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The recording of digital hologram at short distance and reconstruction using convolution approach*

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By adopting in-line lensless Fourier setup and phase-shifting technique, we recorded the phase-shifting digital hologram at short distance. As the Fresnel diffraction condition is no longer valid, the convolution approach is chosen for the reconstruction. However, the simulated reference wave for the reconstruction would suffer from severe undersampling due to the comparatively large pixel size. To solve this problem, sinc-interpolation is introduced to get the pixel-size of the hologram reduced prior to the reconstruction. The experimental results show that an object image of high fidelity is obtained with this method.

Keywords: short-distance digital holography, convolution algorithm, sinc-interpolation
PACC: 4240K, 4240H, 4230V

1. Introduction

Digital holography is a novel imaging method that records a hologram using digital camera and reconstructs the complex wavefields with digital computation. Because it involves no chemical processing and can obtain not only intensities but also phase distributions, this technique has found extensive applications.[1−3] Nevertheless, because of relatively low resolution and small imaging area of the video camera devices, a major problem of this technique is to improve the resolution of the reconstructed image. A high resolution reconstruction requires the recording of the hologram with a large numerical aperture (NA) of the object. Either of two ways can be used to increase the numerical aperture, namely shortening the object–sensor distance[4] or enlarging the sensor size using aperture synthesis.[5]

To satisfy the sampling theorem at short distance, it is very important to choose a proper recording setup. By adopting the in-line lensless Fourier setup,[6] Nyquist sampling condition can be easily satisfied, and if the size of object is sufficiently small, the recording distance can be much shortened, thus more high-frequency information of the object can be recorded and the resolution can be improved. Furthermore, introducing the phase-shifting technique into in-line lensless Fourier holography, the DC term and virtual image can be eliminated.[7]

However, with large NA, the Fresnel diffraction condition is no longer valid and the Fresnel algorithm would result in a blurred object image because of the spherical aberration. In this case a digital hologram can generally be reconstructed by evaluating the Kirchhoff–Fresnel or Sommerfeld integral, but with very low calculation efficiency. Thus the pursuit of more efficient reconstruction algorithms is urgent.[8−11] In Ref.[8], a modified Fresnel algorithm was proposed in which the paraxial approximation was made at the object plane. And in Ref.[9], Lu *et al* proved that higher-order phase terms can be sufficiently compensated for under certain conditions and the Fourier algorithm is still available for the reconstruction even if the Fresnel diffraction condition is not satisfied. Nevertheless, both of these methods would put a strict limitation on the size of the recording objects. Particularly under high NA conditions, only a fraction of the whole object field can be reconstructed without aberration. In Ref.[12], the convolution was directly derived from Fresnel–Kirchhoff integration without approximation, and being based on Fast Fourier Transform, it is calculation speed is fast. However, when applying this algorithm to the re-

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construction of high NA hologram, the simulated reference wave would suffer from severe under-sampling due to the comparatively large pixel size. To solve this problem, Ref.[10] proposed a novel method using linear-interpolation to reduce the pixel distance of the recorded hologram. But the linear-interpolation, being a low-order polynomial interpolation, has low accuracy and would induce artefacts in the reconstructed image.

In this paper, the recording of high NA holograms by adopting the in-line lensless Fourier setup is presented and in order to improve the reconstruction quality of the convolution approach, sinc-interpolation is applied to make ideal expansion of the hologram.

2. Recording of the phase-shifting holograms at short distance

Figure 1 shows the profile of the in-line lensless Fourier setup for recording small object at short distance. The point R, which is located at the centre of the object field, is the source of the spherical reference wave. In real experiment, this in-line setup can be realized by a beamsplitter. It can be seen that, in spite of the large incident angles of the object

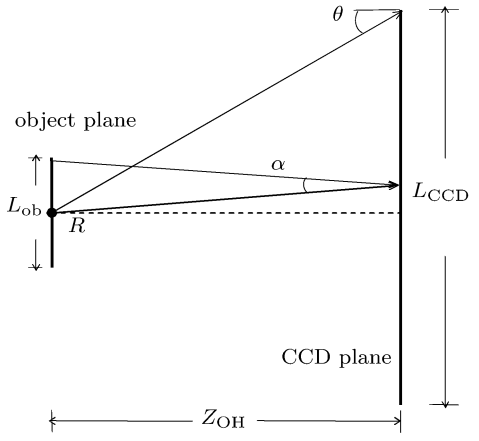


Fig.1. Illustration of the in-line lensless Fourier setup for recording small object at short distance.

wave and reference wave, the object-reference angle α, which determines the frequency of the interference fringe (α/λ), can be reduced by limiting the object extent. Denoting the object size by $L_{\rm ob}$, the recording distance by $Z_{\rm OH}$, and noting that the maximum object-reference angle is $\alpha = 2\tan^{-1}(L_{\rm ob}/(4Z_{\rm OH}))$, by imposing the Nyquist sampling law, we get the relation between the allowable field size and the recording distance

$$L_{\rm ob} = \frac{\lambda}{\Delta} Z_{\rm OH}, \tag{1}$$

where λ is the wavelength of the operating light and Δ is the pixel spacing of the image sensor. According to this equation, when the object–CCD distance is shortened to increase the recording numerical aperture, the size of object field is reduced linearly.

Assuming that $O(x_{\rm H},\ y_{\rm H})$ is the complex amplitude of the diffracted object wave on the image sensor and $R(x_{\rm H},\ y_{\rm H})$ is the complex amplitude of the spherical reference wave, the recorded intensity hologram by the image sensor can be written as

$$\begin{aligned} I(x_{\rm H}, y_{\rm H}) =& |O(x_{\rm H}, y_{\rm H})|^2 + |R|^2 + O(x_{\rm H}, y_{\rm H})R^* \\ &+ O^*(x_{\rm H}, y_{\rm H})R. \end{aligned} \tag{2}$$

We use four-step phase-shifting technique to eliminate the zero-order image and the conjugate image. The phase of the reference wave was shifted sequentially by 0, $\pi/2$, π, $3\pi/2$, and four corresponding in-line holograms $I_n(x_{\rm H},\ y_{\rm H})$ (n=1, 2, 3, 4) are recorded. Multiplying the four holograms by $\exp[{\rm j}(n-1)\pi/2]$ respectively, and then adding them together, we obtain a four-step phase-shifting hologram

$$\begin{aligned} I_{\rm ps}(x_{\rm H}, y_{\rm H}) =& \frac{1}{4}\sum_{n=1}^{4} I_n \exp\left[{\rm j}\,(n-1)\,\pi/2\right] \\ =& O(x_{\rm H}, y_{\rm H})R^*(x_{\rm H}, y_{\rm H}). \end{aligned} \tag{3}$$

3. Reconstruction algorithm

In Fresnel holography, where the object extent and the hologram extent are much smaller than the recording distance, by taking Fourier transform of $I_{\rm ps}$, we would get the object image.[13] However when recording an object at short distance, Fresnel approximation is not accurate anymore and higher order phase term has to be considered. Strictly, the object can be reconstructed precisely using Fresnel–Kirchhoff integral Eq.(4), but the direct numerical processing of Fresnel–Kirchhoff integral without approximation is time consuming. Thus

$$\begin{aligned} O(x_{\rm I}, y_{\rm I}) =& \iint_{\Sigma_{\rm H}} I_{\rm ps}(x_{\rm H}, y_{\rm H}) R(x_{\rm H}, y_{\rm H}) \\ &\times g(x_{\rm I} - x_{\rm H}, y_{\rm I} - y_{\rm H}) {\rm d}x_{\rm H} {\rm d}y_{\rm H}, \end{aligned} \tag{4}$$

where $x_{\rm I}$ and $y_{\rm I}$ are the coordinates of the image plane, $R(x_{\rm H}, y_{\rm H})$ is the simulated reference wave and $g(x_{\rm I}-x_{\rm H}, y_{\rm I}-y_{\rm H})$ is the impulse response of free space propagation, which is given by

$$g(x_{\mathrm{I}}-x_{\mathrm{H}},y_{\mathrm{I}}-y_{\mathrm{H}})=\frac{\mathrm{i}}{\lambda}\frac{\exp\left[-\mathrm{i}\frac{2\pi}{\lambda}\sqrt{Z_{\mathrm{OH}}^2+(x_{\mathrm{H}}-x_{\mathrm{I}})^2+(y_{\mathrm{H}}-y_{\mathrm{I}})^2}\right]}{\sqrt{Z_{\mathrm{OH}}^2+(x_{\mathrm{H}}-x_{\mathrm{I}})^2+(y_{\mathrm{H}}-y_{\mathrm{I}})^2}}. \quad (5)$$

The convolution approach uses the fact that the diffraction integral Eq.(4), viewed as a superposition integral, represents a linear shift-invariant system that can be written as a convolution, which then can be calculated by the Fourier transform

$$\begin{aligned}O(x_{\mathrm{I}},y_{\mathrm{I}}) =&[I_{\mathrm{ps}}(x_{\mathrm{H}},y_{\mathrm{H}})R(x_{\mathrm{H}},y_{\mathrm{H}})]*g(x_{\mathrm{H}},y_{\mathrm{H}})\\ =&F^{-1}\{F[I_{\mathrm{ps}}(x_{\mathrm{H}},y_{\mathrm{H}})R(x_{\mathrm{H}},y_{\mathrm{H}})]\\ &\times G(f_x,f_y)\}, \end{aligned} \quad (6)$$

where $*$, F and F^{-1} denote the convolution operation, Fourier Transform and its inverse, respectively. $G(f_x,\ f_y)$ is the transfer function, which has the form

$$G(f_x,f_y)=\exp[-\mathrm{i}kZ_{\mathrm{OH}}(1-\lambda^2f_x^2-\lambda^2f_y^2)^{1/2}]. \quad (7)$$

According to the sampling character of the CCD, the four-step phase-shifting digital hologram I_{ps}, the simulated reference wave $R(x_{\mathrm{H}},\ y_{\mathrm{H}})$ and $G(f_x,\ f_y)$ in Eq.(6) should be changed to discrete form, and the Fourier transforms are realized by the discrete Fourier transform algorithm. However, due to the short recording distance, the simulated reference wave $R(x_{\mathrm{H}},\ y_{\mathrm{H}})$ would reach the CCD with very high spatial frequency, thus under-sampling would occur and high frequency information would be lost,[14] as shown in Fig.2. So, in order to apply the convolution approach when the object is placed near the CCD, the sample distance has to be reduced.

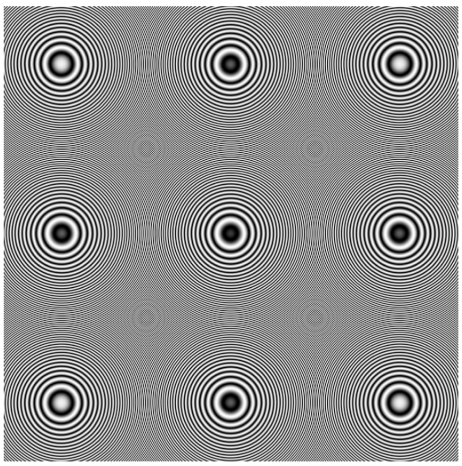


Fig.2. The real part of the simulated reference wave $R(x_{\mathrm{H}},\ y_{\mathrm{H}})$ sampled by a CCD sensor with 576×576 pixels and pixel size 10 μm×10 μm at distance Z_{OH}=30 mm. Due to the under-sampling, the high frequency part of the spherical wave degrades to eight sets of circular fringes on the edge of the CCD.

As the hologram is recorded digitally, the reduction of the sample distance is not a problem by using pixel interpolation. As shown in Fig.1, the maximum spatial frequency of the reference wave on the CCD plane is

$$f_{\max}=\frac{\sin\theta}{\lambda}=\left(\lambda\sqrt{\left(\frac{2Z_{\mathrm{OH}}}{L_{\mathrm{CCD}}}\right)^2+1}\right)^{-1}. \quad (8)$$

In order to make the simulated reference wave $R(x_{\mathrm{H}},\ y_{\mathrm{H}})$ satisfy Nyquist sampling condition, the new pixel distance Δ' after interpolation must meet $\Delta'\le 1/(2f_{\max})$, and combining Eq.(8), we finally get the ratio of the original pitch to the new one,

$$R=\frac{\Delta}{\Delta'}\ge 2\Delta\left(\lambda\sqrt{\left(\frac{2Z_{\mathrm{OH}}}{L_{\mathrm{CCD}}}\right)^2+1}\right)^{-1}. \quad (9)$$

In Ref.[9], the pixel distance of the hologram is reduced by linear interpolation. But linear interpolation has low interpolation accuracy and would introduce artifacts into the reconstructed image. In view of the sampling theorem, sinc-interpolation[15] is the most accurate method for representing bandlimited signals. For this reason, we adopt sinc-interpolation. Figure 3 shows an efficient sinc-interpolation algorithm[15] used for accelerating the interpolation speed. Based on the fact that the frequency spectrum of I_{ps} is the same as that of its ideal amplification I_{MPS} except that the frequency spectrum of the latter is surrounded by additional zeros, we directly get the frequency spectrum of I_{MPS} by padding the spectrum of I_{ps} with an appropriate number of zeros, and then by taking a inverse Fourier transform of the padded spectrum to get I_{MPS}.

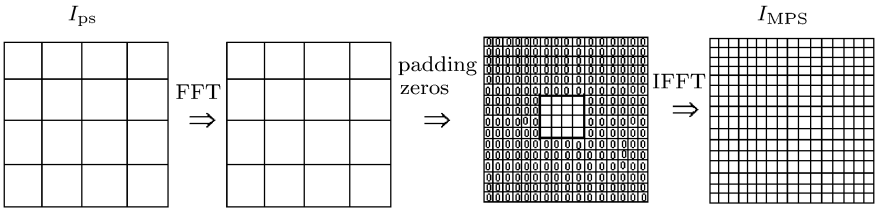


Fig.3. Flow diagram of the fast sinc-interpolation implemented by the signal spectrum zero-padding algorithm.

The flow chart of reconstruction is shown in Fig.4. First, the four-step phase-shifting digital hologram I_{ps}, which is derived by Eq.(3), is expanded $R \times R$ times using sinc-interpolation to get a pixel-size reduced hologram I_{MPS}, then the reference wave $R(x_H,\ y_H)$ can be simulated without under-sampling and the subsequent reconstruction is completed using convolution approach.

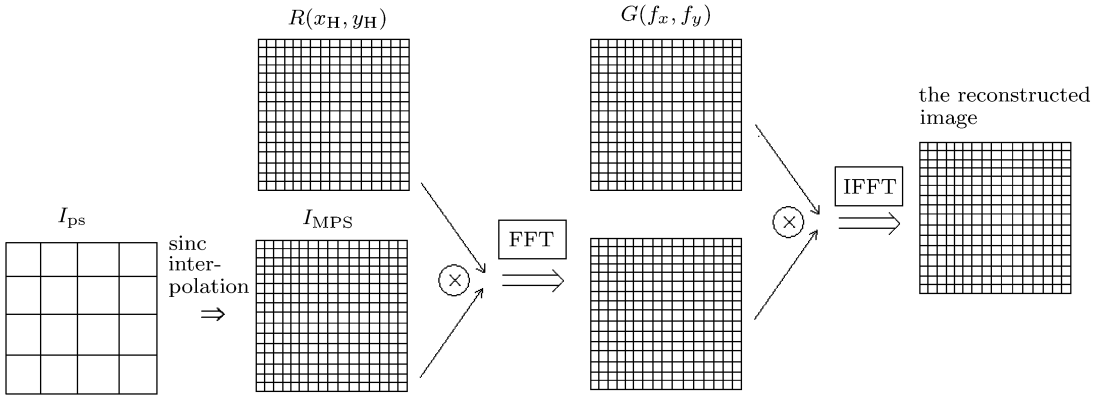


Fig.4. The flow chart of the reconstruction by the convolution approach.

4. Experiment and results

The experimental setup shown in Fig.5 is basically a Mach–Zehnder interferometer configuration. The light beam from a He–Ne laser is split into two beams by a splitter BS_1. One illuminates the object after being reflected by mirror M_1. This wave is regarded as the object wave. The other beam is reflected and phase-shifted by the reflector M_2 which is moved by a piezoelectric transducer (PZT) controlled by a computer. And then, the beam is converted to a spherical reference wave by a microscope objective(40×, 0.65 NA). A pinhole of a diameter of 20 μm placed at the focus of the microscope objective acts as a spatial filter. The spherical reference beam recombines with the object beam at the second beam splitter BS_2. By adjusting the tilt of BS_2 and the position of the object, the object and the reference wave point are made in-line and coplanar. Then the phase-shifted interference patterns are recorded by a CCD image sensor with 576(H)×576(V) pixels and pixel size 10 μm×10.8 μm. In the experiment, the CCD sensor is placed as close as possible to the object to give a minimal recording distance.

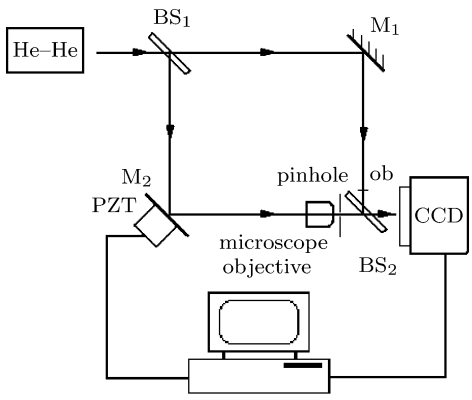


Fig.5. Experimental setup for recording in-line phase-shifting digital hologram at short distance.

A Chinese standard No.3 resolution test pattern is recorded at a distance of 24 mm. At this recording distance, the allowable field size L_{ob}=1.52 mm. In order to have an object of permitted size, a square aperture of the size of 1 mm is attached as a mask, as shown in Fig.6. The 25th element, which consists of four gratings with grating constant d=20 μm, is kept in the aperture. Figure 7 shows one of the recorded phase-shifting holograms. One can see that the third order diffraction pattern of one grating in the 25th element is recorded. This diffraction pattern has a diffraction angle of 0.095 rad, which is in accordance with the recording NA=$L_{CCD}/(2Z_{OH})$ = 0.12.

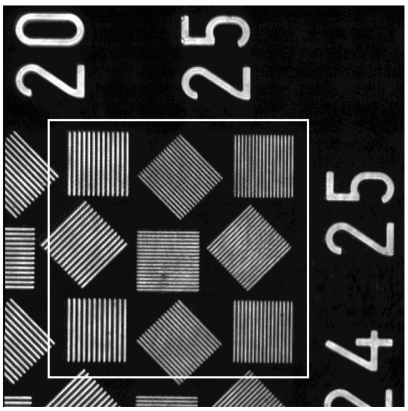


Fig.6. Image of the test pattern obtained with an optical lens. The square denotes the aperture used for masking out the other part of the test pattern.

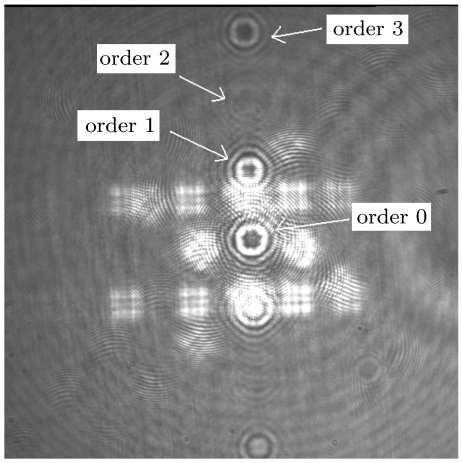


Fig.7. Recorded hologram pattern.

The image reconstructed by the convolution algorithm is shown in Fig.8. In the case of Z_{OH}=24 mm, the result of Eq.(9) is $R \geq 3.76$. Here we choose $R = 4$, so the pixel size after interpolation is reduced to 2.5 μm×2.7 μm and pixel number is expanded to 2304 H×2304 V. The sinc-interpolation shown in Fig.3 and the reconstruction procedure in Fig.4 are all implemented by the Fast Fourier Transform algorithm in Matlab. The total time cost for one processing is about 30 s.

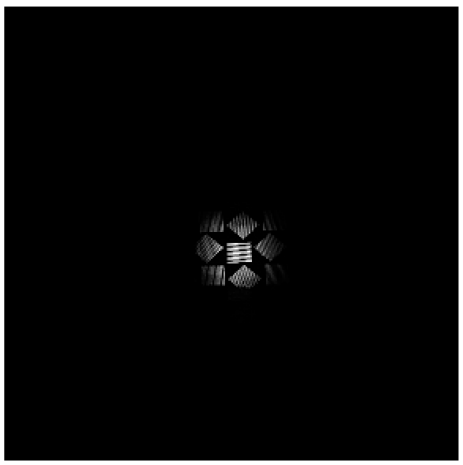


Fig.8. Reconstructed image by the convolution algorithm(pixel size 2.5 μm×2.7 μm and pixel number 2304 H×2304 V).

For the convolution algorithm, the size of reconstructed image equals to that of the CCD sensor.[12] So, the reconstructed object, which is much smaller than the CCD, is a fractional part of the reconstructed image, as shown in Fig.8. The reconstructed detail is shown in Fig.9. One can see that an object image of high fidelity is obtained with the convolution algorithm, while in the result of the Fresnel algorithm shown in Fig.10, the gratings near the boundaries are blurred and the resolution degrades because of the spherical aberration.

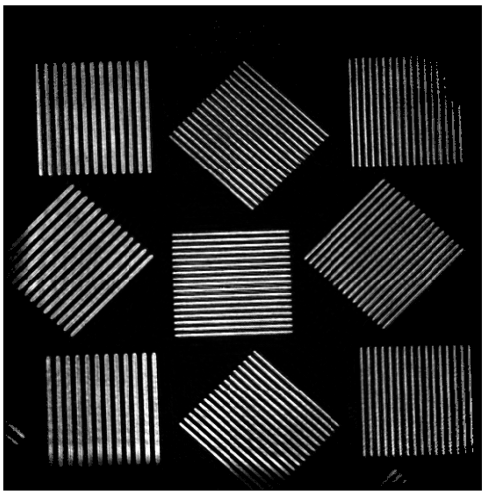


Fig.9. Enlarged part of Fig.8.

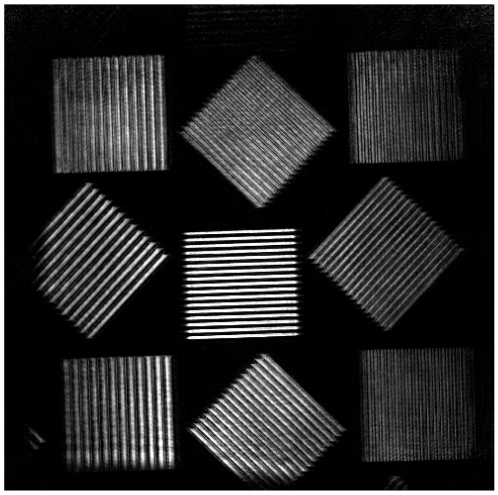


Fig.10. Reconstruction result with the Fresnel algorithm.

5. Conclusions

In summary, the recording of phase-shifting holograms of high-NA has been realized by adopting inline lensless Fourier setup and the phase-shifting technique. As the Fresnel approximation is not accurate anymore, the convolution approach, which is the fast implement of the Fresnel–Kirchhoff integral, is chosen for the reconstruction. To solve the undersampling problem of the simulated reference wave, sinc-interpolation is introduced to get the pixel-size of the hologram reduced prior to the reconstruction. Experimental results show that the quality of the reconstructed image is greatly improved by this reconstruction procedure.

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