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Macroscopic spin and charge transport theory*

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According to the general principle of non-equilibrium thermodynamics, we propose a set of macroscopic transport equations for the spin transport and the charge transport. In particular, the spin torque is introduced as a generalized 'current density' to describe the phenomena associated with the spin non-conservation in a unified framework. The Einstein relations and the Onsager relations between different transport phenomena are established. Specifically, the spin transport properties of the isotropic non-magnetic and the isotropic magnetic two-dimensional electron gases are fully described by using this theory, in which only the macroscopic-spin-related transport phenomena allowed by the symmetry of the system are taken into account.

Keywords: spin torque, spin Hall effect, Einstein relation, Onsager relation
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1. Introduction

Recent intensive theoretical and experimental studies have demonstrated the possibility of directly generating and controlling spins by using the pure electric method in the non-magnetic semiconductors. The key ingredient of the approach is the spin–orbit coupling that is ubiquitously present in various systems. Compared with the conventional approaches such as spin injection[1−4] or using ferromagnetic semiconductors,[5−7] the new approach has the advantage in better integration with the existing semiconductor technology. The most important effect proposed up to date is the spin-Hall effect (SHE),[8−12] i.e. the transverse spin transport induced by an external electric field. Its reciprocal effect, i.e. the inverse spin Hall effect, is also investigated in theory,[13,14] Experimentally, the SHE has been demonstrated to exist due to the spin accumulation at the boundary of sample induced by an electric field, and measured by optical method.[15−19] Electrical measurement, in which the spins are injected from the ferromagnetic electrodes into metal, was also reported.[20,21] These new findings, together with the other known spin-related phenomena such as the spin galvanic effect[22,23] and the spin generation effect,[24,25] demonstrate the feasibility of applying the spin–orbit coupling to the spin control in semiconductors.

In this paper, we investigate the spin-related transport phenomena in a general framework of macroscopic non-equilibrium thermodynamics. In particular, we introduce spin torque into the transport equations as a generalized current density, so that the phenomena associated with the spin non-conservation can be considered in a unified framework. Moreover, a whole set of the spin-related phenomena, such as the spin galvanic effect and the spin generation effect, can now be fitted into the general framework. By adopting such an approach, we avoid the perplexing problems of the microscopic origins of these phenomena, which at present are still highly controversial and under debate. Instead, we treat these phenomena as instances of the macroscopic non-equilibrium processes allowed by the symmetry of the system. The general considerations of such kind are sufficient for us to establish a set of Einstein and Onsager relations in which those seemingly unrelated phenomena are related together. Accordingly, the novel transport phenomenon such as the spin Hall effect becomes another transport phenomenon in a typical conducting material, and the spin Hall coefficient becomes one of the parameters to characterize the transport properties of the material, just like the usual electric (Hall) conductivity.

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The remainder of the present paper is organized as follows. In Section 2 we propose a set of general macroscopic transport equations for spin-charge transports and establish the Einstein relations and the Onsager relations. In Section 3, the general transport theory is applied to the isotropic non-magnetic and the isotropic magnetic two-dimensional electron gases (2DEGs), demonstrating that it can treat any macroscopic system. Section 4 is devoted to the conclusions drawn from the present study.

2. Macroscopic spin and charge transport equations

To describe the spin transport in a macroscopic sample, we introduce two new 'currents densities': spin current density j^{s}, which describes the spatial transport of spin, and the spin torque density $\mathcal{T} \equiv \langle \dot{S} \rangle$, which describes the 'transport' of spin in the $SU(2)$ internal space. Note that the introduction of spin torque is necessary in order to describe the phenomena associated with the spin non-conservation. Both 'current densities', like the usual charge current density j^{c}, can be defined as the time derivative of 'displacement' operators (see Table 1). The linear response of these currents to the non-equilibrium distribution of spin density and charge density can be in general written as

$$\begin{aligned} j_\alpha^{\mathrm{c}}/q =& - D_{\alpha\beta}^{\mathrm{cc}} \nabla_\beta (\Delta n) - D_{\alpha\beta}^{\mathrm{cs}} \nabla_\beta (\Delta S^z) \\ & + G_\alpha^{\mathrm{c}} \Delta S^z, \end{aligned} \tag{1}$$

$$\begin{aligned} j_\alpha^{\mathrm{s}} =& - D_{\alpha\beta}^{\mathrm{sc}} \nabla_\beta (\Delta n) - D_{\alpha\beta}^{\mathrm{ss}} \nabla_\beta (\Delta S^z) \\ & + G_\alpha^{\mathrm{s}} \Delta S^z, \end{aligned} \tag{2}$$

and

$$\begin{aligned} \mathcal{T}^z =& - D_\beta^{\mathcal{T}\mathrm{c}} \nabla_\beta (\Delta n) - D_\beta^{\mathcal{T}\mathrm{s}} \nabla_\beta (\Delta S^z) \\ & + G^{\mathcal{T}} \Delta S^z, \end{aligned} \tag{3}$$

where Δn and ΔS^z denote the deviations of the electron number density and the spin density from their values in local equilibrium, respectively. The subscript $\alpha\,(\beta) = x,\ y,\ z$ denotes the spatial index, and q denotes the charge quantity of carriers. The transport is characterized by a set of coefficients D–s and G–s with the appropriate superscripts: s denotes spin, c represents the charge, and $\mathcal{T}$ refers to the spin torque. The terms proportional to the gradients of Δn and ΔS^z describe the diffusion induced by the non-equilibrium density. The unusual response to the non-equilibrium spin density itself, instead of its gradient, is actually evident experimentally: for instance, the response $j_\alpha^{\mathrm{c}}/q = G_\alpha^{\mathrm{c}} \Delta S^z$ describes the spin galvanic effect present in systems with reduced-symmetry, in which a non-equilibrium spin density gives rise to the charge current; another example of the response of such kind is the spin-relaxation process $\mathcal{T}^z = -\Delta S^z/\tau_{\mathrm{s}}$, in which $G^{\mathcal{T}} = -1/\tau_{\mathrm{s}}$ and τ_{s} is the spin relaxation time. Here, we consider only the z-component of the spin density. For more general circumstances, other components of the spin density ΔS^x and ΔS^y, as well as the corresponding components of the spin current and spin torque, should also be included.[1)] Such a generalization is straightforward.

In the presence of the external fields, the local number (spin) density in equilibrium, corresponding to the linear order of the external field, can be written as

$$\begin{aligned} n_{\mathrm{eq}}(x) =& n_0 - \left(\frac{\partial n}{\partial \mu}\right)_{h_z} q\varphi(x) \\ & + \left(\frac{\partial n}{\partial h_z}\right)_{\mu} h_z(x), \end{aligned} \tag{4}$$

and

$$\begin{aligned} S_{\mathrm{eq}}(x) =& S_0^z - \left(\frac{\partial S^z}{\partial \mu}\right)_{h_z} q\varphi(x) \\ & + \left(\frac{\partial S^z}{\partial h_z}\right)_{\mu} h_z(x), \end{aligned} \tag{5}$$

where $n_0\,(S_0^z)$ is the equilibrium number (spin) density in the absence of the external fields; μ is the chemical potential of the system; $\varphi(x)$ and $h_z(x)$ denote the electric potential and the Zeeman fields, which are coupled to the charge and spin degrees of freedom respectively. The non-equilibrium electron number and spin densities can then be expressed as

$$\Delta n(x) = n(x) - n_{\mathrm{eq}}(x), \tag{6}$$

and

$$\Delta S^z(x) = S^z(x) - S_{\mathrm{eq}}(x). \tag{7}$$

Equations (1)–(7) fully describe the linear spin transport and the linear charge transport for a general system. In the absence of external fields, these

[1)] In the most general form, one also needs to introduce terms proportional to Δn on the right-hand sides of equations. The particle number change rate $\langle \dot{n} \rangle$ should also be treated as a current.

equations describe the 'diffusion' processes, i.e. the transport driven by the inhomogeneous density distribution. The coefficients D–s and G–s in Eqs.(1)–(3) are the 'diffusion' constants. On the other hand, in the presence of external fields while maintaining the homogeneous density distribution, these equations describe the 'conducting' processes driven by the external fields. The latter can also be described phenomenologically by

$$\begin{aligned} j_\alpha^{\mathrm{c}} =& -\sigma_{\alpha\beta}^{\mathrm{cc}}\nabla_\beta\varphi(x) + g\mu_{\mathrm{B}}\sigma_{\alpha\beta}^{\mathrm{cs}}\nabla_\beta h_z(x) \\ &+ \lambda_\alpha^{\mathrm{c}} h_z(x) + \ldots, \end{aligned} \tag{8}$$

$$\begin{aligned} j_\alpha^{\mathrm{s}} =& -\sigma_{\alpha\beta}^{\mathrm{sc}}\nabla_\beta\varphi(x) + g\mu_{\mathrm{B}}\sigma_{\alpha\beta}^{\mathrm{ss}}\nabla_\beta h_z(x) \\ &+ \lambda_\alpha^{\mathrm{s}} h_z(x) + \ldots, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \mathcal{T}^z =& -\sigma_\beta^{\mathcal{T}\mathrm{c}}\nabla_\beta\varphi(x) + g\mu_{\mathrm{B}}\sigma_\beta^{\mathcal{T}\mathrm{s}}\nabla_\beta h_z(x) \\ &+ \lambda^{\mathcal{T}} h_z(x) + \ldots, \end{aligned} \tag{10}$$

where σ-s and λ-s with the appropriate superscripts denote the 'conductivities' for the corresponding processes, and $\ldots$ represents the omitted terms proportional to $\varphi(x)$. It is easy to identify the physical effects from the set of phenomenological equations. For instance, the spin Hall effect corresponds to $j_x^{\mathrm{s}} = -\sigma_{xy}^{\mathrm{sc}}\nabla_y\varphi(x)$, and the inverse spin Hall effect to $j_x^{\mathrm{c}} = g\mu_{\mathrm{B}}\sigma_{xy}^{\mathrm{cs}}\nabla_y h_z(x)$.

Comparing Eqs.(1)–(7) with Eqs.(8)–(10), we can obtain the Einstein relations that relate the 'diffusion' to 'conduction':

$$\sigma_{\alpha\beta}^{\mathrm{cc}} = q^2\left[D_{\alpha\beta}^{\mathrm{cc}}\left(\frac{\partial n}{\partial\mu}\right)_{h_z} + D_{\alpha\beta}^{\mathrm{cs}}\left(\frac{\partial S^z}{\partial\mu}\right)_{h_z}\right], \tag{11}$$

$$\sigma_{\alpha\beta}^{\mathrm{cs}} = \frac{q}{g\mu_{\mathrm{B}}}\left[D_{\alpha\beta}^{\mathrm{cc}}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D_{\alpha\beta}^{\mathrm{cs}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{12}$$

$$\lambda_\alpha^{\mathrm{c}} = -G_\alpha^{\mathrm{c}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}, \tag{13}$$

$$\sigma_{\alpha\beta}^{\mathrm{sc}} = q\left[D_{\alpha\beta}^{\mathrm{sc}}\left(\frac{\partial n}{\partial\mu}\right)_{h_z} + D_{\alpha\beta}^{\mathrm{ss}}\left(\frac{\partial S^z}{\partial\mu}\right)_{h_z}\right], \tag{14}$$

$$\sigma_{\alpha\beta}^{\mathrm{ss}} = \frac{1}{g\mu_{\mathrm{B}}}\left[D_{\alpha\beta}^{\mathrm{sc}}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D_{\alpha\beta}^{\mathrm{ss}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{15}$$

$$\lambda_\alpha^{\mathrm{s}} = -G_\alpha^{\mathrm{s}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}, \tag{16}$$

$$\sigma_{\alpha\beta}^{\mathcal{T}\mathrm{c}} = q\left[D_\beta^{\mathcal{T}\mathrm{c}}\left(\frac{\partial n}{\partial\mu}\right)_{h_z} + D_\beta^{\mathcal{T}\mathrm{s}}\left(\frac{\partial S^z}{\partial\mu}\right)_{h_z}\right], \tag{17}$$

$$\sigma_\beta^{Ts} = \frac{1}{g\mu_{\mathrm{B}}}\left[D_\beta^{Tc}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D_\beta^{Ts}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{18}$$

and

$$\lambda^{\mathcal{T}} = -G^{\mathcal{T}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}. \tag{19}$$

Furthermore, it is easy to identify the generalized forces associated with the 'currents', and their 'displacement' operators corresponding to the 'forces', as shown in Table 1.

Table 1. The 'displacement' operators and the generalized forces corresponding to the different 'current densities'.

'current density' j_i	'displacement' X_i	'force' F_i
j^{c}	qx	$-\nabla\varphi(x)$
j^{s}	$S^z x$	$g\mu_{\mathrm{B}}\nabla h_z(x)$
$\mathcal{T}^z$	S^z	$h_z(x)$

The 'current densities' can be in general defined as $j_i = \mathrm{d}X_i/\mathrm{d}t$.[26] With these correspondences, the Onsager relations can be established for the different conducting processes. In general, the Onsager relation reads[27]

$$\sigma_{ij} = \lambda(X_i)\,\lambda(X_j)\,\sigma_{ji}, \tag{20}$$

where $\lambda(X_i)$ denotes the parity of the displacement operator X_i under the time reversal. By applying the general principle to our spin–charge transport equations, many relations can be established. For instance, the spin Hall coefficient is related to the inverse spin Hall coefficient by[26]

$$\sigma_{\alpha\beta}^{\mathrm{sc}} = -\sigma_{\beta\alpha}^{\mathrm{cs}}. \tag{21}$$

Another example is the relation between the spin generation effect, in which an electric field induces a spin torque, and the spin galvanic effect, i.e.

$$\sigma_\alpha^{\mathcal{T}\mathrm{c}} = -\lambda_\alpha^{\mathrm{c}}, \tag{22}$$

where $\sigma_\alpha^{\mathrm{Tc}}$ is the spin generation coefficient, and $\lambda_\alpha^{\mathrm{c}}$ is related to the spin galvanic effect by Eq.(13).

Equations (1)–(22) provide a full description for the linear charge and spin transports of any system. While some of these results have been discussed in the literature,[28] it is the first time that all the spin- and the charge-related linear transport phenomena have been collected in a unified framework to provide a consistent basis for the theoretical considerations of the different phenomena.

3. Application of the macroscopic spin-charge transport theory

3.1. Isotropic non-magnetic two-dimensional electron gas

The transport equations presented in the last section can be greatly simplified in systems with high symmetries. An important example is an isotropic non-magnetic two-dimensional electron gas (2DEG). For such a system, the transport equations should be invariant under the in-plane rotation and reflections. This requirement yields more symmetry relations between transport coefficients, and it also prohibits some transport processes. It can be shown that $\sigma_{xx}^{\rm cc} = \sigma_{yy}^{\rm cc} \neq 0$, $\sigma_{xx}^{\rm ss} = \sigma_{yy}^{\rm ss} \neq 0$, $\sigma_{xy}^{\rm sc} = -\sigma_{yx}^{\rm sc} = \sigma_{xy}^{\rm cs} = -\sigma_{yx}^{\rm cs} \neq 0$, $\sigma_{xy}^{\rm cc} = -\sigma_{yx}^{\rm cc} \neq 0$, $\sigma_{xy}^{\rm ss} = -\sigma_{yx}^{\rm ss} \neq 0$, $\sigma_{xx}^{\rm sc} = \sigma_{yy}^{\rm sc} = \sigma_{xx}^{\rm cs} = \sigma_{yy}^{\rm cs} \neq 0$ and $\lambda^{\mathcal{T}} \neq 0$. Other coefficients vanish because the corresponding processes are disallowed by the symmetry: for instance, the spin generation effect $\mathcal{T}^z = \sigma_\alpha^{\mathcal{T}c} E_\alpha$ cannot occur because it violates the reflection symmetry. Moreover, the time reversal symmetry for the non-magnetic systems leads to $(\partial S^z/\partial\mu)_{h_z} = (\partial n/\partial h_z)_\mu \equiv 0$. All summarized, the only existing transport coefficients in an isotropic non-magnetic 2DEG are $\sigma_{xx}^{\rm cc} = \sigma_{yy}^{\rm cc} = \sigma^{\rm c}$, $\sigma_{xx}^{\rm ss} = \sigma_{yy}^{\rm ss} = \sigma^{\rm s}$, $\sigma_{xy}^{\rm sc} = -\sigma_{yx}^{\rm sc} = \sigma_{xy}^{\rm cs} = -\sigma_{yx}^{\rm cs}$ and λ^{τ}.

The general spin-charge transport equations for an isotropic non-magnetic 2DEG can be written as

$$\begin{aligned} j^{\rm c} =& -qD^{\rm c}\nabla n(x) - \sigma^{\rm c}\nabla\varphi(x) \\ & - qD^{\rm cs}\hat{z}\times\nabla S^z(x) \\ & + g\mu_{\rm B}\sigma^{\rm cs}\hat{z}\times\nabla h_z(x), \end{aligned} \tag{23}$$

$$\begin{aligned} j^{\rm s} =& -D^{\rm s}\nabla S^z(x) + g\mu_{\rm B}\sigma^{\rm s}\nabla h_z(x) \\ & - qD^{\rm sc}\hat{z}\times\nabla n(x) \\ & - \sigma^{\rm sc}\hat{z}\times\nabla\varphi(x), \end{aligned} \tag{24}$$

and

$$\mathcal{T}^z = -\frac{S^z(x)}{\tau_{\rm s}} + \lambda^{\mathcal{T}} h_z(x), \tag{25}$$

where $D^{\rm c}$ ($D^{\rm s}$) is the charge (spin) diffusion constant, $\sigma^{\rm c}$ ($\sigma^{\rm s}$) is the charge (spin) conductivity, $\sigma^{\rm sc}$ ($\sigma^{\rm cs}$) is the (inverse) spin Hall coefficient, and $\tau_{\rm s}$ is the spin relaxation time with $G^{\mathcal{T}} = -1/\tau_{\rm s}$.

The corresponding Einstein relations read

$$\sigma^{\rm c} = q^2\left(\frac{\partial n}{\partial\mu}\right)_{h_z} D^{\rm c}, \tag{26}$$

$$\sigma^{\rm s} = \frac{1}{g\mu_{\rm B}}\left(\frac{\partial S^z}{\partial h_z}\right)_\mu D^{\rm s}, \tag{27}$$

$$\sigma^{\rm sc} = q\left(\frac{\partial n}{\partial\mu}\right)_{h_z} D^{\rm sc}, \tag{28}$$

$$\sigma^{\rm cs} = \frac{q}{g\mu_{\rm B}}\left(\frac{\partial S^z}{\partial h_z}\right)_\mu D^{\rm cs}, \tag{29}$$

and

$$\lambda^{\mathcal{T}} = -G^{\mathcal{T}}\left(\frac{\partial S^z}{\partial h_z}\right)_\mu. \tag{30}$$

The Onsager relation yields:

$$\sigma^{\rm sc} = \sigma^{\rm cs}. \tag{31}$$

From the above equations, it is obviously indicated that only three independent coefficients (e.g. $\sigma^{\rm s}$, $\sigma^{\rm sc}$ ($\sigma^{\rm cs}$) and $\tau_{\rm s}$) are needed to fully characterize its spin transport. Other spin-related transport coefficients appearing in Eqs.(23)–(25) can be determined through the Einstein relations and the Onsager relations.

3.2. Isotropic magnetic two-dimensional electron gas

For an isotropic magnetic 2DEG, all the transport coefficients each will be a function of the equilibrium magnetization S_0. It is easy to verify that the transport coefficients introduced in the last subsection for the non-magnetic system each are an even function of S_0. In addition to them, there exists a set of anomalous transport phenomena with the transport coefficients each being an odd function of S_0. For instance, an electric field can induce a transverse charge current (anomalous Hall effect), i.e. $j^{\rm c} \propto -\sigma_{\rm A}^{\rm c}(S_0)\,\hat{z}\times\nabla\phi(x)$, where $\sigma_{\rm A}^{\rm c}$ is the anomalous Hall coefficient and has the symmetry $\sigma_{\rm A}^{\rm c}(-S_0) = -\sigma_{\rm A}^{\rm c}(S_0)$. Also, the equilibrium values of $(\partial S^z/\partial\mu)_{h_z} = (\partial n/\partial h_z)_\mu$ no longer vanish.

For such a system, the full spin-charge transport equations can be written as

$$\begin{aligned} j^{\rm c} =& -qD^{\rm c}\nabla n(x) - qD_{\rm A}^{\rm c}\hat{z}\times\nabla n(x) \\ & - \sigma^{\rm c}\nabla\varphi(x) - \sigma_{\rm A}^{\rm c}\hat{z}\times\nabla\varphi(x) \\ & - qD_{\rm A}^{\rm cs}\nabla S^z(x) - qD^{\rm cs}\hat{z}\times\nabla S^z(x) \\ & + g\mu_{\rm B}\sigma_{\rm A}^{\rm cs}\nabla h_z(x) + g\mu_{\rm B}\sigma^{\rm cs}\hat{z}\times\nabla h_z(x), \end{aligned} \tag{32}$$

$$\begin{aligned} j^{\rm s} =& -qD^{\rm s}\nabla S^z(x) - qD_{\rm A}^{\rm s}\hat{z}\times\nabla S^z(x) \\ & + g\mu_{\rm B}\sigma^{\rm s}\nabla h_z(x) + g\mu_{\rm B}\sigma_{\rm A}^{\rm s}\hat{z}\times\nabla h_z(x) \\ & - D_{\rm A}^{\rm sc}\nabla n(x) - D^{\rm sc}\hat{z}\times\nabla n(x) \\ & - \sigma_{\rm A}^{\rm sc}\nabla\varphi(x) - \sigma^{\rm sc}\hat{z}\times\nabla\varphi(x), \end{aligned} \tag{33}$$

and

$$\mathcal{T}^z = -\frac{S^z(x)}{\tau_{\rm s}} + \lambda^{\mathcal{T}} h_z(x), \tag{34}$$

where the coefficients with subscript A indicate the anomalous transport processes.

Correspondingly, the Einstein relations are

$$\sigma^{\rm c} = q^2\left[D^{\rm c}\left(\frac{\partial n}{\partial \mu}\right)_{h_z} + D^{\rm cs}_{\rm A}\left(\frac{\partial S^z}{\partial \mu}\right)_{h_z}\right], \tag{35}$$

$$\sigma^{\rm c}_{\rm A} = q^2\left[D^{\rm c}_{\rm A}\left(\frac{\partial n}{\partial \mu}\right)_{h_z} + D^{\rm cs}\left(\frac{\partial S^z}{\partial \mu}\right)_{h_z}\right], \tag{36}$$

$$\sigma^{\rm cs} = \frac{q}{g\mu_{\rm B}}\left[D^{\rm c}_{\rm A}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D^{\rm cs}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{37}$$

$$\sigma^{\rm cs}_{\rm A} = \frac{q}{g\mu_{\rm B}}\left[D^{\rm c}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D^{\rm cs}_{\rm A}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{38}$$

$$\sigma^{\rm sc} = q\left[D^{\rm sc}\left(\frac{\partial n}{\partial \mu}\right)_{h_z} + D^{\rm s}_{\rm A}\left(\frac{\partial S^z}{\partial \mu}\right)_{h_z}\right], \tag{39}$$

$$\sigma^{\rm sc}_{\rm A} = q\left[D^{\rm sc}_{\rm A}\left(\frac{\partial n}{\partial \mu}\right)_{h_z} + D^{\rm s}\left(\frac{\partial S^z}{\partial \mu}\right)_{h_z}\right], \tag{40}$$

$$\sigma^{\rm s} = \frac{1}{g\mu_{\rm B}}\left[D^{\rm sc}_{\rm A}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D^{\rm s}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{41}$$

$$\sigma^{\rm s}_{\rm A} = \frac{1}{g\mu_{\rm B}}\left[D^{\rm sc}\left(\frac{\partial n}{\partial h_z}\right)_{\mu} + D^{\rm s}_{\rm A}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}\right], \tag{42}$$

and

$$\lambda^{\mathcal{T}} = -G^{\mathcal{T}}\left(\frac{\partial S^z}{\partial h_z}\right)_{\mu}. \tag{43}$$

The Onsager relation yields

$$\sigma^{\rm sc}_{\rm A}(S_0) = -\sigma^{\rm sc}_{\rm A}(-S_0) = \sigma^{\rm cs}_{\rm A}(S_0). \tag{44}$$

4. Conclusions

We have proposed a set of equations describing the macroscopic spin and charge transports. Most importantly, they can be used to treat any macroscopic system in a unified framework because of the introduction of the spin torque density as a generalized current density. The Einstein relations and the Onsager relations are established, where the seemingly unrelated different transport phenomena are related together. The transport properties of a macroscopic sample can be characterized by using a small set of coefficients. For instance, only three independent coefficients are needed to characterize the spin-charge transport of an isotropic non-magnetic 2DEG system.

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