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Ultranarrow absorptive spectral line induced by microwave field*

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This paper investigates the absorptive spectral lines of four-level atomic system driven by a coupling, probe and microwave fields. Due to the perturbation of the microwave field, the original electromagnetically induced transparency is changed to electromagnetically induced absorption and the absorptive spectral line can be very narrow. This ultranarrow spectral line has potential applications to the microwave atomic frequency standard and the measurement of very weak magnetic field.

Keywords: photon–atom interaction, susceptibility, quantum interference phenomenon
PACC: 4250, 3280, 3200

1. Introduction

Light propagation in an atomic ensemble with special controlled methods has been investigated extensively due to its interesting features and potential applications. One of these methods is electromagnetically induced transparency (EIT)[1] which relies on quantum interference to suppress the probe light absorption. It has been extensively studied recently due to its interesting features, such as very steep narrow dispersion relation,[2] ultraslow light speed,[3−6] superluminality,[7] and its potential applications, such as nonlinear optics,[8] quantum information processing[2] and weak magnetic field measurement.[9] Based on the EIT scheme and by introducing an additional atomic state, an electromagnetically induced absorption (EIA)[10−15] system can be formed which is another method to control the light propagation in an atomic medium. A four-level EIA system has been investigated for the generation of giant nonlinearity and low light intensity wave mixing as well as quantum information processing. Reference [16] investigated the generation of a very sharp optical absorptive spectral line by using a weak laser field to disturb the EIT system. This very narrow spectral line, which is much narrower than the spectral line of a usual EIT system, can be used for atomic frequency standard. This system can also be dealt with via a four-level atomic system. Contrary to previous research[16] which used all laser fields to implement an atomic clock, we consider a four-level atomic system to fulfil an atomic standard frequency by using two laser fields and a microwave field to generate an ultranarrow absorption line. Because the relaxation time of the ground hyperfine states is much longer than that of the excited hyperfine states, so we can expect much narrow spectral line in this system which may be narrower than that of all light systems. Although this kind of four-level system has been studied[10−13] and several interesting phenomena have been found, such as narrow optical spectra and EIA, these investigations focus on all laser fields interacting with atoms or scanning a probe laser light to get an absorption line. For the all-light implemented atomic clock, far-off detuned lights should be applied to achieve a very narrow absorption line at the expense of the signal-to-noise ratio. In our study we use only one laser device to produce a coupling and a probe light with the same frequency. We scan a weak microwave to generate an absorption line, while the coupling and probe lights are interacting with atomic ensemble. Although this scheme is similar to the case for scanning probe light

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while the coupling and the microwave are used resonantly, the noises induced by laser light can be avoided by using microwave and the refraction indices are very different. As far as we know, the study based on EIT scheme by scanning a weak microwave to generate a very narrow spectral line has not been done. In this way the required optical devices will be simple and this is the advantage of our scheme.

2. Theory and simulations

We consider a four-level atomic system shown in Fig.1.

$|1\rangle$ is the excited state and $|2\rangle$, $|3\rangle$, $|4\rangle$ are ground states.

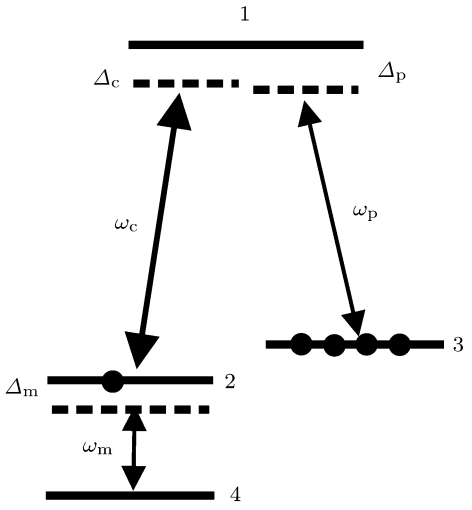


Fig.1. Four-level atomic structure. $|1\rangle$ is the excited state and $|2\rangle$, $|3\rangle$, $|4\rangle$ are ground states. $\omega_{\rm p}$, $\omega_{\rm c}$, and $\omega_{\rm m}$ are probe, coupling, and microwave frequencies respectively. $\Delta_{\rm p}$, $\Delta_{\rm c}$ and $\Delta_{\rm m}$ are probe, coupling, and microwave detunings respectively.

As a realistic system, laser lights drive the transitions of the ^{87}Rb D1 line. $|1\rangle$ can be a Zeeman state of the excited hyperfine state of an alkaline metal atom, such as Rubidium $|F_{\rm e}=1,\ m_{\rm F}=1\rangle$. $|2\rangle$ and $|3\rangle$ are the Zeeman states of the same ground hyperfine state which can be $|2\rangle=|F_{\rm g}=2,\ m_{\rm F}=0\rangle$ and $|3\rangle=|F_{\rm g}=2,\ m_{\rm F}=2\rangle$. $|4\rangle$ is another hyperfine state which can be $|F_{\rm g}=1,\ m_{\rm F}=0\rangle$. $|2\rangle$ and $|4\rangle$ are the states of a clock. Initially all the populations of the atoms are pumped into the state $|3\rangle$. As in conventional EIT, the probe light drives the transition $|1\rangle\leftrightarrow|3\rangle$ and the coupling light is applied to the $|1\rangle\leftrightarrow|2\rangle$ transition. The atomic levels $|2\rangle$ and $|4\rangle$ transition is driven by a microwave field. $\omega_{\rm p}$, $\omega_{\rm c}$, and $\omega_{\rm m}$ are the probe, coupling, and microwave frequencies respectively. $\Delta_{\rm p}$, $\Delta_{\rm c}$, and $\Delta_{\rm m}$ are the probe, coupling, and microwave detunings respectively whose definitions will be given later. Here we consider that atomic ensemble is of a pencil-like shape, so we use one dimensional description. The three fields are given as

$$E_{\rm p}(z,t)=E_{\rm p0}{\rm e}^{-{\rm i}(\omega_{\rm p}t-k_{\rm p}z)}+{\rm c.c}, \tag{1}$$

$$E_{\rm c}(z,t)=E_{\rm c0}{\rm e}^{-{\rm i}(\omega_{\rm c}t-k_{\rm c}z)}+{\rm c.c.} \tag{2}$$

$$E_{\rm m}(z,t)=E_{\rm m0}{\rm e}^{-({\rm i}\omega_{\rm m}t-k_{\rm m}z)}+{\rm c.c.}, \tag{3}$$

where $E_{\rm p0}$, $E_{\rm c0}$, $E_{\rm m0}$, $k_{\rm p}$, $k_{\rm c}$, and $k_{\rm m}$ are the amplitudes, and wave vectors of the three fields respectively. The Hamiltonian of the system is

$$H=H_0+H_{\rm I}, \tag{4}$$

where H_0 is the free part of the Hamiltonian given as

$$\begin{aligned}H_0=&\sum_{j=1}^{N}(\hbar\omega_1|1_j\rangle\langle 1_j|+\hbar\omega_2|2_j\rangle\langle 2_j|\\&+\hbar\omega_3|3_j\rangle\langle 3_j|+\hbar\omega_4|4_j\rangle\langle 4_j|),\end{aligned} \tag{5}$$

and the interaction part of the Hamiltonian is

$$\begin{aligned}H_{\rm I}=-\sum_{j=1}^{N}&(\hbar\Omega_{\rm p}{\rm e}^{-{\rm i}\omega_{\rm p}t}|1_j\rangle\langle 3_j|+\hbar\Omega_{\rm c}{\rm e}^{-{\rm i}\omega_{\rm c}t}|1_j\rangle\langle 2_j|\\&+\hbar\Omega_{\rm m}{\rm e}^{-{\rm i}\omega_{\rm m}t}|2_j\rangle\langle 4_j|+{\rm h.c.})\end{aligned} \tag{6}$$

with $\Omega_{\rm p}$, $\Omega_{\rm c}$ and $\Omega_{\rm m}$ are the Rabi frequencies of the three fields respectively which are defined by $\Omega=\wp_{ij}E/\hbar$. Assuming that the atomic ensemble is dense, we find that there are a great number of atoms within a macroscopically small volume. The collective slowly varying atomic operators can be described as

$$\sigma_{\mu\nu}=\frac{1}{N}\sum_{j=1}^{N}|\mu_j\rangle\langle\nu_j|{\rm e}^{-{\rm i}\omega_{\mu\nu}t}. \tag{7}$$

With the above Hamiltonian we can obtain equations of motion for the atomic density matrix elements under rotating wave approximation:

$$\dot\sigma_{11}=-(\gamma_1+\gamma_2)\sigma_{11}+{\rm i}\Omega_{\rm p}(\sigma_{31}-\sigma_{13})+{\rm i}\Omega_{\rm c}(\sigma_{21}-\sigma_{12}), \tag{8}$$

$$\dot\sigma_{22}=\gamma_1\sigma_{11}+{\rm i}\Omega_{\rm c}(\sigma_{12}-\sigma_{21})+{\rm i}\Omega_{\rm m}(\sigma_{42}-\sigma_{24}), \tag{9}$$

$$\dot\sigma_{33}=\gamma_2\sigma_{11}+{\rm i}\Omega_{\rm p}(\sigma_{13}-\sigma_{31}), \tag{10}$$

$$\dot\sigma_{44}={\rm i}\Omega_{\rm m}(\sigma_{24}-\sigma_{42}), \tag{11}$$

$$\dot\sigma_{12}=-\Gamma_{12}\sigma_{12}-{\rm i}\Omega_{\rm c}(\sigma_{11}-\sigma_{22})-{\rm i}\Omega_{\rm m}\sigma_{14}, \tag{12}$$

$$\dot\sigma_{13}=-\Gamma_{13}\sigma_{13}+{\rm i}\Omega_{\rm p}(\sigma_{33}-\sigma_{11})+{\rm i}\Omega_{\rm c}\sigma_{23}, \tag{13}$$

$$\dot\sigma_{14}=-\Gamma_{14}\sigma_{14}+{\rm i}\Omega_{\rm p}\sigma_{34}+{\rm i}\Omega_{\rm c}\sigma_{24}-{\rm i}\Omega_{\rm m}\sigma_{12}, \tag{14}$$

$$\dot\sigma_{23}=-\Gamma_{23}\sigma_{23}+{\rm i}\Omega_{\rm c}\sigma_{13}-{\rm i}\Omega_{\rm p}\sigma_{21}+{\rm i}\Omega_{\rm m}\sigma_{43}, \tag{15}$$

$$\dot{\sigma}_{43} = -\Gamma_{34}\sigma_{34} - \mathrm{i}\Omega_{\mathrm{p}}\sigma_{41} + \mathrm{i}\Omega_{\mathrm{m}}\sigma_{23}, \tag{16}$$

$$\dot{\sigma}_{\mu\nu} = \dot{\sigma}^{*}_{\nu\mu}, \tag{17}$$

where the generalized decay rates are

$$\Gamma_{12} = \mathrm{i}\Delta_{\mathrm{c}} + \gamma_{12}, \tag{18}$$

$$\Gamma_{13} = \mathrm{i}\Delta_{\mathrm{p}} + \gamma_{13}, \tag{19}$$

$$\Gamma_{14} = \mathrm{i}(\Delta_{\mathrm{c}} + \Delta_{\mathrm{m}}) + \gamma_{14}, \tag{20}$$

$$\Gamma_{23} = \mathrm{i}(\Delta_{\mathrm{p}} - \Delta_{\mathrm{c}}) + \gamma_{23}, \tag{21}$$

$$\Gamma_{43} = \mathrm{i}(\Delta_{\mathrm{p}} - \Delta_{\mathrm{c}} - \Delta_{\mathrm{m}}) + \gamma_{43}, \tag{22}$$

with $\Delta_{\mathrm{p}} = \omega_{13} - \omega_{\mathrm{p}}$, $\Delta_{\mathrm{c}} = \omega_{12} - \omega_{\mathrm{c}}$, $\Delta_{\mathrm{m}} = \omega_{24} - \omega_{\mathrm{m}}$. Because the probe field and the microwave field are very weak, we keep terms of all orders in the strong coupling field, but only the linear terms in the probe field. This is equivalent to using

$$\sigma_{33}^{(0)} = 1,\ \sigma_{42}^{(0)} = 0,\ \sigma_{21}^{(0)} = 0,\ \sigma_{41}^{(0)} = 0. \tag{23}$$

Thus we obtain

$$\dot{\sigma}_{13} = -\Gamma_{13}\sigma_{13} + \mathrm{i}\Omega_{\mathrm{p}} + \mathrm{i}\Omega_{\mathrm{c}}\sigma_{23}, \tag{24}$$

$$\dot{\sigma}_{23} = -\Gamma_{23}\sigma_{23} + \mathrm{i}\Omega_{\mathrm{c}}\sigma_{13} + \mathrm{i}\Omega_{\mathrm{m}}\sigma_{43}, \tag{25}$$

$$\dot{\sigma}_{43} = -\Gamma_{43}\sigma_{43} + \mathrm{i}\Omega_{\mathrm{m}}\sigma_{23}. \tag{26}$$

Therefore, we obtain

$$\sigma_{13} = \mathrm{i}\Omega_{\mathrm{p}}\frac{\Gamma_{23}\Gamma_{43} + \Omega_{\mathrm{m}}^2}{\Gamma_{13}\Gamma_{23}\Gamma_{43} + \Gamma_{13}\Omega_{\mathrm{m}}^2 + \Gamma_{43}\Omega_{\mathrm{c}}^2}. \tag{27}$$

The linear susceptibility is given as

$$\chi = \mathrm{i}\beta\frac{\Gamma_{23}\Gamma_{43} + \Omega_{\mathrm{m}}^2}{\Gamma_{13}\Gamma_{23}\Gamma_{43} + \Gamma_{13}\Omega_{\mathrm{m}}^2 + \Gamma_{43}\Omega_{\mathrm{c}}^2}, \tag{28}$$

where $\beta = (3/8\pi)N\gamma_{13}\lambda^3$. The susceptibility is similar to that of Ref.[10], but the difference is that we consider the changes of the microwave parameters. For the case of all optical transitions, it has been dealt in Ref.[12]. One absorption peak located at $[\Delta(\gamma_{23}\gamma_{43} + \Omega_{\mathrm{m}}^2)]/(\gamma_{13}\gamma_{23} + \Omega_{\mathrm{c}}^2)$ when there is one-photon detuning $\Delta = \Delta_{\mathrm{p}} = \Delta_{\mathrm{c}}$. Under the conditions $\gamma_{23}\gamma_{43} \ll \Omega_{\mathrm{m}}^2$ and $\gamma_{13}\gamma_{23} \ll \Omega_{\mathrm{c}}^2$, the half width is

$$\Gamma_{\mathrm{half}} = \frac{\Omega_{\mathrm{m}}^2[\gamma_{43}(\gamma_{23}^2\Delta^2 + \Omega_{\mathrm{c}}^4) + (\gamma_{23}\Delta^2 + \gamma_{13}\Omega_{\mathrm{c}}^2)\Omega_{\mathrm{m}}^2]}{\Omega_{\mathrm{m}}^2(\Omega_{\mathrm{c}}^4 - \gamma_{23}^2\Delta^2) + \gamma_{23}\gamma_{43}(\gamma_{23}^2\Delta^2 + \Omega_{\mathrm{c}}^4)}. \tag{29}$$

When the laser lights are applied resonantly, the calculated susceptibility of the system is shown in Fig.2, where all the parameters are scaled as the spontaneous emission rate. It can be seen that when the microwave field is resonant the absorption reaches its maximum. With the increase of the detuning of microwave, the absorption is decreasing. The maximum of the absorption is $(\gamma_{23}\gamma_{43} + \Omega_{\mathrm{m}}^2)/[\gamma_{43}\Omega_{\mathrm{c}}^2 + \gamma_{13}(\gamma_{23}\gamma_{43} + \Omega_{\mathrm{m}}^2)]$ at $\Delta_{\mathrm{m}} = 0$ for the optical resonant interactions.

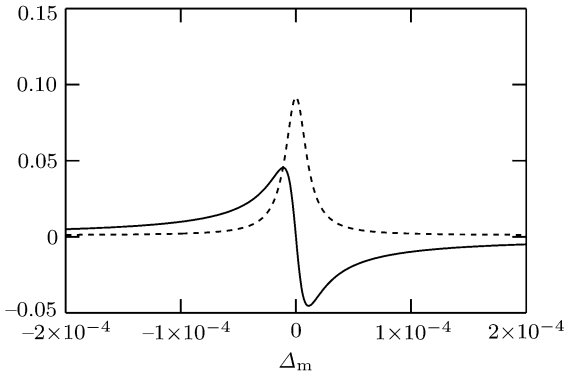


Fig.2. The susceptibility of the system. The solid line is the dispersion and the dashed line is the absorption. $\gamma_{13} = 1.0$, $\gamma_{23} = 0.001$, $\gamma_{43} = 10^{-5}$, $\Omega_{\mathrm{c}} = 1.0$, $\Omega_{\mathrm{m}} = 10^{-3}$, $\Delta_{\mathrm{p}} = \Delta_{\mathrm{c}} = 0$.

For comparison, we also computed the susceptibility when the probe field is detuned and plotted the results in Fig.3. Although the absorptions for the two cases are same, the refractions are very different. The group velocity of the probe light is slow when we scan the probe field. The half width of the absorption for our scheme is

$$\Gamma_{\mathrm{half}} = \frac{\gamma_{43}\Omega_{\mathrm{c}}^2 + \gamma_{13}\Omega_{\mathrm{m}}^2}{\Omega_{\mathrm{c}}^2} \tag{30}$$

for $\Delta = 0$, $\gamma_{23}\gamma_{43} \ll \Omega_{\mathrm{m}}^2$ and $\gamma_{13}\gamma_{23} \ll \Omega_{\mathrm{c}}^2$. Because the absorption line is created in the EIT window, so its width should be smaller than the width of the EIT.[17] Because of $\gamma_{43} \ll \gamma_{13}$ and $\Omega_{\mathrm{m}} \ll \Omega_{\mathrm{c}}$, the absorption spectral line will be very narrow. For an example, as a realistic EIT system, the line width is determined by γ_{23} which is around 1kHz. According to Eq.(30) and for $\gamma_{43} = 10^{-5}$, the contribution to

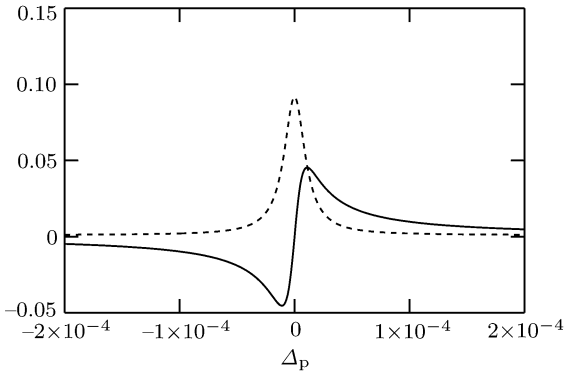


Fig.3. The susceptibility of the system. The solid line is the dispersion and the dashed line is the absorption. $\gamma_{13} = 1.0$, $\gamma_{23} = 0.001$, $\gamma_{43} = 10^{-5}$, $\Omega_{\mathrm{c}} = 1.0$, $\Omega_{\mathrm{m}} = 10^{-3}$, $\Delta_{\mathrm{c}} = 0$, $\Delta_{\mathrm{m}} = 0$.

the line width of this part is less than 1 kHz. In realistic experimental study, the Rabi frequency of probe light is around MHz, but the Rabi frequency of the microwave field is around kHz. So the contribution to the line width of the second part of Eq.(30) can also be less than 1 kHz. Therefore we can obtain very narrow absorption line in our system. In addition, because of the weak probe field, the scheme will not suffer too much optical pumping effects. It is the advantage that the absorption spectral line is applied to atomic frequency reference. For this kind of microwave clock, its accuracy will be affected by many factors, such as doppler, power, collision, pressure and magnetic field frequency shifts as well as laser light field, microwave field and electronic noises, which will be compared with experimental results and discussed elsewhere.

3. Conclusions

In conclusion, we have closely investigated the absorptive spectra of four-level atomic ensemble interacting with the laser fields and a microwave field. Our results clearly show that the line width of the spectral line can be very narrow. Due to its relative simplicity of optics and very narrow spectral line, the scheme has the perspective on atomic frequency standard and the measurement of very weak magnetic field.

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