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Adaptive synchronization in an array of asymmetric coupled neural networks*

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This paper investigates the global synchronization in an array of linearly coupled neural networks with constant and delayed coupling. By a simple combination of adaptive control and linear feedback with the updated laws, some sufficient conditions are derived for global synchronization of the coupled neural networks. The coupling configuration matrix is assumed to be asymmetric, which is more coincident with the realistic network. It is shown that the approaches developed here extend and improve the earlier works. Finally, numerical simulations are presented to demonstrate the effectiveness of the theoretical results.

Keywords: coupled neural networks, adaptive synchronization, Lyapunov functional, delayed coupling

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1. Introduction

Over the past few decades, neural networks have been applied to various signal processing problems such as optimization, image processing, and associative memory design. These applications strongly depend on the dynamic behaviour of the network. Recently, dynamic behaviours of neural networks with time delays have been extensively investigated.[1−7] It has been recognized that the neural networks can exhibit some complicated dynamics and even chaotic behaviours if the network parameters and time delays are appropriately chosen.[7] In recent years, arrays of coupled systems have attracted much attention of researchers in different research fields.[8−17] The coupled neural networks are easy to physically implement and have promising applications especially in secure communication and image processing.[18,19]

Since its introduction by Pecora and Carroll,[20] synchronization of coupled chaotic systems has been extensively studied due to its potential applications in creating secure communication systems.[21−30] References [25–29] paid much attention to synchronization in an array of linearly coupled neural networks. Reference [25] investigated the synchronization of an array of linearly coupled neural networks with delays. Reference [26] derived a sufficient condition for global synchronization of the coupled identical delayed neural networks based on the Lyapunov functional method and Hermitian matrix theory. Reference [27] investigated the global synchronization of an array of delayed neural networks with constant and delayed coupling. On the other hand, among all the approaches for the synchronization of chaotic systems, a newly developed technique based on adaptive control has received increasing attention.[30−34] Reference [30] deduced several adaptive synchronization criteria for an uncertain complex dynamical network by using Lyapunov stability theory. Reference [33] proposed some novel approaches of adaptive synchronization and parameters identification for the delayed neural networks by using the invariance principle of functional differential equations. To the best of our knowledge, the adaptive synchronization in an array of coupled neural networks has received little research attention.

Motivated by the above discussions, in this paper we investigate the global synchronization in an array of coupled neural networks with constant and delayed coupling. Combining the dynamical error feedback theory with adaptive control, we derive some global synchronization criteria for the coupled neural networks based on the Lyapunov functional method. The

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coupling configuration matrix of the coupled system is assumed to be asymmetric. The obtained conditions are expressed in terms of linear matrix inequalities (LMIs), which can be efficiently verified by the Matlab LMI toolbox.

The rest of this paper is organized as follows. In the next section, problem formulation and some preliminaries are introduced. In Section 3, some sufficient criteria to ensure the globally asymptotically synchronization for an array of coupled neural networks are derived. In Section 4, simulations are given to illustrate the effectiveness of the main results. Finally, conclusions are drawn in Section 5.

Notation: Throughout this paper, $\boldsymbol{R}^n$ and $\boldsymbol{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. $\boldsymbol{I}_n$ denotes the $n\times n$ identity matrix and diag$\{\cdot\}$ denotes the diagonal matrix. For real symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the notation $\boldsymbol{X}\geq\boldsymbol{Y}$ (respectively, $\boldsymbol{X}>\boldsymbol{Y}$) means that the matrix $\boldsymbol{X}-\boldsymbol{Y}$ is positive semi-definite (respectively, positive definite). $\|\cdot\|$ stands for the Euclidean norm.

2. Model description and preliminaries

Consider an isolated delayed neural network (DNN), which can be described by the following differential equation:[7]

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& -\boldsymbol{D}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}(t))\\ &+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}(t-\sigma))+\boldsymbol{I}(t),\end{aligned}\tag{1}$$

where $\boldsymbol{x}(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^{\mathrm{T}}\in\boldsymbol{R}^n$ is the state vector of the neural network; $\boldsymbol{D}=\mathrm{diag}\{d_1,d_2,\ldots,d_n\}>0$ is a diagonal matrix with positive diagonal entries, $d_r>0$, $r=1,2,\ldots,n$; $\boldsymbol{A}=(a_{rj})_{n\times n}$ and $\boldsymbol{B}=(b_{rj})_{n\times n}$ denote the connection weight matrix and the delayed connection weight matrix, respectively. The constant $\sigma>0$ represents the transmission delay. $\boldsymbol{g}(\boldsymbol{x}(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_n(x_n(t))]^{\mathrm{T}}\in\boldsymbol{R}^n$ is the activation function vector, and $\boldsymbol{I}(t)=[I_1(t),I_2(t),\ldots,I_n(t)]^{\mathrm{T}}\in\boldsymbol{R}^n$ is an external input vector.

In this paper, we investigate a dynamical system consisting of N linearly coupled identical delayed neural networks, which is given as follows:

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t) =& -\boldsymbol{D}\boldsymbol{x}_i(t)+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}_i(t))+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}_i(t-\sigma))\\ &+\boldsymbol{I}(t)+c\sum_{j=1}^{N}G_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)\\ &+c\sum_{j=1}^{N}G_{ij}\boldsymbol{\Gamma}_\tau\boldsymbol{x}_j(t-\tau)+\boldsymbol{u}_i(t),\\ &i=1,2,\ldots,N,\end{aligned}\tag{2}$$

where $\boldsymbol{x}_i(t)=[x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t)]^{\mathrm{T}}\in\boldsymbol{R}^n$, $i=1,2,\ldots,N$, is the state vector of the ith delayed neural network at time t; $\boldsymbol{g}(\boldsymbol{x}_i(t))=[g_1(x_{i1}(t)),g_2(x_{i2}(t)),\ldots,g_n(x_{in}(t))]^{\mathrm{T}}\in\boldsymbol{R}^n$, $i=1,2,\ldots,N$, and $g_r(\cdot)$ $(r=1,2,\ldots,n)$ are activation functions. $\boldsymbol{u}_i(t)=[u_{i1}(t),u_{i2}(t),\ldots,u_{in}(t)]^{\mathrm{T}}\in\boldsymbol{R}^n$, $i=1,2,\ldots,N$ are the controllable inputs. The constant $c>0$ represents the coupling strength. τ denotes the coupling delay. Moreover, $\boldsymbol{\Gamma}=\mathrm{diag}\{\gamma_1,\gamma_2,\ldots,\gamma_n\}$ and $\boldsymbol{\Gamma}_\tau=\mathrm{diag}\{\gamma_1^\tau,\gamma_2^\tau,\ldots,\gamma_n^\tau\}$ to be determined are constant inner coupling matrices of the neural networks at time t and $t-\tau$. $\boldsymbol{G}=(G_{ij})_{N\times N}$ is the coupling configuration matrix representing the coupling strength and the topological structure of the system with

$$\begin{aligned}&G_{ij}\geq 0,\quad i\neq j,\\ &G_{ii}=-\sum_{j=1,j\neq i}^{N}G_{ij}.\end{aligned}\tag{3}$$

The initial condition associated with Eq.(2) is

$$\begin{aligned}&x_{ir}(s)=\phi_{ir}(s),\quad -\max\{\sigma,\tau\}\leq s\leq 0,\\ &\forall r=1,2,\ldots,n,\quad i=1,2,\ldots,N.\end{aligned}$$

Throughout this paper, the activation functions $g_r(\cdot)$ $(r=1,2,\ldots,n)$ are assumed to satisfy the following condition:

(A_1) There exist constants $l_r>0$ $(r=1,2,\ldots,n)$ such that

$$\begin{aligned}&|g_r(u)-g_r(v)|\leq l_r|u-v|,\\ &\forall\, u,\, v\in R,\ r=1,2,\ldots,n;\end{aligned}\tag{4}$$

or

(A_2) There exist constants $l_r>0$ $(r=1,2,\ldots,n)$ such that

$$\begin{aligned}&0\leq\frac{g_r(u)-g_r(v)}{u-v}\leq l_r,\\ &\forall\, u,\, v\in R,\ r=1,2,\ldots,n.\end{aligned}\tag{5}$$

Remark 1 The assumption (A_1) implies that the activation functions are continuous but not always monotonic. However, the activation functions satisfying the assumption (A_2) are continuous and monotonic activation functions. Therefore, the class of

functions satisfying the assumption (A_1) is larger than the class of functions satisfying the assumption (A_2).

To give our main results in the next section, we need the following definitions and lemma.

Definition 1[9] The set $\boldsymbol{S} = \{(\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_N^{\mathrm{T}}(t))^{\mathrm{T}} \in \boldsymbol{R}^{nN} : \boldsymbol{x}_i(t) = \boldsymbol{x}_j(t) \text{ for } i, j = 1, 2, \ldots, N\}$ is called synchronization manifold, where $\boldsymbol{x}_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^{\mathrm{T}} \in \boldsymbol{R}^n$ $(i = 1, 2, \ldots, N)$ is the state of node i.

Definition 2[11] Synchronization manifold S is said to be globally asymptotically stable for the coupled system (2), equivalently, the coupled system (2) is globally asymptotically synchronized, if the following condition holds:

$$\lim_{t\to\infty} \|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\| = 0, \quad i, j = 1, 2, \ldots, N.$$

Lemma 1[4] For any vectors $\boldsymbol{a}$, $\boldsymbol{b} \in \boldsymbol{R}^n$ and positive definite matrix $\boldsymbol{Y} \in \boldsymbol{R}^{n\times n}$, the following inequality holds:

$$2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b} \le \boldsymbol{a}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{a} + \boldsymbol{b}^{\mathrm{T}}\boldsymbol{Y}^{-1}\boldsymbol{b}.$$

3. Main results

In this section, based on the Lyapunov theory, we establish the following sufficient criteria for adaptive synchronization of system (2).

When the coupled system (2) achieves synchronization, namely, when the states of system (2) satisfy

$$\lim_{t\to\infty} \|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\| = 0, \quad \forall\, i, j = 1, 2, \ldots, N,$$

the coupling controllable terms should vanish:

$$c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}_\tau\boldsymbol{x}_j(t-\tau) + \boldsymbol{u}_i(t) = 0,$$
$$\text{for all} \quad i = 1, 2, \ldots, N.$$

Let $\boldsymbol{s}(t)$ be a synchronous solution of the coupled system (2), that is,

$$\dot{\boldsymbol{s}}(t) = -\boldsymbol{D}\boldsymbol{s}(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{s}(t)) + \boldsymbol{B}\boldsymbol{g}(\boldsymbol{s}(t-\sigma)) + \boldsymbol{I}(t), \tag{6}$$

where $\boldsymbol{s}(t)$ can be an equilibrium point, a periodic orbit, or a chaotic orbit.

Define error vector

$$\boldsymbol{e}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{s}(t), \quad i = 1, 2, \ldots, N, \tag{7}$$

then the error dynamical system is governed as follows:

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) = {} & -\boldsymbol{D}\boldsymbol{e}_i(t) + \boldsymbol{A}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t)) + \boldsymbol{B}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\sigma)) \\ & + c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}_\tau\boldsymbol{e}_j(t-\tau) \\ & + \boldsymbol{u}_i(t), \quad i = 1, 2, \ldots, N,\end{aligned} \tag{8}$$

where

$$\begin{aligned}&\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t)) = \boldsymbol{g}(\boldsymbol{x}_i(t)) - \boldsymbol{g}(\boldsymbol{s}(t)),\\ &\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\sigma)) = \boldsymbol{g}(\boldsymbol{x}_i(t-\sigma)) - \boldsymbol{g}(\boldsymbol{s}(t-\sigma)),\\ &i = 1, 2, \ldots, N.\end{aligned}$$

It follows, respectively, from the assumptions (A_1) and (A_2) that

$$|\widetilde{g}_r(u)| \le l_r|u|, \tag{9}$$

$$\begin{aligned}&0 \le u\widetilde{g}_r(u) \le l_r u^2,\\ &|\widetilde{g}_r(u)| \le l_r|u|,\end{aligned} \tag{10}$$

for $\forall\, u \in R$ and $r = 1, 2, \ldots, n$.

In the sequence, we denote $\boldsymbol{L} = \mathrm{diag}\{l_1, l_2, \ldots, l_n\} > 0$.

Theorem 1 Let the adaptive controllers

$$\begin{aligned}&\boldsymbol{u}_i(t) = -m_i\boldsymbol{e}_i(t) - n_i\boldsymbol{e}_i(t-\tau),\\ &i = 1, 2, \ldots, N,\end{aligned} \tag{11}$$

and the following updating laws

$$\begin{aligned}&\dot{m}_i = \rho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad \dot{n}_i = \varrho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t-\tau),\\ &i = 1, 2, \ldots, N,\end{aligned} \tag{12}$$

in which ρ_i and ϱ_i, $i = 1, 2, \ldots, N$ are positive constants. Under the assumption (A_1), the synchronization manifold $\boldsymbol{S}$ of the coupled system (2) is globally asymptotically stable, if there exist positive definite matrices $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, positive diagonal matrices $\boldsymbol{R} = \mathrm{diag}\{r_1, r_2, \ldots, r_n\} > 0$, $\boldsymbol{Q} = \mathrm{diag}\{q_1, q_2, \ldots, q_n\} > 0$, $\widehat{\boldsymbol{M}} = \mathrm{diag}\{\widehat{m}_1, \widehat{m}_2, \ldots, \widehat{m}_N\} > 0$ and $\widehat{\boldsymbol{N}} = \mathrm{diag}\{\widehat{n}_1, \widehat{n}_2, \ldots, \widehat{n}_N\} > 0$, such that the following LMIs hold:

$$\begin{bmatrix} \boldsymbol{\Sigma}_i & 0 \\ 0 & \frac{1}{2}\boldsymbol{L}Z_2^{-1}\boldsymbol{L} - \boldsymbol{R} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N, \tag{13}$$

and

$$\begin{bmatrix} c\gamma_j\dfrac{\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}}{2} + q_j\boldsymbol{I}_N - \dfrac{1}{2}\widehat{\boldsymbol{M}} & \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G} - \dfrac{1}{2}\widehat{\boldsymbol{N}} \\ \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G}^{\mathrm{T}} - \dfrac{1}{2}\widehat{\boldsymbol{N}} & -q_j\boldsymbol{I}_N \end{bmatrix} < 0,$$
$$j = 1, 2, \ldots, n, \tag{14}$$

where

$$\boldsymbol{\Sigma}_i = -\frac{1}{2}\widehat{m}_i\boldsymbol{I}_n - \boldsymbol{D} + \frac{1}{2}\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_1^{-1}\boldsymbol{L} + \boldsymbol{R}.$$

Proof Define a Lyapunov functional candidate for system (8) as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(m_i-\widehat{m}_i)^2}{\rho_i} + \frac{1}{2}\sum_{i=1}^{N}\frac{(n_i-\widehat{n}_i)^2}{\varrho_i} + \sum_{i=1}^{N}\int_{t-\sigma}^{t}\boldsymbol{e}_i^{\mathrm{T}}(s)R\boldsymbol{e}_i(s)\mathrm{d}s + \sum_{i=1}^{N}\int_{t-\tau}^{t}\boldsymbol{e}_i^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{e}_i(s)\mathrm{d}s. \tag{15}$$

Calculating the time derivative of $V(t)$ along the trajectories of system (8), we obtain

$$\begin{aligned}\dot{V}(t) =& \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\dot{\boldsymbol{e}}_i(t) + \sum_{i=1}^{N}\frac{(m_i-\widehat{m}_i)\dot{m}_i}{\rho_i} + \sum_{i=1}^{N}\frac{(n_i-\widehat{n}_i)\dot{n}_i}{\varrho_i} \\ &+ \sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-\sigma)\boldsymbol{R}\boldsymbol{e}_i(t-\sigma)\right] \\ &+ \sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-\tau)\boldsymbol{Q}\boldsymbol{e}_i(t-\tau)\right] \\ =& -\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{D}\boldsymbol{e}_i(t) + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t)) \\ &+ \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{B}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\sigma)) \\ &+ c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}G_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) \\ &+ c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}G_{ij}\boldsymbol{\Gamma}_\tau\boldsymbol{e}_j(t-\tau) \\ &- \sum_{i=1}^{N}\widehat{m}_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \sum_{i=1}^{N}\widehat{n}_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t-\tau) \\ &+ \sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-\sigma)\boldsymbol{R}\boldsymbol{e}_i(t-\sigma)\right] \\ &+ \sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-\tau)\boldsymbol{Q}\boldsymbol{e}_i(t-\tau)\right].\end{aligned} \tag{16}$$

By Lemma 1 and making use of Eq.(9), we can obtain

$$\begin{aligned}&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t)) \\ &\le \frac{1}{2}\sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}}\boldsymbol{e}_i(t) + \widetilde{\boldsymbol{g}}^{\mathrm{T}}(\boldsymbol{e}_i(t))\boldsymbol{Z}_1^{-1}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t))\right] \\ &\le \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}}\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{L}\boldsymbol{Z}_1^{-1}\boldsymbol{L}\boldsymbol{e}_i(t),\end{aligned} \tag{17}$$

and

$$\begin{aligned}&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{B}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\sigma)) \\ &\le \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}}\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-\sigma)\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L}\boldsymbol{e}_i(t-\sigma).\end{aligned} \tag{18}$$

Substituting Eqs.(17) and (18) into Eq.(16), we have

$$\begin{aligned}\dot{V}(t) \le& -\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{D}e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}}\boldsymbol{e}_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{L}\boldsymbol{Z}_1^{-1}\boldsymbol{L}\boldsymbol{e}_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}}\boldsymbol{e}_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-\sigma)\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L}\boldsymbol{e}_i(t-\sigma) \\ &- \frac{1}{2}\sum_{i=1}^{N}\widehat{m}_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ &+ \sum_{i=1}^{N}\left[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-\sigma)\boldsymbol{R}\boldsymbol{e}_i(t-\sigma)\right] \\ &+ \sum_{j=1}^{n}c\gamma_j\tilde{\boldsymbol{e}}^j(t)^{\mathrm{T}}\boldsymbol{G}\tilde{\boldsymbol{e}}^j(t) \\ &+ \sum_{j=1}^{n}c\gamma_j^{\tau}\tilde{\boldsymbol{e}}^j(t)^{\mathrm{T}}\boldsymbol{G}\tilde{\boldsymbol{e}}^j(t-\tau) \\ &- \frac{1}{2}\sum_{j=1}^{n}\tilde{\boldsymbol{e}}^j(t)^{\mathrm{T}}\widehat{\boldsymbol{M}}\tilde{\boldsymbol{e}}^j(t) - \sum_{j=1}^{n}\tilde{\boldsymbol{e}}^j(t)^{\mathrm{T}}\widehat{\boldsymbol{N}}\tilde{\boldsymbol{e}}^j(t-\tau) \\ &+ \sum_{j=1}^{n}\left[q_j\tilde{\boldsymbol{e}}^j(t)^{\mathrm{T}}\tilde{\boldsymbol{e}}^j(t) - q_j\tilde{\boldsymbol{e}}^j(t-\tau)^{\mathrm{T}}\tilde{\boldsymbol{e}}^j(t-\tau)\right] \\ =& \sum_{i=1}^{N}\begin{bmatrix}\boldsymbol{e}_i(t)\\ \boldsymbol{e}_i(t-\sigma)\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Theta}_1\begin{bmatrix}\boldsymbol{e}_i(t)\\ \boldsymbol{e}_i(t-\sigma)\end{bmatrix} \\ &+ \sum_{j=1}^{n}\begin{bmatrix}\tilde{\boldsymbol{e}}^j(t)\\ \tilde{\boldsymbol{e}}^j(t-\tau)\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Theta}_2\begin{bmatrix}\tilde{\boldsymbol{e}}^j(t)\\ \tilde{\boldsymbol{e}}^j(t-\tau)\end{bmatrix},\end{aligned} \tag{19}$$

where $\boldsymbol{e}_i(t) = [e_i^1(t), e_i^2(t), \ldots, e_i^n(t)]^{\mathrm{T}}$,

$$\tilde{\boldsymbol{e}}^j(t) = [e_1^j(t), e_2^j(t), \ldots, e_N^j(t)]^{\mathrm{T}},$$
$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \boldsymbol{\Sigma}_i & 0 \\ 0 & \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L} - \boldsymbol{R} \end{bmatrix}, i = 1, 2, \ldots, N,$$

in which

$$\boldsymbol{\Sigma}_i = -\frac{1}{2}\widehat{m}_i\boldsymbol{I}_n - \boldsymbol{D} + \frac{1}{2}\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_1^{-1}\boldsymbol{L} + \boldsymbol{R},$$

and

$$\boldsymbol{\Theta}_2 = \begin{bmatrix} c\gamma_j\dfrac{\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}}{2} + q_j\boldsymbol{I}_N - \dfrac{1}{2}\widehat{\boldsymbol{M}} & \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G} - \dfrac{1}{2}\widehat{\boldsymbol{N}} \\ \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G}^{\mathrm{T}} - \dfrac{1}{2}\widehat{\boldsymbol{N}} & -q_j\boldsymbol{I}_N \end{bmatrix},$$
$$j = 1, 2, \ldots, n.$$

From the LMIs (13) and (14), clearly, Eq.(19) implies that $\dot{V}(t) \leq 0$. That is, the synchronization manifold S of dynamical system (2) is globally asymptotically stable under the adaptive controllers (11) and updating laws (12). The proof of Theorem 1 is completed.

If the inner delayed coupling matrix $\boldsymbol{\Gamma}_\tau = 0$, system (2) reduces to

$$\dot{\boldsymbol{x}}_i(t) = -\boldsymbol{D}\boldsymbol{x}_i(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}_i(t)) + \boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}_i(t-\sigma)) + \boldsymbol{I}(t) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t) + \boldsymbol{u}_i(t), \quad i = 1, 2, \ldots, N. \tag{20}$$

Then, for the error vector

$$\boldsymbol{e}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{s}(t), \quad i = 1, 2, \ldots, N,$$

the error dynamical system is given as

$$\dot{\boldsymbol{e}}_i(t) = -\boldsymbol{D}\boldsymbol{e}_i(t) + \boldsymbol{A}\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t)) + \boldsymbol{B}\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\sigma)) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) + \boldsymbol{u}_i(t), \quad i = 1, 2, \ldots, N. \tag{21}$$

Corollary 1 Let the adaptive controllers

$$\boldsymbol{u}_i(t) = -m_i\boldsymbol{e}_i(t), \quad i = 1, 2, \ldots, N, \tag{22}$$

and the following updating laws

$$\dot{m}_i = \rho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad i = 1, 2, \ldots, N \tag{23}$$

in which ρ_i, $i = 1, 2, \ldots, N$ are positive constants. Under the assumption (A_1), the synchronization manifold $\boldsymbol{S}$ of the coupled system (20) is globally asymptotically stable, if there exist positive definite matrices $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, positive diagonal matrices $\boldsymbol{R} = \mathrm{diag}\{r_1, r_2, \ldots, r_n\} > 0$, $\boldsymbol{Q} = \mathrm{diag}\{q_1, q_2, \ldots, q_n\} > 0$, and $\widehat{\boldsymbol{M}} = \mathrm{diag}\{\widehat{m}_1, \widehat{m}_2, \ldots, \widehat{m}_N\} > 0$, such that the following LMIs hold:

$$\begin{bmatrix} \boldsymbol{\Sigma}_i^* & 0 \\ 0 & \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L} - \boldsymbol{R} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N,$$

and

$$\begin{bmatrix} c\gamma_j\dfrac{\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}}{2} + q_j\boldsymbol{I}_N - \dfrac{1}{2}\widehat{\boldsymbol{M}} & 0 \\ 0 & -q_j\boldsymbol{I}_N \end{bmatrix} < 0, \quad j = 1, 2, \ldots, n,$$

where

$$\boldsymbol{\Sigma}_i^* = -\frac{1}{2}\widehat{m}_i\boldsymbol{I}_n - \boldsymbol{D} + \frac{1}{2}\boldsymbol{A}\boldsymbol{Z}_1\boldsymbol{A}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}} + \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_1^{-1}\boldsymbol{L} + \boldsymbol{R}.$$

Proof In Theorem 1, let $\widehat{n}_i = 0$, $i = 1, 2, \ldots, N$, $\gamma_j^{\tau} = 0$, $j = 1, 2, \ldots, n$, then its proof is straightforward and hence can be omitted.

Theorem 2 Let the adaptive controllers

$$\boldsymbol{u}_i(t) = -m_i\boldsymbol{e}_i(t) - n_i\boldsymbol{e}_i(t-\tau), \quad i = 1, 2, \ldots, N, \tag{24}$$

and the following updating laws

$$\dot{m}_i = \rho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad \dot{n}_i = \varrho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t-\tau), \quad i = 1, 2, \ldots, N, \tag{25}$$

in which ρ_i and ϱ_i, $i = 1, 2, \ldots, N$ are positive constants. Under the assumption (A_2), the synchronization manifold $\boldsymbol{S}$ of the coupled system (2) is globally asymptotically stable, if there exist a positive definite matrix $\boldsymbol{Z}_2$, positive diagonal matrices $\boldsymbol{R} = \mathrm{diag}\{r_1, r_2, \ldots, r_n\} > 0$, $\boldsymbol{Q} = \mathrm{diag}\{q_1, q_2, \ldots, q_n\} > 0$, $\widehat{\boldsymbol{M}} = \mathrm{diag}\{\widehat{m}_1, \widehat{m}_2, \ldots, \widehat{m}_N\} > 0$, and $\widehat{\boldsymbol{N}} = \mathrm{diag}\{\widehat{n}_1, \widehat{n}_2, \ldots, \widehat{n}_N\} > 0$, such that the following LMIs hold:

$$\begin{bmatrix} \boldsymbol{\Omega}_i & 0 \\ 0 & \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L} - \boldsymbol{R} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N,$$

and

$$\begin{bmatrix} c\gamma_j\dfrac{\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}}{2} + q_j\boldsymbol{I}_N - \dfrac{1}{2}\widehat{\boldsymbol{M}} & \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G} - \dfrac{1}{2}\widehat{\boldsymbol{N}} \\ \dfrac{1}{2}c\gamma_j^{\tau}\boldsymbol{G}^{\mathrm{T}} - \dfrac{1}{2}\widehat{\boldsymbol{N}} & -q_j\boldsymbol{I}_N \end{bmatrix} < 0,$$
$$j = 1, 2, \ldots, n,$$

where

$$\boldsymbol{\Omega}_i = -\frac{1}{2}\widehat{m}_i\boldsymbol{I}_n - \boldsymbol{D} + \frac{\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}}{2}\boldsymbol{L} + \frac{1}{2}\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}} + \boldsymbol{R}.$$

Proof Similar to the approach for proving Theorem 1, we construct the Lyapunov function (15). According to the assumption (A_2), we have

$$\begin{aligned}&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\widetilde{\boldsymbol{g}}(\boldsymbol{e}_i(t))\\ \le&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{L}\boldsymbol{e}_i(t)\\ =&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\frac{\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}}{2}\boldsymbol{L}\boldsymbol{e}_i(t),\end{aligned}\tag{26}$$

then Theorem 2 can be easily obtained if one substitutes Eq.(17) by Eq.(26) in calculating the time derivative of the $V(t)$ along the trajectory of the system (8).

Similar to the approach for proving Corollary 1, we can obtain Corollary 2 from Theorem 2.

Corollary 2 Let the adaptive controllers

$$\boldsymbol{u}_i(t) = -m_i\boldsymbol{e}_i(t), \quad i = 1, 2, \ldots, N, \tag{27}$$

and the following updating laws

$$\dot{m}_i = \rho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad i = 1, 2, \ldots, N, \tag{28}$$

in which ρ_i, $i = 1, 2, \ldots, N$ are positive constants. Under the assumption (A_2), the synchronization manifold $\boldsymbol{S}$ of the coupled system (20) is globally asymptotically stable, if there exist a positive definite matrix $\boldsymbol{Z}_2$, positive diagonal matrices $\boldsymbol{R} = \mathrm{diag}\{r_1, r_2, \ldots, r_n\} > 0$, $\boldsymbol{Q} = \mathrm{diag}\{q_1, q_2, \ldots, q_n\} > 0$, and $\widehat{\boldsymbol{M}} = \mathrm{diag}\{\widehat{m}_1, \widehat{m}_2, \ldots, \widehat{m}_N\} > 0$, such that the following LMIs hold:

$$\begin{bmatrix}\boldsymbol{\Omega}_i^* & 0\\ 0 & \frac{1}{2}\boldsymbol{L}\boldsymbol{Z}_2^{-1}\boldsymbol{L} - \boldsymbol{R}\end{bmatrix} < 0, \quad i = 1, 2, \ldots, N,$$

and

$$\begin{bmatrix}c\gamma_j\dfrac{\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}}{2} + q_j\boldsymbol{I}_N - \dfrac{1}{2}\widehat{\boldsymbol{M}} & 0\\ 0 & -q_j\boldsymbol{I}_N\end{bmatrix} < 0,$$
$$j = 1, 2, \ldots, n,$$

where

$$\boldsymbol{\Omega}_i^* = -\frac{1}{2}\widehat{m}_i\boldsymbol{I}_n - \boldsymbol{D} + \frac{\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}}{2}\boldsymbol{L} + \frac{1}{2}\boldsymbol{B}\boldsymbol{Z}_2\boldsymbol{B}^{\mathrm{T}} + \boldsymbol{R}.$$

Remark 2 In Refs.[25, 26, 28], the authors studied the synchronization of the coupled connected neural networks without delayed coupling. However, as discussed in Section 1, time delays should be considered in order to simulate the realistic networks. In this paper, we investigate the dynamical coupled systems with constant and delayed coupling. Therefore, the system considered in this paper is more general than that considered in the literature.

Remark 3 In Ref.[27], the authors discussed the global synchronization of N identical delayed neural networks with the symmetric coupling configuration matrix. However, the assumption that the coupling configuration matrix should be symmetric is removed in this paper. Hence our results are less restrictive.

Remark 4 In order to achieve synchronization of the coupled system (2), the variable feedback strength related to the synchronization errors will be automatically adapted to a suitable strength and the adaptive synchronization scheme is simple to implement in practical engineering design. It is worth noting that we add the time-delay adaptive feedback control term $-n_i\boldsymbol{e}_i(t-\tau)$ in the coupled system (2). However, such a time delayed feedback term was not considered in the adaptive synchronization scheme studied in Refs.[30, 31]. Hence, our results extend and improve the previous results.

4. A simulation example

Consider a 2-dimensional neural networks:[7]

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& -\boldsymbol{D}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}(t))\\ &+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}(t-\sigma)) + \boldsymbol{I}(t),\end{aligned}\tag{29}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ is the state vector of the network and $\boldsymbol{g}(\boldsymbol{x}(t)) = [\tanh(x_1), \tanh(x_2)]^{\mathrm{T}}$. The system parameters of Eq.(29) are

$$\boldsymbol{D} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad \boldsymbol{A} = \begin{pmatrix}2.0 & -0.1\\ -5.0 & 3.0\end{pmatrix},$$
$$\boldsymbol{B} = \begin{pmatrix}-1.5 & -0.1\\ -0.2 & -2.5\end{pmatrix}, \ \boldsymbol{I}(t) = \begin{pmatrix}0\\ 0\end{pmatrix}, \ \sigma = 1.$$

The model (29) has a chaotic attractor as shown in Fig.1 with the initial condition $x_1(s) = 0.4$, $x_2(s) = 0.6$, $\forall s \in [-1, 0]$. Now, consider a dynamical system

consisting of five linearly coupled identical neural networks (29). The state equation of the entire system is

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t) = & -\boldsymbol{D}\boldsymbol{x}_i(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}_i(t)) + \boldsymbol{B}g(\boldsymbol{X}_i(t-\sigma)) \\ & + \boldsymbol{I}(t) + c\sum_{j=1}^{5} G_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t) \\ & + c\sum_{j=1}^{5} G_{ij}\boldsymbol{\Gamma}_\tau\boldsymbol{x}_j(t-\tau) + \boldsymbol{u}_i(t), \\ & i = 1, 2, \ldots, 5, \end{aligned} \tag{30}$$

where $\boldsymbol{x}_i(t) = [x_{i1}(t),\, x_{i2}(t)]^{\mathrm{T}}$ $(i = 1, 2, \ldots, 5)$ are the state variables of the ith neural network.

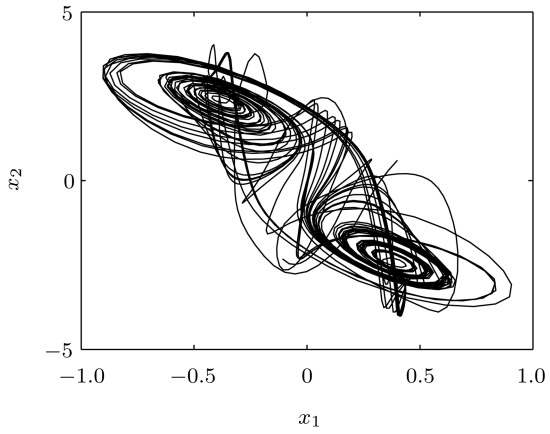


Fig.1. Chaotic trajectory of the model (29).

The asymmetric coupling configuration matrix is taken as

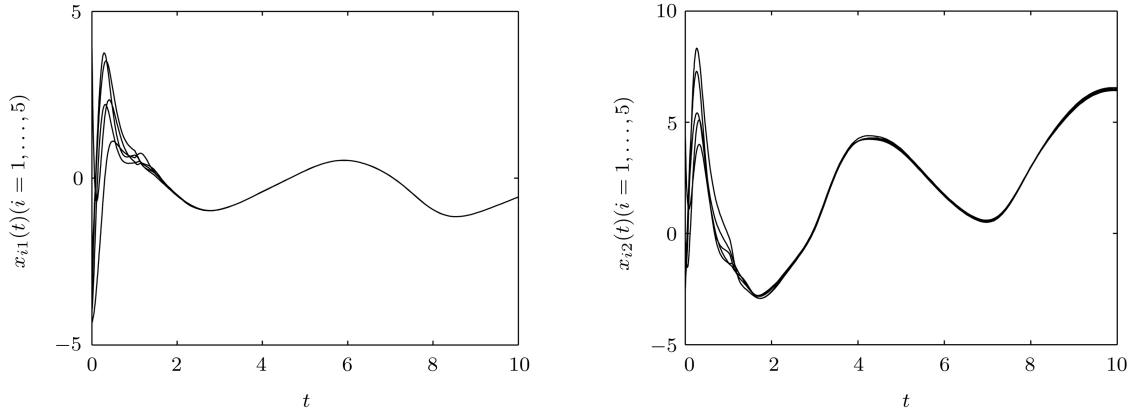
$$\boldsymbol{G} = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & 1 & -3 & 1 & 0 \\ 0 & 1 & 2 & -4 & 1 \\ 2 & 0 & 0 & 1 & -3 \end{bmatrix}.$$

The state coupling matrix $\boldsymbol{\Gamma}$ and delayed coupling matrix $\boldsymbol{\Gamma}_\tau$ are respectively chosen as $\boldsymbol{\Gamma} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$, $\boldsymbol{\Gamma}_\tau = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}$. Moreover, let the coupling delay $\tau = 1$, $\rho_i = \varrho_i = 1$, $i = 1, 2, \ldots, 5$.

Obviously, the coupling configuration matrix $\boldsymbol{G}$ is asymmetric, therefore from the main results in Ref.[27], we cannot judge whether the coupled system (30) is synchronized under the given conditions. However, in this paper, Theorems 1 and 2 can be used to treat this case.

For example, let $\boldsymbol{Z}_1 = \boldsymbol{Z}_2 = \boldsymbol{I}$, then there exist the following feasible solutions for the LMIs (13) and (14) in Theorem 1: $\boldsymbol{R} = \begin{bmatrix} 1.2526 & 0 \\ 0 & 1.1598 \end{bmatrix}$, $\boldsymbol{M} = 50\boldsymbol{I}$, $\boldsymbol{Q} = \begin{bmatrix} 1.5692 & 0 \\ 0 & 1.2363 \end{bmatrix}$, $\boldsymbol{N} = 10\boldsymbol{I}$. According to Theorem 1, the synchronization manifold $\boldsymbol{S}$ of the coupled system (30) is globally asymptotically stable.

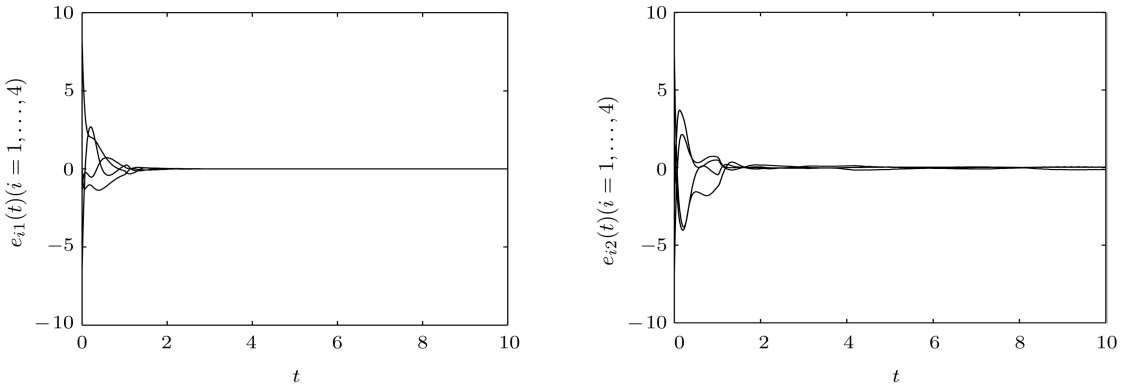
When the initial values of the system (30) are randomly chosen from $[-5, 5]$, Fig.2 shows the time responses of state variables $x_{i1}(t)$ and $x_{i2}(t)$ $(i = 1, 2, \ldots, 5)$ of the coupled system (30).



(a) $x_{i1}\,(i = 1, 2, \ldots, 5)$ (b) $x_{i2}\,(i = 1, 2, \ldots, 5)$

Fig.2. Synchronized states x_i $(i = 1, 2, \ldots, 5)$.

Figure 3 depicts the trajectories of the synchronization errors $\boldsymbol{e}_i(t)$, $i = 1, 2, \ldots, 4$, where $\boldsymbol{e}_i(t) = [e_{i1}(t), e_{i2}(t)]^{\mathrm{T}}$ and $\boldsymbol{e}_{i,j}(t) = x_{i,j}(t) - x_{i+1,j}(t)$ for $i = 1, 2, \ldots, 4$, $j = 1, 2$. From Fig.3, it is easy to see that the errors between the synchronized states converge to zero. Thus, the synchronization manifold $\boldsymbol{S}$ of the coupled system (30) is globally asymptotically stable under the given adaptive controller.



(a) $e_{i1}\,(i = 1, 2, \ldots, 4)$ (b) $e_{i2}(i = 1, 2, \ldots, 4)$

Fig.3. Synchronization errors of the system (30).

5. Conclusions

This paper considers the adaptive synchronization in an array of linearly coupled neural networks. Some novel synchronization conditions are presented for the coupled system by using Lyapunov functional method. The coupling configuration matrix of the coupled system can be assumed to be asymmetric. Our results improve and generalize some existing ones and the new conditions are easy to check and apply in practice. Finally, a numerical example is given to show the effectiveness and feasibility of the obtained results.

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