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Material parameter equation for rotating elliptical spherical cloaks*

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By using the coordinate transformation method, we have deduced the material parameter equation for rotating elliptical spherical cloaks and carried out simulation as well. The results indicate that the rotating elliptical spherical cloaking shell, which is made of meta-materials whose permittivity and permeability are governed by the equation deduced in this paper, can achieve perfect invisibility by excluding electromagnetic fields from the internal region without disturbing any external field.

Keywords: invisibility, cloaks, meta-materials
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1. Introduction

An interesting application of meta-materials is suggested in Refs.[1, 2], where described is an exotic lens, called cloaking shell, which is so designed that it can exclude an electromagnetic (EM) field from a transparent object without disturbing any external field, thus an object hidden in the cloak is perfectly invisible to the outside. This finding has been verified through both simulations and experiments.[3,4] The analyses of the interactions between EM waves and such cloaks as well as the scatterings of EM waves are also performed.[5−7] A modified motion equation is proposed,[8] which is invariant under the coordinate transformation, and it opens up possibilities for achieving the perfect invisibility cloaking in acoustic waves. Then, cylindrical acoustic cloaks[9] and spherical acoustic cloaks[10] are reported, separately.

The design of such cloaks is mostly based on the coordinate transformation method,[1] and the optical conformal mapping method can also be used under the geometrical optics limit.[2] Now the material parameter equations (MPEs) for cylindrical cloaks and spherical cloaks have been proposed by Pendry *et al*,[1] and MPE for elliptical cylindrical cloaks has been obtained by us in another paper.[11]

In the present paper, we apply the coordinate transformation method to deducing the MPE for rotating elliptical spherical (RES) cloaks, and present the simulations and discussions in the following sections.

2. Theoretical deduction

To build the MPE for a cloak with a given shape, an appropriate curvilinear coordinate system should be chosen first as a new coordinate system where the spatial distortion associated with the cloak shape can be described and the transformation between the original Cartesian coordinates and the new curvilinear coordinates can be performed correctly. For objects that are of RES shape, the RES coordinates $S(x, y, z)$ are generally selected as the new curvilinear coordinates $S'(\xi, \eta, \phi)$. The relationship between Cartesian coordinates $S(x, y, z)$ and curvilinear coordinates $S'(\xi, \eta, \phi)$ is as follows:

$$x = c\xi\eta, \quad y = \rho\cos\phi, \quad \text{and} \quad z = \rho\sin\phi, \tag{1}$$

where

$$\rho = c\sqrt{(1+\xi^2)(1-\eta^2)},$$

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and c is the focal length of the ellipse which is rotating around its long axis (the x axis) to form an RES. In the RES coordinates, spatial point (ξ, η, ϕ) can be located on a series of RES shells with the same focal length if we assume c to be a constant. Characterized by ξ, the RES shells fall into two regions, i.e. $[1, \xi_1]$ and $[\xi_1, \xi_2]$. According to the coordinate transformation method, fields excluded from region $[1, \xi_1]$ into region $[\xi_1, \xi_2]$ correspond to a spatial compression from the elliptical spherical region $[1, \xi_2]$ into the rotating annular region $[\xi_1, \xi_2]$ and to a spatial distortion that can be described as transformation between the original Cartesian coordinates and the coordinates of new RES system. The spatial compression can be expressed mathematically as

$$\xi' = \xi_1 + (\xi - 1)\frac{\xi_2 - \xi_1}{\xi_2 - 1}, \quad \eta' = \eta, \quad \text{and} \quad \phi' = \phi, \tag{2}$$

where ξ_1 and ξ_2 are the inner and outer shell coordinate parameters of the RES cloak, respectively.

Performing the coordinate transformation based on Eqs.(1) and (2), we obtain the MPE for an RES cloak as

$$\varepsilon'_{\xi'} = \mu'_{\xi'} = \frac{1 + \alpha^2}{1 + \xi'^2}\frac{\xi_2 - \xi_1}{\xi_2 - 1},$$

$$\varepsilon'_{\eta'} = \mu'_{\eta'} = \frac{\xi_2 - 1}{\xi_2 - \xi_1},$$

and

$$\varepsilon'_{\phi'} = \mu'_{\phi'} = \frac{\alpha^2 + \eta'^2}{\xi'^2 + \eta'^2}\frac{1 + \xi'^2}{1 + \alpha^2}\frac{\xi_2 - 1}{\xi_2 - \xi_1}, \tag{3}$$

where $\alpha = (\xi' - \xi_1)[(\xi_2 - 1)/(\xi_2 - \xi)] + 1$, and

$$\xi_1 \le \xi' \le \xi_2, \quad -1 \le \eta' \le 1. \tag{4}$$

Note that Eq.(3) is obtained on condition that the cloak is embedded in vacuum; however, it can also be used in non-vacuum cases by being multiplied with the relative permittivity and the relative permeability of the outer medium correspondingly. Suppose that the outer medium is symmetrical and isotropic, and that its relative permittivity and relative permeability are ε and μ, respectively, thus Eq.(3) can be rewritten as follows:

$$\varepsilon'_{\xi'}(\mu'_{\xi'}) = \frac{1 + \alpha^2}{1 + \xi'^2}\frac{\xi_2 - \xi_1}{\xi_2 - 1}\varepsilon(\mu),$$

$$\varepsilon'_{\eta'}(\mu'_{\eta'}) = \frac{\xi_2 - 1}{\xi_2 - \xi_1}\varepsilon(\mu),$$

$$\varepsilon'_{\phi'}(\mu'_{\phi'}) = \frac{\alpha^2 + \eta'^2}{\xi'^2 + \eta'^2}\frac{1 + \xi'^2}{1 + \alpha^2}\frac{\xi_2 - 1}{\xi_2 - \xi_1}\varepsilon(\mu). \tag{5}$$

The deduction above implies that, with the relative permittivity and permeability governed by Eqs.(3) and (5) respectively, the central elliptical spherical region $[1, \xi_1]$ can be cloaked by an RES cloak region $[\xi_1, \xi_2]$, which requires that the cloaking shell should be constructed by position-dependent, anisotropic meta-materials, and that the relative permittivity and relative permeability should be designed to be less than one at some positions.

3. Simulation and discussion

Equations (3) and (5) are the strict analytical expressions without any approximation, so they hold true theoretically. To further verify them and research the cloaking performance, we carry out numerical simulation by using a commercial EM simulation software, COMSOL$^{\text{TM}}$, and the results and discussion are given in the following.

In the simulation, we design an RES cloak, which is constructed with a meta-material, whose permittivity and permeability are governed by Eq.(5), and located in a perfect electric conductor (PEC) waveguide. Theoretically, all types of EM waves are suitable for observing the cloaking performance; however, plane EM waves are mostly selected because they can be analysed more easily. The modes of EM waves are restricted by the waveguide size. The TE_{10} mode requires that the wavelength and the waveguide size be approximately matched with each other. If there exists no medium in the waveguide, the wavelength of the TE_{10} wave and the size of the RES cloak are approximately matched with each other, so an inaccuracy would be brought about for the cloaking performance. To avoid such an inaccuracy, we fill the waveguide with a magnetic medium, whose refractive index is n. In this way, the wavelength of EM in the waveguide reduces to a one- nth wavelength in the free space.

Furthermore, we assume the cloaking meta-material to be lossless and the inner shell of the RES cloak to be PEC boundary. Thus, we can pay more attention to the distribution of the EM waves and explore the cloaking performance under ideal conditions.

Figure 1 shows the electric-field distribution when an EM wave is incident on the cloak in its propagation

direction along the major axis (the x axis) of elliptical sphere. From the figure, we find that the EM wave fronts remain planar and the scatting is quite small outside the cloaking shell; while inside the cloaking shell, the phase fronts are warped, which causes the power-flow to bend around the inner shell. In short, the EM fields are smoothly excluded from the internal region with minimal scatterings, and the cloaking effect is quite obvious.

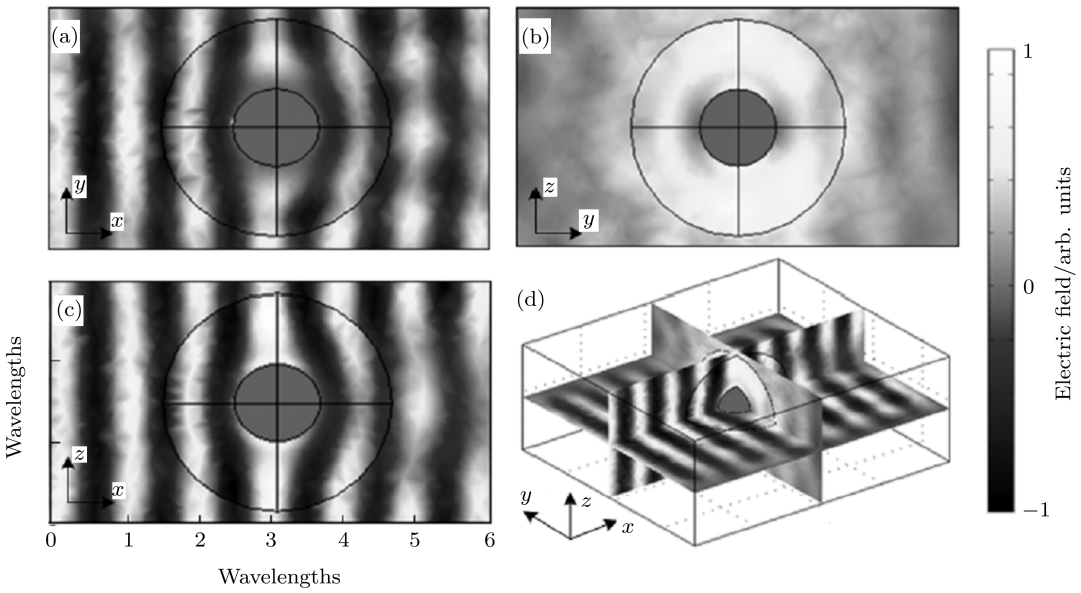


Fig.1. The electric-field distributions of a 5.16 GHz TE_{10} wave guided by a waveguide, in which an RES cloak is embedded in a medium with relative permittivity and relative permeability being 1 and 2 respectively. In the figure, three reference planes are selected to be (a) $z = 0$, (b) $x = 0$ and (c) $y = 0$. Panel (d) depicts a whole three-dimensional profile of the cloaking effect, showing the electric-field distributions in the three reference planes.

We also perform numerical simulation to investigate the cloaking effect of spherical cloaks and make a comparison between spherical sphere cloaks and RES cloaks. It is found that the scatting of RES cloaks is more significant than that of sphere cloaks. By improving the computational method and increasing the number of elements in the simulated field, the scatting of RES cloaks can reduce slightly, but cannot be eliminated completely. Such a difference between RES cloaks and sphere cloaks is caused by the difference between their symmetry properties. To verify this speculation, we reduce the RES to a spherical cloak and carry out the simulation on the same configuration as the above, and find that the scatting reduces to a minimal degree and the cloaking performance is as good as that of spherical cloak. Such a comparison gives the immediate evidence that there is a strong coherence between the MPEs of RES and spherical cloak, and it also shows that the scattering is very sensitive to the symmetry of cloak shapes.

4. Conclusions

We have deduced an MPE for RES cloak theoretically and explored the cloaking effect of RES cloak numerically. The results show that both the theoretical analyses and the numerical simulations are correct. In the simulations, field scattering is quite significant in the case where the RES eccentricity is large: this problem can be solved by improving the computational method and increasing the number of field elements. In the present paper, we offer another three-dimensional MPE besides that of spherical sphere cloaks, and open up possibilities for constructing more complex and multiplex three-dimensional cloaks by using simple models such as spherical cloak and RES cloak.

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