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Modulational instability of incoherently coupled beams in azobenzene-containing polymer with photoisomerization nonlinearity*

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The modulational instability of two incoherently coupled beams in azobenzene-containing polymer with photoisomerization nonlinearity is investigated analytically and numerically. Our results show that as a precursor to spatial optical soliton formation, modulational instability can be adjusted and controlled by the wavelength combinations of the signal and background beams. We also discuss the dependences of strength of modulational instability on intensities of two signal beams and background beam. These findings make it possible to predict the formation of incoherently coupled soliton pairs in azobenzene-containing polymer.

Keywords: modulational instability, azobenzene-containing polymer, photoisomerization nonlinearity

PACC: 4265S, 4270G

1. Introduction

Nowadays, nonlinear physics has been intensively studied and a lot of nonlinear phenomena have been found in quite different fields of modern physics, such as self-organization, spontaneous order, solitons, and chaos.[1−3] One of the most interesting problems of nonlinear optics is the formation of optical solitons in a nonlinear medium.[4] The development of the all-optical device in which light can be used to control light is one of the goals of modern nonlinear optics; and spatial optical solitons (SOSs) may serve as a powerful means for all-optical circuits in which light is guided and controlled by light.[5] SOS forms when a very narrow optical beam induces (through self-focusing) a waveguide structure and guides itself in the waveguide. A broad optical beam propagating through a nonlinear medium can undergo spatial instabilities, leading to spontaneous patterns or filaments in some parameter region where bright SOSs occur. Such an instability is the modulational instability (MI). The relation between MI and solitons is best manifested in the fact that the patterns or filaments that emerge from the MI process are actually trains of almost ideal solitons. Therefore, MI can be considered as a precursor to soliton formation. The MI has been already described in the early times of nonlinear optics development with a perfect degree of spatial and temporal coherence in a Kerr medium.

Recently, the nonlinearities of photorefractive and polymer materials have aroused the particular interest of scientists, for they require low power densities (about mW/cm^2). SOSs in these materials have become a hot research subject due to their applications in all-optical signal processing and switching.[6,7] Even more importantly, incoherent SOSs (partially spatially incoherent SOSs and both temporally and spatially incoherent SOSs) have been observed in these materials and subsequently investigated theoretically.[8−12] After the discovery of incoherent SOSs, it is natural to further investigate whether the MI also exists for incoherent beams because it is always believed to be a coherent process. Before long, it has been shown theoretically and experimentally that an incoherent light beam including partially spatially incoherent and incoherent 'white' light beam can also undergo the MI.[13−18] Very recently, incoherent MI has been found to occur in nonlocal nonlinear media such as liquid crystals, and it can also emerge in nonlinear photonic lattices.[19−21] The purpose of this paper is to investigate the MI of two incoherently coupled beams in azobenzene-containing polymer (ACP) with photoiso-

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merization nonlinearity analytically and numerically.

2. Modulational instability

Azobenzene-containing polymer is one of the typical photoisomerization materials. Usually, azobenzene derivatives have two forms of geometric isomers, i.e. the 'trans' and the 'cis' forms. Photoisomerization, namely photoinduced chromophores trans $\leftrightarrow$ cis isomerization, is a very fascinating process, which makes it possible to use light to manipulate the optical properties of ACPs. Some interesting phenomena such as intensity-dependent change of the refractive index, photoinduced birefringence and photoinduced dichroism in ACPs have been found from the photoisomerization process.[22−24] The intensity-dependent change of the refractive index corresponding to several combinations of polarizations of the signal and background beams has been studied in detail.[25] As shown in Ref.[25], for a linearly polarized signal beam and an unpolarized background beam, the change of the first-order dielectric susceptibility $\chi^{(1)}$ is

$$\begin{aligned}&\Delta\chi^{(1)}\\&= 4\pi ST_0\left[\frac{\alpha I+\gamma}{\beta I}\left(1-\sqrt{\frac{\alpha I+\gamma+1/3}{\beta I}}\right.\right.\\&\left.\left.\times\tan^{-1}\sqrt{\frac{\beta I}{\alpha I+\gamma+1/3}}\right)-\frac{\gamma}{3(\gamma+1/3)}\right],\end{aligned}\tag{1}$$

and the corresponding change of refractive index is $\Delta n = \Delta\chi^{(1)}/2n_0$, where $\alpha = q_{\rm Cs}\sigma_{\rm Cs}/q_{\rm Tb}\sigma_{\rm Tb}$, $\beta = q_{\rm Ts}\sigma_{\rm Ts}/q_{\rm Tb}\sigma_{\rm Tb}$, $\gamma = (q_{\rm Cb}\sigma_{\rm Cb} + K/I_{\rm b})/(q_{\rm Tb}\sigma_{\rm Tb})$, and $I = I_{\rm s}/I_{\rm b}$; $q_{\rm Ts}$ and $q_{\rm Cs}$ are the quantum yields of the signal beam for trans-to-cis and cis-to-trans transitions, respectively; $\sigma_{\rm Ts}$ and $\sigma_{\rm Cs}$ are the absorption cross sections of the signal beam in the trans-to-cis and cis-to-trans transitions, respectively; $I_{\rm s}$ is the intensity of the signal beam. The parameters with subscript b such as $q_{\rm Tb}$, $q_{\rm Cb}$, $\sigma_{\rm Tb}$, $\sigma_{\rm Cb}$ and $I_{\rm b}$ are those corresponding to the background beam. K is the thermal relaxation coefficient of the cis-to-trans transition; S is a constant for a given material; and T_0 is the concentration of molecules in the trans form in darkness.

We can see that the form of photoisomerization nonlinearity is very complex. However, we can use a simple model, i.e. the saturable nonlinearity model, to describe this process as follows:

$$\Delta\chi^{(1)} = 4\pi ST_0\frac{r\cdot I}{1+s\cdot I},\tag{2}$$

where s and r/s represent the saturation strength and saturation change of refractive index. The positive (negative) value of r corresponding to self-focusing (self-defocusing) can be obtained by choosing some special combinations of wavelengths of signal and background beams.[25]

In this paper, we consider two incoherently coupled signal beams with the same polarization, propagating along the z-axis direction in a bulk ACP material under the irradiation of an unpolarized background beam. The signal beams are allowed to diffract only along the x-axis direction. In this case, the intensity-dependent change of $\chi^{(1)}$ is

$$\begin{aligned}\Delta\chi^{(1)} = 4\pi ST_0\times&\left[\frac{\alpha_1 I_1+\alpha_2 I_2+\gamma}{\beta_1 I_1+\beta_2 I_2}\left(1-\sqrt{\frac{\alpha_1 I_1+\alpha_2 I_2+\gamma+1/3}{\beta_1 I_1+\beta_2 I_2}}\right.\right.\\&\left.\left.\times\tan^{-1}\sqrt{\frac{\beta_1 I_1+\beta_2 I_2}{\alpha_1 I_1+\alpha_2 I_2+\gamma+1/3}}\right)-\frac{\gamma}{3(\gamma+1/3)}\right].\end{aligned}\tag{3}$$

We can use a simple formula to replace the above one, namely,

$$\Delta\chi^{(1)} = 4\pi ST_0\frac{r_1\cdot I_1+r_2\cdot I_2}{1+s_1\cdot I_1+s_2\cdot I_2},\tag{4}$$

where the subscripts 1 and 2 represent signal beams 1 and 2, respectively.

The propagation dynamics of signal beams can be described by the following generalized coupled nonlinear Schrödinger equation (NLSE) in the paraxial approximation and the slowly varying amplitude approximation:

$$\begin{aligned}&{\rm i}2k_1n_1\frac{\partial E_1}{\partial z}+\frac{\partial^2E_1}{\partial x^2}+k_1^2\Delta\chi^{(1)}\cdot E_1=0,\\&{\rm i}2k_2n_2\frac{\partial E_2}{\partial z}+\frac{\partial^2E_2}{\partial x^2}+k_2^2\Delta\chi^{(1)}\cdot E_2=0,\end{aligned}\tag{5}$$

where k_j $(j=1,2)$ and n_j are the vacuum wave number and the linear refractive index of the jth signal beam, respectively. E_j $(j=1,2)$ describes the complex envelope of the optical field of the jth signal beam. So the mathematical model for two incoherently coupled beams in an ACP with a uniform background irradiation can be written as follows:

$$\begin{aligned}\frac{\partial u_1}{\partial \zeta} &= \frac{\mathrm{i}}{L_1}\left(\frac{1}{2}\frac{\partial^2 u_1}{\partial \xi^2} + b_1 \frac{r_1 |u_1|^2 + r_2 |u_2|^2}{1 + s_1 |u_1|^2 + s_2 |u_2|^2} u_1\right),\\ \frac{\partial u_2}{\partial \zeta} &= \frac{\mathrm{i}}{L_2}\left(\frac{1}{2}\frac{\partial^2 u_2}{\partial \xi^2} + b_2 \frac{r_1 |u_1|^2 + r_2 |u_2|^2}{1 + s_1 |u_1|^2 + s_2 |u_2|^2} u_2\right),\end{aligned} \tag{6}$$

where u_j $(j=1,2)$ is the modal field amplitude of the jth signal beam and $|u_j|^2 = I_j/I_\mathrm{b}$, with I_j and I_b being the intensities of the signal and background beams, respectively; $\zeta = z/z_0$, $\xi = x/x_0$, $L_j = n_j k_j x_0^2/z_0$ and $b_j = 2\pi S T_0 k_j^2 x_0^2$ $(j=1,2)$, with x_0 and z_0 being arbitrary spatial scales.

The steady-state solution of Eq.(6) in the simplest case of plane wave has the form

$$\begin{aligned}u_1 &= f \cdot \exp(\mathrm{i}\Gamma_1 \zeta),\\ u_2 &= g \cdot \exp(\mathrm{i}\Gamma_2 \zeta),\end{aligned} \tag{7}$$

where positive quantities f and g represent the normalized intensities of signal beams, and Γ_1 and Γ_2 satisfy the following equations:

$$\begin{aligned}\Gamma_1 &= \frac{b_1}{L_1} \cdot \frac{r_1 f^2 + r_2 g^2}{1 + s_1 f^2 + s_2 g^2},\\ \Gamma_2 &= \frac{b_2}{L_2} \cdot \frac{r_1 f^2 + r_2 g^2}{1 + s_1 f^2 + s_2 g^2}.\end{aligned} \tag{8}$$

To investigate the MI of this plane wave solution, we introduce the following perturbation to the stationary solution of Eq.(7)

$$\begin{aligned}u_1 &= [f + \sigma_1(\xi,\zeta)]\exp(\mathrm{i}\Gamma_1\zeta),\\ u_2 &= [g + \sigma_2(\xi,\zeta)]\exp(\mathrm{i}\Gamma_2\zeta),\end{aligned} \tag{9}$$

where $\sigma_1(\xi,\zeta)$ and $\sigma_2(\xi,\zeta)$ are small perturbations. When Eq.(9) is substituted into Eq.(6), with the higher-order terms in σ_1 and σ_2 dropped, we can obtain the following linearized equations:

$$\begin{aligned}\frac{\partial \sigma_1}{\partial \zeta} &= \frac{\mathrm{i}}{L_1}\left[\frac{1}{2}\frac{\partial^2 \sigma_1}{\partial \xi^2} + b_1 \cdot V_{11}(\sigma_1 + \sigma_1^*)\right.\\ &\quad \left. + b_1 \cdot V_{12}(\sigma_2 + \sigma_2^*)\right] = 0,\\ \frac{\partial \sigma_2}{\partial \zeta} &= \frac{\mathrm{i}}{L_2}\left[\frac{1}{2}\frac{\partial^2 \sigma_2}{\partial \xi^2} + b_2 \cdot V_{22}(\sigma_2 + \sigma_2^*)\right.\\ &\quad \left. + b_2 \cdot V_{21}(\sigma_1 + \sigma_1^*)\right] = 0,\end{aligned} \tag{10}$$

where

$$\begin{aligned}V_{11} &= \frac{\left[r_1 + (r_1 s_2 - r_2 s_1) g^2\right] f^2}{(1 + s_1 f^2 + s_2 g^2)^2},\\ V_{12} &= \frac{\left[r_2 + (r_2 s_1 - r_1 s_2) f^2\right] fg}{(1 + s_1 f^2 + s_2 g^2)^2},\\ V_{22} &= \frac{\left[r_2 + (r_2 s_1 - r_1 s_2) f^2\right] g^2}{(1 + s_1 f^2 + s_2 g^2)^2},\\ V_{11} &= \frac{\left[r_1 + (r_1 s_2 - r_2 s_1) g^2\right] fg}{(1 + s_1 f^2 + s_2 g^2)^2},\end{aligned}$$

and the asterisk represents complex conjugation.

Now we assume that the perturbation term takes the form

$$\begin{aligned}\sigma_j(\xi,\zeta) &= c_j \cdot \cos(K\zeta + \Omega\xi)\\ &\quad + \mathrm{i}d_j \cdot \sin(K\zeta + \Omega\xi),\\ &\quad (j=1,2),\end{aligned} \tag{11}$$

where K and Ω are the dimensionless wave number and spatial frequency of the perturbation wave, respectively. Insert expression (11) into Eq.(10), then we will obtain a set of four homogeneous equations as follows:

$$\begin{pmatrix} -K & -\dfrac{\Omega^2}{L_1} & 0 & 0\\ \dfrac{\Omega^2}{L_1} - \dfrac{2b_1 V_{11}}{L_1} & K & -\dfrac{2b_1 V_{12}}{L_1} & 0\\ 0 & 0 & -K & -\dfrac{\Omega^2}{L_2}\\ -\dfrac{2b_2 V_{21}}{L_2} & 0 & \dfrac{\Omega^2}{L_2} - \dfrac{2b_2 V_{22}}{L_2} & K \end{pmatrix} \begin{pmatrix} c_1\\ d_1\\ c_2\\ d_2 \end{pmatrix} = 0. \tag{12}$$

The non-trivial solution of Eq.(12) exists only when the following dispersion relation holds:

$$K^4 - T \cdot K^2 + D = 0, \tag{13}$$

where

$$T = \frac{\Omega^2}{L_1}\left(\frac{\Omega^2}{L_1} - \frac{2b_1V_{11}}{L_1}\right) + \frac{\Omega^2}{L_2}\left(\frac{\Omega^2}{L_2} - \frac{2b_2V_{22}}{L_2}\right),$$

$$D = \frac{\Omega^4}{L_1L_2}\left(\frac{\Omega^2}{L_1} - \frac{2b_1V_{11}}{L_1}\right)\left(\frac{\Omega^2}{L_2} - \frac{2b_2V_{22}}{L_2}\right) - 4\left(\frac{\Omega^2}{L_1L_2}\right)^2 b_1b_2V_{12}V_{21}.$$

If K has an imaginary part for some values of Ω, the plane wave solution is unstable, for σ_1 and σ_2 increases exponentially. This phenomenon is referred to as MI, for it leads to the modulation of the steady-state amplitude. So, Eq.(13) describes the MI of two incoherently coupled beams. The instability growth rate (or MI gain) is given by $G(\Omega) = |\mathrm{Im}(K)|$, where Im denotes the imaginary part. There are two cases here.

(a) If quantity $T^2 - 4D \geq 0$, Eq.(13) yields two real solutions for K^2, namely,

$$K_{\pm}^2 = \frac{1}{2}\left(T \pm \sqrt{T^2 - 4D}\right);$$

(b) If quantity $T^2 - 4D < 0$, Eq.(13) yields two solutions for K^2, namely,

$$K_{\pm}^2 = \frac{1}{2}\left(T \pm \mathrm{i}\sqrt{4D - T^2}\right).$$

3. Simulations and results

In the following, we numerically study the MI of two broad incoherently coupled beams in an ACP with photoisomerization nonlinearity. We take disperse red 1 (DR1)-doped polymethylmethacrylate (PMMA) (dopant concentration 5% by weight) described in Refs.[25, 26] as the working material, whose ST_0 is about 10^{-2} and linear refractive index is about 1.5 in the visible range. In our simulations, we choose the wavelengths of signal beams to be 514.5 and 632.8 nm and corresponding parameters $\alpha_1 \approx 3$ and $\beta_1 \approx 1$, and $\alpha_2 \approx 0.1$ and $\beta_2 \approx 0.1$, respectively. Three typical values of γ are chosen to be 0.2, 1 and 2, which correspond to different wavelengths of background beam. It is known that for the case of single signal beam, a 514.5 nm signal beam will undergo focusing for the values of γ ranging from 0.2 to 1 while defocusing for $\gamma = 2$; and a 632.8 nm signal beam will undergo focusing for $\gamma = 0.2$ while defocusing for the values of γ ranging from 1 to 2. Unless otherwise stated, we choose spatial scales $x_0 = 10\,\mu$m and $z_0 = 1$ mm in our simulations. Now, we study the MI of two broad incoherently coupled beams in detail at $\gamma = 0.2$, 1 and 2, respectively.

3.1. $\gamma = 0.2$

First, we use nonlinear least-squares fit algorithm to find the values of parameters γ_1, γ_2, s_1 and s_2 in expression (4) according to expression (3). We obtain $r_1 = 1.022$, $r_2 = 0.025$, $s_1 = 6.625$, and $s_2 = 0.290$. Now, the MI of two incoherently coupled beams can be analysed based on Eq.(13).

Some typical MI gain spectra for different values of M with $f = 0.02$ are shown in Fig.1(a), where $M = g/f$. The curves for maximum MI gain versus M with different values of f are shown in Fig.1(b). Obviously, the curves for maximum MI gain do not decrease monotonically with M for some values of f but they have peak values at some appropriate values of M (M_p), for example, $M_\mathrm{p} = 50$ for $f = 0.02$ and $M_\mathrm{p} = 18$ for $f = 0.05$. In addition, the value of M_p corresponding to peak value of maximum of MI gain decreases monotonically with the value of f increasing, and the peak value of the maximum of MI gain appears at $M_\mathrm{p} = 0$, when the value of f exceeds a certain value (about 0.098). We can also see that MI is suppressed when the value of M is large enough, which means that we can use a strong signal beam to suppress the MI of the other weak signal beam. This is due to the fact that with the increase of M, the photoisomerization nonlinearity of ACP becomes saturated, and so the MI of coupled beams is suppressed. It is known that a large MI gain is favourable for obtaining SOSs. Here, we have two methods to arrive at the maximum MI gain. One is to adjust the intensity of the other signal beam I_2 ($M^2 = I_2/I_1$) for the given intensities of one signal beam and background beam ($I_1/I_\mathrm{b} = f^2$), and the other is to adjust the intensity of the background beam I_b($f^2 = I_1/I_\mathrm{b}$) for the given intensities of two signal beams ($I_2/I_1 = M^2$).

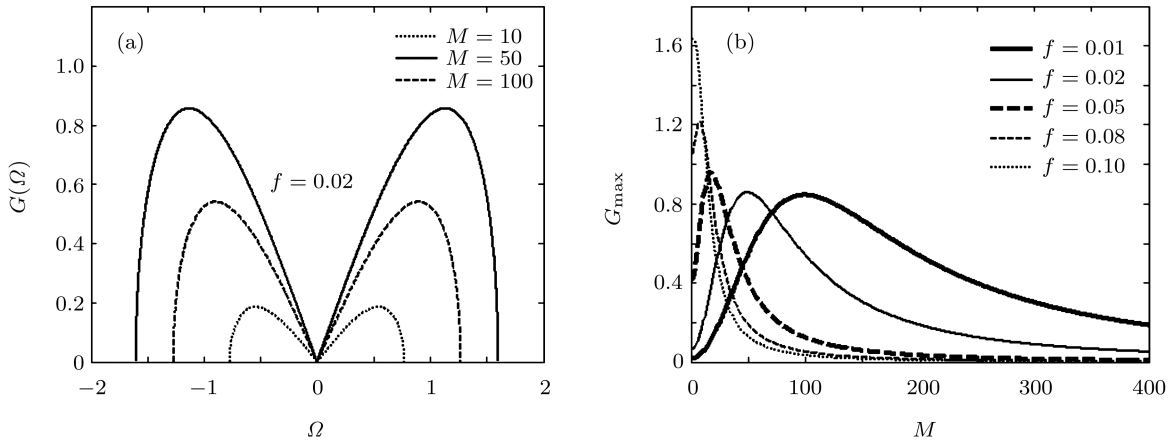


Fig.1. Modulational instability gain spectra for different values of M at $f = 0.02$ (a) and the curves for maximum modulational instability gain versus M for different values of f (b).

3.2. $\gamma = 1$

In this case, $r_1 = 0.075$, $r_2 = -0.004$, $s_1 = 2.545$, and $s_2 = 0.106$. Some typical MI gain spectra as a function of f for $M = 1$ are shown in Fig.2(a). It is evident that we can obtain the incoherently coupled bright-bright soliton pairs though the 632.8nm signal beam is defocused in this case. It is found that the MI gain decreases monotonically with M increasing and disappears when $M > 5.3$ (see Fig.2(b)), which indicates that the combination of 632.8 nm ($r_2 < 0$) and 514.5 nm ($r_1 > 0$) signal beams can effectively suppress, even completely cancel the MI. This is due to not the saturation of photoisomerization nonlinearity but the defocusing effect of 632.8 nm signal beam. Comparing Fig.2(b) with Fig.1(b), one can see that the MI is suppressed strongly when the 632.8 nm signal beam exists. Moreover, the weaker the 514.5 nm light is (i.e. the smaller the value of f becomes), the less the MI will be, and the stronger the 632.8 nm light is (i.e. the stronger the M becomes.), the less the MI will be.

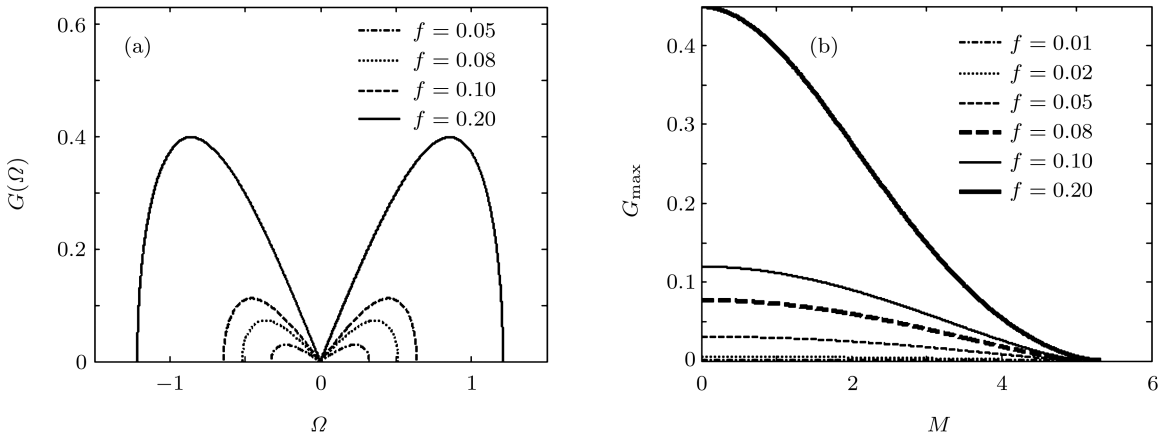


Fig.2. Modulational instability gain spectra for different values of f with $M = 1$ (a) and the curves for maximum modulational instability gain versus M for different values of f (b).

3.3. $\gamma = 2$

In this case, $r_1 = -0.011$, $r_2 = -0.006$, $s_1 = 1.687$, and $s_2 = 0.083$. We cannot find the MI for two broad incoherently coupled beams. So it can support not the bright–bright incoherent coupled soliton pairs but the dark–dark incoherently coupled soliton pairs.

4. Conclusion

We have investigated the modulational instability of two incoherently coupled beams in DR1-doped PMMA with a background irradiation both analytically and numerically. Our results show that as a precursor to soliton formation, the modulational instabil-

ity can be adjusted and controlled by the wavelength combinations of the signal and background beams. We have also discussed the dependences of the strength of modulational instability on the intensities of 632.8 nm and 514.5 nm signal beams with a given background beam. For $\gamma = 0.2$, the MI can be strengthened or suppressed, depending on the ratio between intensities of two signal beams. For $\gamma = 1$, the wavelength combination of two signal beams can effectively suppress even completely cancel the MI. For $\gamma = 2$, two incoherently coupled signal beams are modulationally stable.

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