Vol 18 No 1, January 2009
1674-1056/2009/18(01)/0195-04

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Generation of superpositions of coherent states for an atomic sample in cavity QED*

Zheng Shi-Biao(郑仕标)†

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, China

(Received 2 May 2008; revised manuscript received 15 May 2008)

This paper proposes a scheme for generation of superpositions of coherent states of the effective bosonic mode in a collection of atoms. In the scheme an atomic sample interacts with a slightly detuned cavity mode and a resonant strong classical field. Under certain conditions the atomic system evolves from a coherent state to a superposition of coherent states.

Keywords: superposition of coherent states, atomic sample, cavity QED
PACC: 4250

In recent years, much attention has been paid to superpositions of macroscopically distinguishable states, i.e. the so-called Schrödinger cat states.[1] Though such superposition states are formed by states having closest classical analogues but they may exhibit various nonclassical properties due to the quantum interference between the coherent components.[2] Schrödinger cat states are of significance for exploring the boundary between quantum and classical worlds and understanding the decoherence effect in quantum information. The macroscopic superposition states also provide a test for quantum measurement theory. In the measurement model, the coupling between a macroscopic apparatus and a microscopic system results in their entanglement and produces a quantum superposition state of the whole system. Schemes have been proposed for generation of such superposition states in cavity QED.[3−5] These superposition states have been experimentally realized for a cavity field.[6]

In addition to the cat states of harmonic oscillators and electromagnetic fields, there has been interest in another kind of superposition state, associated with a collection of two-level atoms. It has been shown that superpositions of two atomic coherent states may also exhibit nonclassical properties, such as oscillations in the atomic population and squeezed fluctuations.[7] The composite atom–field system is a qualified candidate for generating quantum states of the atomic system. Recently, Zhang *et al* have studied the dynamics of a cavity QED system, in which a collection of atoms are trapped in a double-well potential and interact dispersively with a damped and driven cavity mode.[8] Gerry and Grobe have proposed a method for generating superpositions of atomic coherent states via dispersive interaction of the atomic system with a cavity field followed by state reduction.[7] Ling and Guo have shown that a large number of atoms may evolve to a superposition of two atomic coherent states by resonantly interacting with a quantized field.[9] This method requires that the field initially is prepared in a superposition of two coherent states. Agarwal *et al* have shown that, under certain conditions, the effective interaction between atoms in a dispersive cavity leads to the superposition of atomic coherent states.[10] The scheme requires that the detuning between the atomic system and the cavity mode is much larger than the atom–cavity coupling strength. Furthermore, the detuning should increase as the number of atoms increases and thus the required interaction time increases with the number of atoms.

In this paper we propose an alternative scheme for generation of superpositions of collective atomic coherent states through interaction with a cavity mode and a strong classical field. Unlike the scheme of Ref.[9], our scheme does not require that the cavity mode is initially prepared in a superposition state. In contrast with the schemes of Refs.[7] and [10], the detuning between the atomic system and the cav-

*Project supported by the National Natural Science Foundation of China (Grant No 10674025) and the Doctoral Foundation of the Ministry of Education of China (Grant No 20070386002).
†E-mail: sbzheng@pub5.fz.fj.cn

ity mode is of the order of the atom–cavity coupling strength and the operating speed is independent of the number of the atoms, which is important in view of decoherence.

Suppose that an atomic sample involving N atoms is trapped in a single-mode cavity and driven by a classical field. Each atom has one excited state $|e\rangle$ and one ground state $|g\rangle$. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$)[11,12]

$$\begin{aligned} H =&\omega_0 \sum_{j=1}^{N} S_{z,j} + \omega_a a^+ a \\ &+ \sum_{j=1}^{N} [g(a^+ S_j^- + a S_j^+) \\ &+ \Omega(S_j^+ \mathrm{e}^{-\mathrm{i}\omega t} + S_j^- \mathrm{e}^{\mathrm{i}\omega t})], \end{aligned} \tag{1}$$

where

$$\begin{aligned} &S_j^+ = |e_j\rangle \langle g_j|, \ S_j^- = |g_j\rangle \langle e_j|, \\ &S_{z,j} = \frac{1}{2}(|e_j\rangle \langle e_j| - |g_j\rangle \langle g_j|), \end{aligned} \tag{2}$$

a^+ and a are the creation and annihilation operators for the cavity mode, respectively, g is the atom–cavity coupling strength, Ω is the Rabi frequency of the classical field, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Under the condition $\omega_0 = \omega$, the interaction Hamiltonian, in the interaction picture, is

$$\begin{aligned} H_\mathrm{i} =& \sum_{j=1}^{N} [g(\mathrm{e}^{i\delta t} a^+ S_j^- + \mathrm{e}^{\mathrm{i}\delta t} a S_j^+) \\ &+ \Omega(S_j^+ + S_j^-)], \end{aligned} \tag{3}$$

where $\delta = \omega_0 - \omega_\mathrm{a}$. The free Hamiltonian $\omega_0 \sum_{j=1}^{N} S_{z,j} + \omega_\mathrm{a} a^+ a$ is used for the transformation to the interaction picture.

Define the new atomic basis

$$\begin{aligned} |+_j\rangle &= \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle), \\ |-_j\rangle &= \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle), \end{aligned} \tag{4}$$

then we can rewrite H_i as

$$\begin{aligned} H_\mathrm{i} = \sum_{j=1}^{N} \Big\{ & g\Big[\mathrm{e}^{i\delta t} a^+ \Big(\sigma_{z,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^-\Big) \\ &+ \mathrm{e}^{\mathrm{i}\delta t} a \Big(\sigma_{z,j} + \frac{1}{2}\sigma_j^- - \frac{1}{2}\sigma_j^+\Big)\Big] + 2\Omega\sigma_{z,j}\Big\}, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\sigma_j^+ = |+_j\rangle \langle -_j|, \ \sigma_j^- = |-_j\rangle \langle +_j|, \\ &\sigma_{z,j} = \frac{1}{2}(|+_j\rangle \langle +_j| - |-_j\rangle \langle -_j|). \end{aligned} \tag{6}$$

The time evolution of this system is decided by Schrödinger's equation:

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H_\mathrm{i}|\psi(t)\rangle. \tag{7}$$

Performing the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_0 t}|\psi'(t)\rangle \tag{8}$$

with

$$H_0 = 2\Omega \sum_{j=1}^{N} \sigma_{z,j}, \tag{9}$$

we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H_\mathrm{i}'|\psi'(t)\rangle, \tag{10}$$

where

$$\begin{aligned} H_\mathrm{i}' = \sum_{j=1}^{N} g\Big[& \mathrm{e}^{-\mathrm{i}\delta t} a^+ \Big(\sigma_{z,j} + \frac{1}{2}\sigma_j^+ \mathrm{e}^{2\mathrm{i}\Omega t} - \frac{1}{2}\sigma_j^- \mathrm{e}^{-2\mathrm{i}\Omega t}\Big) \\ &+ \mathrm{e}^{\mathrm{i}\delta t} a \Big(\sigma_{z,j} + \frac{1}{2}\sigma_j^- \mathrm{e}^{-2\mathrm{i}\Omega t} - \frac{1}{2}\sigma_j^+ \mathrm{e}^{2\mathrm{i}\Omega t}\Big)\Big]. \end{aligned} \tag{11}$$

Under the condition $\Omega \gg \delta, g$, we can neglect the terms oscillating fast,[11] then H_i' reduces to

$$H_\mathrm{i}' = \sum_{j=1}^{N} g(\mathrm{e}^{-\mathrm{i}\delta t} a^+ + \mathrm{e}^{\mathrm{i}\delta t} a)\sigma_{z,j}. \tag{12}$$

Set

$$\begin{aligned} b &= \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |-_j\rangle \langle +_j|, \\ b^+ &= \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |+_j\rangle \langle -_j|, \\ n_b &= \sum_{j=1}^{N} |+_j\rangle \langle +_j|, \end{aligned} \tag{13}$$

then we have

$$\begin{aligned} &\big[b, b^+\big] = 1 - \frac{2}{N} n_b, \\ &\big[n_b, b^+\big] = b^+, \\ &[n_b, b] = -b. \end{aligned} \tag{14}$$

Suppose that N is much larger than 1, and where $\bar{n}_b$ is the average number of the atoms in the state $|+\rangle$. Then b, b^+ can be regarded as the bosonic operators and the atomic system can be regarded as a bosonic system.[9] The Hamiltonian can be rewritten as

$$H_\mathrm{i}' = g(\mathrm{e}^{-\mathrm{i}\delta t} a^+ + \mathrm{e}^{\mathrm{i}\delta t} a)(b^+ b - N/2). \tag{15}$$

The evolution operator for Hamiltonian $H_{\rm i}'$ can be written as[13]

$$\begin{aligned} U'(t) =& {\rm e}^{-{\rm i}A(t)(b^+b-N/2)^2}{\rm e}^{-{\rm i}B(t)(b^+b-N/2)a} \\ & \times {\rm e}^{-{\rm i}C(t)(b^+b-N/2)a^+}. \end{aligned} \tag{16}$$

Using the Schrödinger equation

$${\rm i}\frac{{\rm d}U'(t)}{{\rm d}t} = H_{\rm i}U'(t), \tag{17}$$

we obtain

$$\begin{aligned} B(t) =& \int_0^t g{\rm e}^{{\rm i}\delta t'}{\rm d}t' = \frac{g}{{\rm i}\delta}({\rm e}^{{\rm i}\delta t}-1), \\ C(t) =& \int_0^t g{\rm e}^{-{\rm i}\delta t'}{\rm d}t' = \frac{g}{-{\rm i}\delta}({\rm e}^{-{\rm i}\delta t}-1), \\ A(t) =& {\rm i}\int_0^t B(t')g{\rm e}^{-{\rm i}\delta t'}{\rm d}t' \\ =& \frac{g^2}{\delta}\left[t+\frac{1}{{\rm i}\delta}({\rm e}^{-{\rm i}\delta t}-1)\right]. \end{aligned} \tag{18}$$

Under the condition

$$\delta t = 2\pi, \tag{19}$$

we have

$$U'(t) = {\rm e}^{-{\rm i}g^2t(b^+b-N/2)^2/\delta}. \tag{20}$$

The whole evolution operator of the system is given by

$$\begin{aligned} U(t) =& {\rm e}^{-{\rm i}H_0t}U'(t) \\ =& {\rm e}^{{\rm i}N\Omega t-{\rm i}N^2g^2t/(4\delta)-{\rm i}(2\Omega-Ng^2/\delta)tb^+b-{\rm i}g^2t(b^+b)^2/\delta}. \end{aligned} \tag{21}$$

Assume that the atomic sample is initially in the coherent state

$$|\psi(0)\rangle = |\alpha\rangle = {\rm e}^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{22}$$

then we have

$$|\psi(t)\rangle = {\rm e}^{-|\alpha|^2/2}\sum_n \frac{\alpha' n}{\sqrt{n!}}{\rm e}^{-{\rm i}n^2g^2t/\delta}|n\rangle, \tag{23}$$

where

$$\alpha' = \alpha{\rm e}^{-{\rm i}(2\Omega-Ng^2/\delta)t}. \tag{24}$$

We here have discarded the common phase factor ${\rm e}^{{\rm i}N\Omega t-{\rm i}N^2g^2t/4\delta}$. Setting

$$g^2t/\delta = \pi/2, \tag{25}$$

we can rewrite $|\psi(t)\rangle$ as[14]

$$\begin{aligned} |\psi(t)\rangle =& \frac{1}{\sqrt{2}}{\rm e}^{-|\alpha|^2/2}\sum_{n=1}^{\infty}\left[{\rm e}^{-{\rm i}\pi/4}\frac{\alpha' n}{\sqrt{n!}}\right. \\ & \left.+{\rm e}^{{\rm i}\pi/4}\frac{(-\alpha')^n}{\sqrt{n!}}\right]|n\rangle \\ =& \frac{1}{\sqrt{2}}({\rm e}^{-{\rm i}\pi/4}|\alpha'\rangle+{\rm e}^{-{\rm i}\pi/4}|-\alpha'\rangle). \end{aligned} \tag{26}$$

By this way we obtain a superposition of two coherent states with same amplitude but opposite phases for the collective atomic mode. Setting $\delta = 2g$ and $gt = \pi$, we find that Eqs.(19) and (25) can be satisfied.

It is necessary to give a brief discussion on the experimental feasibility of the proposed scheme. For the Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is about $T_{\rm r} = 3\times10^{-2}\,{\rm s}$, and the coupling constant is $g = 2\pi\times25\,{\rm kHz}$,[15] then the required interaction time is $t = 2\times10^{-6}\,{\rm s}$, and the decoherence rate of the atomic sample is given by $|\alpha|^2/T_{\rm r}$.[6] Set $|\alpha|^2 = 100$. Thus the infidelity induced by the atomic spontaneous emission is about $|\alpha|^2 t/T_{\rm r} \simeq 0.67\times10^{-2}$. Suppose the cavity field is initially in the vacuum state, then the average photon number during the interaction is of the order of $\bar{n}\sim g^2/\delta^2 \sim 1$. Very recently, an ultrahigh fineness Fabry–Perot resonator with a damping time $T_{\rm c} = 0.13\,{\rm s}$ has been built.[16] The infidelity induced by the cavity decay is about $\bar{n}t/T_{\rm c} \sim 1.54\times10^{-5}$. With the whole decoherence effect being considered the fidelity is about 0.99.

In conclusion, we have proposed a scheme for generation of superpositions of coherent states for a collective atomic mode in cavity QED. The scheme does not require that the cavity field is initially in a superposition state. The detuning between the atomic system and the cavity mode does not need to be much larger than the atom–cavity coupling strength and the required interaction time is independent of the number of the atoms. This is important for the suppression of decoherence. With available experimental techniques, the scheme might be realizable.

References

[1] Schrödinger E 1935 *Naturwissenschaften* **23** 807

[2] Yia Y and Guo G 1989 *Phys. Lett.* A **136** 281

[3] Brune M, Haroche S, Raimond J M, Davidovich L and Zagury N 1992 *Phys. Rev.* A **45** 1992

[4] Zhou L, Song H S, Li C and Guo Y Q 2003 *Chin. Phys.* **12** 45

[5] Liu X, Fang M F and Liu A L 2000 *Acta Phys. Sin.* **49** 1707 (in Chinese)

[6] Brune M, Hagley E, Dreyer J, Maitre X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **77** 4887

[7] Gerry C C and Grobe R 1997 *Phys. Rev.* A **56** 2390

[8] Zhang J M, Liu W M and Zhu D L 2008 *Phys. Rev.* A **77** 033620

[9] Ling T and Guo G C 1997 *J. Opt. Soc. Am.* B **14** 1537

[10] Agarwal G S, Puri R R and Singh R P 1997 *Phys. Rev.* A **56** 2249

[11] Solano E, Agarwal A S and Walther H 2003 *Phys. Rev. Lett.* **90** 027903

[12] Zheng S B 2002 *Phys. Rev.* A **66** 060303

[13] Sϕrensen A and Mϕlmer K 2000 *Phys. Rev.* A **62** 022311

[14] Yurk B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13

[15] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024

[16] Kuhr S, Gleyzes S, Guerlin C, Bernu J, Hoff U B, Deleglise S, Osnaghi S, Brune M, Raimond J M and Haroche S 2006 quant-ph/0612138