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Optical soliton with group delay in microring coupled-resonator optical waveguides*

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This paper derives the dispersion relation of microring coupled-resonator optical waveguides (CROWs) without any approximation by using the transfer matrix method. Based on the established dispersion relation of CROWs it obtains the slow group velocity and dispersion coefficient. It finds that the effect of dispersion on optical pulses can be adjusted to balance the effect of nonlinearity by changing coupling coefficient or loss, so optical soliton with group delay can be obtained in microring CROWs. The optical soliton with group delay is of great significance for applications of microring CROWs in delay lines and optical buffers of future all-optical communication systems.

Keywords: optical waveguide, optical soliton, group delay
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1. Introduction

Since microring coupled-resonator optical waveguides (CROWs) based on coupling of optical micro-resonators were proposed in 1999,[1] much work[2−7] has been done on them and has accelerated their development. Present research on microring CROWs mainly focuses on their applications in optic communication, such as filtering,[8−12] multi/demultiplexing,[13,14] wavelength altering,[15] modulating,[16] switching[17] and so on. Also microring CROWs have the potential application for optical delay, because optical delay is usually implemented by using fibre loops in present optic communication systems but these devices are bulky and do not offer a compact solution. In microring CROWs input light pulses spend the bulk of their time circulating within each resonator, so large group delay can be obtained. And specially the footprint of microring CROWs can be very small by using special fabrication technology even on a chip.[18] So we can use microring CROWs to make on-chip optical buffers which might play an important role in future all-optical communication systems and computer systems.

It is well known that the optical soliton is a special type of wave packet appearing due to the balance between dispersion and nonlinearity. Optical soliton can propagate undistortedly over a long distance with large message capacity and hence is considered as the most promising candidate for applications in optic communication. Obviously optical soliton in microring CROWs is propitious for applications of microring CROWs in optic communication.

Therefore we establish the dispersion relation of microring CROWs without approximation by using transfer matrices and obtain slow group velocity and dispersion coefficient. By changing coupling coefficient or loss we find that the effect of dispersion can be adjusted to balance the nonlinearity effect, so we can obtain optical soliton with group delay in microring CROWs. The optical soliton with group delay is of great significance for applications of microring CROWs in optical buffers of future all-optical communication systems.

2. Theoretical analysis of dispersion

As shown in Fig.1,[2,19] the coupling and propagation of the field can be described as shown below by

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matrices[20−22]

$$\begin{bmatrix} a'_n \\ b'_n \end{bmatrix} = \frac{1}{k} \begin{bmatrix} -t & 1 \\ -|k|^2 - |t|^2 & t^* \end{bmatrix} \begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} = C \begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \begin{bmatrix} 0 & \alpha e^{-i\beta\pi R} \\ \frac{1}{\alpha} e^{i\beta\pi R} & 0 \end{bmatrix} \begin{bmatrix} a'_n \\ b'_n \end{bmatrix} = T \begin{bmatrix} a'_n \\ b'_n \end{bmatrix}, \quad (2)$$

where t and k are the transmission and the coupling coefficients of the field amplitude respectively, $\beta = \frac{n_e \omega}{c}$ is the propagation constant, n_e is the effective index of microrings, ω is the radian frequency, c is the velocity of light in vacuum, and R is the microring radius, α is the loss of amplitude field in microrings over distance πR. From Fig.1 we know that the field is periodic,[2] so according to Bloch's theorem we have

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = e^{-iK\Lambda} \begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix}, \quad (3)$$

where K is the propagation constant of the Bloch wave, and Λ is the periodicity.

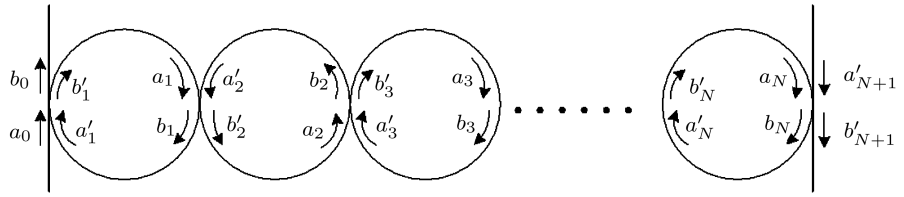


Fig.1. Travelling wave in CROW, where a_n, a'_n, b_n, and b'_n are the amplitudes of optical field.

Combining Eqs.(1), (2) and (3) we obtain

$$\begin{aligned} &-2\mathrm{Re}(k)[\mathrm{Re}(k)\sin(K\Lambda) - \mathrm{Im}(k)\cos(K\Lambda)] \\ &= \mathrm{Re}(k)\left[\alpha(|k|^2 + |t|^2) + \frac{1}{\alpha}\right]\sin(\beta\Lambda) \\ &\quad +\mathrm{Im}(k)\left[\alpha(|k|^2 + |t|^2) - \frac{1}{\alpha}\right]\cos(\beta\Lambda). \end{aligned} \quad (4)$$

Equation (4) is the dispersion relation of microring CROWs and can be rewritten as

$$C\sin(K\Lambda - \theta) = \sin(\beta\Lambda + \phi), \quad (5)$$

where

$$k = |k|\, e^{i\theta}, \quad C = \frac{-2\mathrm{Re}(k)\,|k|}{\sqrt{A^2 + B^2}},$$
$$\sin\phi = \frac{B}{\sqrt{A^2 + B^2}}, \quad \cos\phi = \frac{A}{\sqrt{A^2 + B^2}},$$
$$A = \mathrm{Re}(k)\left[\alpha(|k|^2 + |t|^2) + \frac{1}{\alpha}\right],$$

and

$$B = \mathrm{Im}(k)\left[\alpha(|k|^2 + |t|^2) - \frac{1}{\alpha}\right]. \quad (6)$$

It is obvious that the spectrum of a CROW consists of a periodic series of passbands shown in Fig.2. The resonant frequencies are $\Omega = m\mathrm{FSR}$, where FSR $= \frac{c}{n_e R}$, which is the free spectral range, corresponding to $\beta\Lambda = m\pi$, m is an integer.

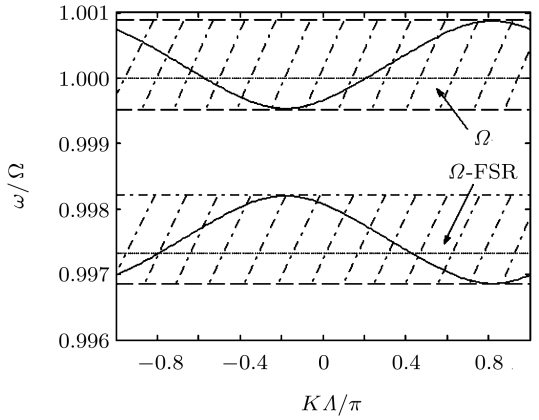


Fig.2. Two adjacent passbands denoted by imaginal lines near the resonant frequencies Ω and Ω-FSR with n_e = 1.49, R = 60μm, $k = 0.7e^{4.14i}$, $|t|$ = 0.65, α = 0.9. Ω corresponds to m = 372.

It is well known that the total field propagating through a periodic structure consists of an infinite set of spatial harmonics. By solving Eq.(5) we obtain the

phase constant of the qth spatial harmonic

$$K_q = \frac{1}{\Lambda}\left\{ \pm \cos^{-1}\left[\frac{1}{C}\sin(\beta\Lambda+\phi) + q2\pi + \frac{\pi}{2} + \theta\right]\right\}. \quad (7)$$

Differentiating K_q in the above equation, we obtain the slow group velocity of these harmonics,

$$\upsilon_{\rm g} = \frac{c}{n_{\rm e}}\frac{\sqrt{C^2-\sin^2(\beta\Lambda+\phi)}}{\cos(\beta\Lambda+\phi)}. \quad (8)$$

From Eqs.(7) and (8) we can see that these harmonics have different phase velocities but the same group velocity. In order to express that the group velocity is reduced with respect to the phase velocity $\upsilon = \dfrac{c}{n_{\rm e}}$, we define the slowing ratio of microring CROWs as

$$S = \frac{\upsilon}{\upsilon_{\rm g}} = \frac{\cos(\beta\Lambda+\phi)}{\sqrt{C^2-\sin^2(\beta\Lambda+\phi)}}. \quad (9)$$

From Eq.(7), the second order dispersion coefficient $K_{q,2}$ can be obtained

$$K_{q,2} = \frac{{\rm d}^2K_q}{{\rm d}\omega^2} = \frac{n_{\rm e}^2\Lambda}{c^2}\frac{\cos(\beta\Lambda+\phi)}{\sqrt{C^2-\sin^2(\beta\Lambda+\phi)}} \times \tan(\beta\Lambda+\phi)\frac{1-C^2}{C^2-\sin^2(\beta\Lambda+\phi)}. \quad (10)$$

It should be noted that the higher order dispersion coefficients $K_{q,n} = \dfrac{{\rm d}^nK_q}{{\rm d}\omega^n}$ are rather less than the second order dispersion coefficient $K_{q,2}$ when $\dfrac{\omega-\Omega}{\rm FSR}\pi \neq -\phi$. Here we ignore the impacts of the higher order dispersion coefficients on optical pulses with $\dfrac{\omega-\Omega}{\rm FSR}\pi \neq -\phi$.

3. The generation of optical soliton with group delay

Based on the results of the slow group velocity and dispersion coefficient in Section 2 we can quantify the effects of dispersion and nonlinearity on optical pulses respectively. From Eq.(10) it is obvious that $K_{q,2}$ is negative in the lower frequency branch $\omega - \Omega < -\phi\dfrac{\rm FSR}{\pi}$, which means that the anomalous dispersion takes place. So we can use that dispersion regime to generate optical soliton. The dispersion length $L_{\rm d} = T_0^2/|K_{q,2}|$, where T_0 is the half width at $1/e$ intensity of optical pulses. It is well known that over the length $L_{\rm d}$ the effects of $K_{q,2}$ on optical pulses become significant.

The effect of nonlinearity in microring CROWs can be conveniently described by the effective nonlinear constant $\gamma_{\rm eff}$. Then $\gamma_{\rm eff} = \dfrac{1}{2}\gamma S^2$ and $\gamma_{\rm eff} = \gamma S^2$ correspond to self-phase modulation (SPM) and cross phase modulation (XPM) respectively,[23] where γ is the nonlinear constant of the waveguide. So the nonlinear length $L_{\rm NL} = 1/(\gamma_{\rm eff}P_0)$, where P_0 is the input peak power of optical pulses.

It is well known that when the effect of nonlinearity, such as SPM or XPM, counteracts the effect of anomalous dispersion, optical soliton generates. If the dispersion length equals the nonlinear length $\dfrac{T_0^2}{|K_{q,2}|} = \dfrac{1}{\gamma_{\rm eff}P_0}$, the fundamental optical soliton can be obtained. From Eqs.(9) and (10) and the expressions of $L_{\rm d}$ and $L_{\rm NL}$ it can be seen that by changing coupling coefficient or loss, we can adjust $L_{\rm d}$ and $L_{\rm NL}$ to satisfy the equation $\dfrac{T_0^2}{|K_{q,2}|} = \dfrac{1}{\gamma_{\rm eff}P_0}$ to obtain the fundamental optical soliton. It should be noted that the transmittance is very important for the optical soliton, because if the transmittance is too low the optical soliton in microring CROWs may be senseless. But that problem can be well solved by choosing wavelengths near resonant wavelengths with high transmittance.[6,19] For example, for a optical pulse with $\gamma = 2\times10^{-4}/({\rm m\cdot mW})$, $T_0 = 10\,{\rm ps}$ and the central wavelength $\lambda = 1550\,{\rm nm}$ which is resonant wavelength, in the case of SPM the input peak power P_0 of the fundamental soliton is shown in Fig.3(a) depending on α and $|k|$. Meanwhile the soliton is delayed, it is shown in Fig.3(b).

So by changing coupling coefficient or loss the delayed fundamental soliton can be obtained for different input peak power P_0. Also that result adapts to T_0 or γ, which plays the same role of P_0 in the equation $\dfrac{T_0^2}{|K_{q,2}|} = \dfrac{1}{\gamma_{\rm eff}P_0}$. It should be noted that the adjustable range of $L_{\rm d}$ is very large due to the term $\tan(\beta\Lambda+\phi)$ in Eq.(10), so we could obtain high order soliton by changing coupling coefficient or loss when $L_{\rm d} = N^2L_{\rm NL}$ is satisfied, where N is a positive integer.

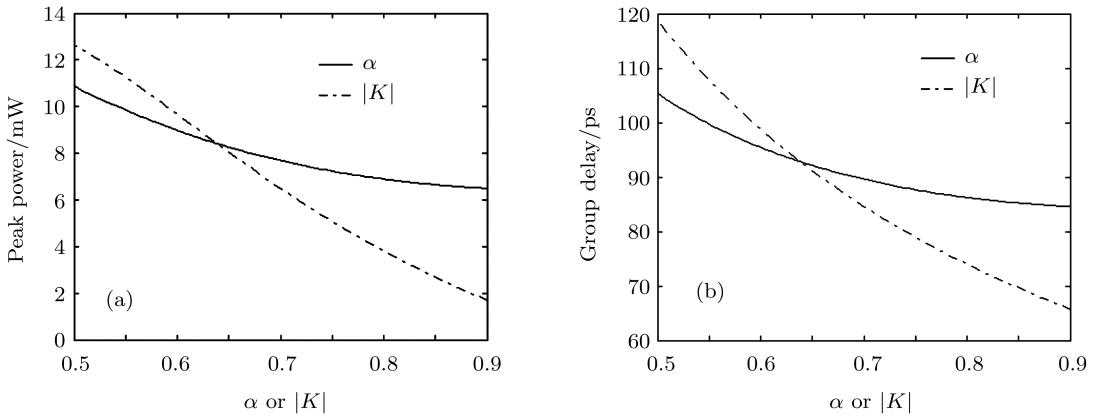


Fig.3. (a) The input peak power of the fundamental soliton. (b) Group delay through 20 microring resonators depending on α with $|k| = 0.7$ (the solid line) and $|k|$ with $\alpha = 0.9$ (the dash-dot line). Here $n_e = 1.49$, $R = 60\mu$m, $|t| = 0.65$.

4. Conclusions

Based on the established dispersion relation of microring CROWs we obtain the slow group velocity and dispersion coefficient. We find that by changing coupling coefficient or loss, the effect of dispersion can be adjusted in a large range to balance the nonlinear effect on optical pulses, so optical soliton with group delay can be obtained in microring CROWs. Optical soliton with group delay is of great significance for applications of microring CROWs in delay lines and optical buffers of future all-optical communication systems.

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