect of different magnetic fields on nuclear energy generation rate for the helium burning under the conditions in density $\rho/\mu_e = 10^6 \text{mol}\cdot\text{cm}^{-3}$ and temperatures $T_9 = 0.01 - 0.2$.

We can see that the stronger the magnetic fields are, the greater the effect on nuclear energy generation rate is. And the magnetic fields effect on the nuclear energy generation rate can be reduced with the increasing temperature. The superstrong magnetic field has an important effect on the nuclear energy generation rate, especially in the lower temperature range ($T_9 < 0.05$), the enough strong magnetic field can increase the nuclear energy generation rate by many orders of magnitude. The magnetic field can increase the nuclear energy generation rate by a factor of 6.3, 46 and 2.5×10^3 at $B = 10^{10}$ T, 10^{11} T and 10^{12} T, respectively.

5. Summary

In this paper, we discussed the influence of superstrong magnetic fields on nuclear energy generation rates on magnetar surfaces. The obtained result shows that the effect of superstrong magnetic fields on the nuclear energy generation rates can be significant, especially in the lower temperature range, which can increase the rates by many orders of magnitude. The increase of nuclear energy generation rates undoubtedly increases the temperature on the magnetar surfaces, which then influence the cooling and the x-ray luminosity observation of magnetars. Furthermore, the increase of rates also affects the nuclear reaction rates on the surface of magnetars, it affects the theoretical predictions for the element abundance of the magnetar surfaces. This would substantially influence the internal physics of the magnetars, and thus greatly affect the evolution of magnetars. Then the conclusion we obtained has significant influence on further study of the magnetars, especially for the cooling, the x-ray luminosity observation and the evolution of the magnetars.

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