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Stochastic resonance in a parallel array of linear elements

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This paper studies stochastic resonance (SR) phenomenon in a parallel array of linear elements with noise. Employing the signal-to-noise ratio (SNR) theory, it obtains the output SNR, and investigates the effects on the output SNR of the system with signal-independent noise and signal-dependent noise respectively. Numerical results show: the curve of the output SNR is monotone with signal-independent noise; whereas SR appears with signal-dependent noise. Moreover, the output SNR enhances rapidly with the increase of N which is the number of elements in this parallel array linear system. This result may provide smart array of simple linear sensors which are capable of acting as noise-aided amplifiers.

Keywords: linear sensor, signal-to-noise ratio theory, stochastic resonance, signal-dependent noise

PACC: 0540, 0250

1. Introduction

Stochastic resonance (SR) has gradually been observed in a variety of fields[1−15] since it was introduced[1] about 30 years ago. In a broad sense, SR indicates an effect of noised-enhanced signal transmission that can be depicted by the quantity of signal-to-noise ratio (SNR) (or other measures of the impact of the transmission) displaying a nonmonotonic behaviour with the noise intensity and peaking at a maximum value for an optimum noise level.[10] And in recent years, there have been many theoretical developments on SR, such as the adiabatic limit theory,[15] the linear-response theory,[16,17] the perturbation theory,[18] the residence-time distribution[19] and the unified coloured noise approximation.[20−23] So far, most systems in which people can observe the SR effect essentially are nonlinear dynamic systems with potential barriers or with thresholds. In this case, the essence of SR is that the information-carrying signal by itself is too weak to overcome a barrier or a threshold in the response of the system. Attendance of the noise then empowers some type of cooperation between signal and noise, so as to hurdle the threshold or barrier, and conduce a response bearing stronger relation to the signal in virtue of assistance from the noise.[2]

With the development of SR, it has been extended far and wide. Recently, Chapeau-Blondeau and Godivev[10] have extended the notion of SR from dynamic system to static (or memoryless) nonlinear system, for example, they observed SR in two-threshold nonlinear system, in diodelike nonlinear system, and so on. Then in 2006, Chapeau-Blondeau and Rousseau[24] also found that noise can amplify SNR in parallel arrays of sensors with hard saturation, and in that paper, authors also pointed out that SR cannot appear in linear parallel system with additive noise. Now, there are more and more studies about SR in static systems, but most of those investigations focus on nonlinear sensors. Up to now, there are few papers discussing SR in linear sensor. At the same time, many of these investigations have been dealt with the assumption that the noise is signal-independent, it is well established that noise may arise in a signal-dependent[25] or multiplicative fashion, for example, signal-dependent noise is characteristic of the propagation or initiation of nerve impulses at synapses through the quantal release of neurotransmitter. Furukawa *et al*[26−27] have shown experimentally that the variation of the neurotransmitter release into a synaptic cleft is proportional to the intensity of the stimulus, and the variance is signal dependent, which cannot be described by using an additive noise process. This provides motivation to investigate the role of signal-dependent noise in the transmission of information. In 2007, Nikitin *et al*[25] found that signal-

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dependent noise could enhance information transmission in an array of nonlinear elements. Then signal-dependent noise has been introduced into the study of SR in parallel arrays of system. So in this paper we studied the SR phenomenon by applying the SNR theory presented by Chapeau-Blondeau and Rousseau[24] in a parallel linear system with signal-independent noise and signal-dependent noise respectively. We found that SR appears in parallel linear system with signal-dependent noise. This result proves another instance of the linear phenomenon of SR where multiplicative noise enhances the signal.

2. The SNR theory in a parallel array of elements

In this section, we will introduce the SNR theory presented by Chapeau-Blondeau *et al.*[2,10,24] Consider the following system realizing the input–output transformation:

$$y_i(t) = h[s(t) + \xi(t) + \eta_i(t)], \quad i = 1, 2, \ldots, N, \quad (1)$$

where t is time-variable, the input information $x(t) = s(t) + \xi(t)$ is applied onto a parallel array of N identical sensors, each sensor of the array is endowed with the same input–output static characteristic $h(.)$, $s(t)$ denotes a periodic signal with period T_s, $\xi(t)$ is the perturbation embedding in $s(t)$, and its probability density function is $f_\xi(x)$. The N noises $\eta_i(t)(i = 1, 2, ..., N)$ are arousing by the system $h(.)$, and they are mutually independent and identically distributed with cumulative distribution $F_\eta(u)$ and probability density function $f_\eta(u) = \mathrm{d}F_\eta(u)/\mathrm{d}u$. Moreover, we assume that $x(t)$ and $\eta_i(t)i = 1, 2, ..., N$ are unaided. Commonly $h(.)$ is a linear or nonlinear function defined on real numbers.

The steady-state response $y(t)$ (with the inputs $x(t) = s(t) + \xi(t)$, which have been applied since $t \to -\infty$) of Eq.(1) is obtained by averaging the outputs of all the sensors,[10,24] as

$$y(t) = \frac{1}{N}\sum_{i=1}^{N} y_i(t). \quad (2)$$

Generally, in Eq.(1), for the random input $\xi(t)$ and $\eta_i(t)$, $i = 1, 2, ..., N$, the output $y(t)$ will be a random signal. On the other hand, since $s(t)$ is deterministic with period T_s, the output signal $y(t)$ is a cyclostationary random signal. The transmission of $s(t)$ by the array is assessed by the output SNR which is standard in SR studies.[10,24] Because $s(t)$ denotes a periodic signal, the output SNR is defined as the power contained in the output spectral line at the fundamental $1/T_s$ divided by the power contained in the noise background in a small frequency band ΔB around $1/T_s$,[10,24]

$$\mathrm{SNR} = \frac{\left|\overline{Y_1}\right|^2}{\langle \mathrm{var}[y(t)]\rangle \, \Delta t \Delta B}, \quad (3)$$

where $\left|\overline{Y_1}\right|^2$ denotes the power of $y(t)$ contained in the output spectral line at the frequency $1/T_s$, $|...|$ shows absolute value, and $\overline{Y_1}$ is the Fourier coefficient at the fundamental of the T_s-periodic nonstationary output expectation $E[y(t)]$, i.e.

$$\overline{Y_1} = \left\langle E[y(t)] \exp\left(-\mathrm{i}\frac{2\pi}{T_s}t\right)\right\rangle \quad (4)$$

with the time average defined as

$$\langle \ldots \rangle = \frac{1}{T_s}\int_0^{T_s} \ldots \mathrm{d}t, \quad (5)$$

and the stationarized output time average of variance $\langle \mathrm{var}[y(t)]\rangle$ was adopted to measure the magnitude of the continuous noise background in the output spectrum, with the nonstationary variance given by $\mathrm{var}[y(t)] = E[y^2(t)] - [E[y(t)]]^2$ at a fixed time t.[10,24] And Δt in Eq.(3) is the time resolution of the measurement (i.e., the signal sampling period in a discrete time implementation).[2] In this paper we take $s(t) = A + A\sin(2\pi t/T_s)$, A is the amplitude, and let $\Delta t \Delta B = 10^{-3}$.[2]

We find hat the key is to compute the mean $E[y(t)]$ and variance $\langle \mathrm{var}[y(t)]\rangle$ in order to induce the expression of the output SNR of Eq.(3).

According to Eq.(2), at time t, for a fixed given value x of the input $x(t)$, there is the conditional expectations[10,24,28]

$$E[y(t)|x] = E[y_i(t)|x], \quad (6)$$

and

$$E[y^2(t)|x] = \frac{1}{N}E[y_i^2(t)|x] + \frac{N-1}{N}E^2[y_i(t)|x], \quad (7)$$

because of Eq.(1), one has for any i,[24]

$$E[y_i(t)\,|x\,] = \int_{-\infty}^{+\infty} h(x+u) f_\eta(u)\mathrm{d}u, \quad (8)$$

and

$$E[y_i^2(t)\,|x\,] = \int_{-\infty}^{+\infty} h^2(x+u) f_\eta(u)\mathrm{d}u. \quad (9)$$

Equations (6) and (7) are both independent of i since the $\eta_i(t)$ are independent and identically distributed. When $N = \infty$, Eq.(7) can be simply accessible by letting $E[y^2(t)\,|x\,] = E^2[y_i(t)\,|x\,]$.

Then since $x(t) = s(t)+\xi(t)$, the probability density for $x(t)$ is $f_\xi(x - s(t))$, so we have

$$E[y(t)] = \int_{-\infty}^{+\infty} E[y(t)\,|x\,]f_\xi(x - s(t))\mathrm{d}x, \tag{10}$$

and

$$E[y^2(t)] = \int_{-\infty}^{+\infty} E[y^2(t)\,|x\,]f_\xi(x - s(t))\mathrm{d}x. \tag{11}$$

$$\mathrm{var}[y(t)] = E[y^2(t)] - [E[y(t)]]^2. \tag{12}$$

So we can obtain the expression of the mean $E[y(t)]$ and variance $\langle \mathrm{var}[y(t)]\rangle$ by combining Eqs.(6)–(12), then according to Eqs.(4), (5) and (3), the expression of the output SNR of system(1) will be gained.

3. The output SNR with signal-independent noise

In this part, we will study the case with $h(.)$ which is linear, for convenience to compute, we take $h(u) = cu$, where $c \neq 0$ is a constant. In order to illustrate the SR phenomenon of Eq.(1), we take the case where $\xi(t)$ is a zero-mean uniform noise over $[-a,\ a]$, and $\eta_i(t)$ $(i = 1, 2, \ldots, N)$ are uniform noise over $[-b+m, b+m](m \in R$ is a constant), and $a,\ b \neq 0$.

In other words, next we will study the output SNR of the following parallel array linear system

$$y_i(t) = h[s(t) + \xi(t) + \eta_i(t)],\quad i = 1, 2, \ldots, N, \tag{1}$$

$$h(u) = cu,\quad c \neq 0 \ \text{ is constant}, \tag{13}$$

where noises $\xi(t)$ and $\eta_i(t)$ $(i = 1, 2, \ldots, N)$ have the following probability density functions $f_\xi(x)$ and $f_{\eta_i}(x)$ $(i = 1, 2, \ldots, N)$

$$f_\xi(x) = \begin{cases} \dfrac{1}{2a},\ x \in [-a, a], \\ 0, \quad \text{others}, \end{cases} \tag{14}$$

$$f_{\eta_i}(x) = \begin{cases} \dfrac{1}{2b},\ x \in [-b+m, b+m] \\ 0, \quad \text{others} \end{cases},\quad i = 1, 2, ..., N. \tag{15}$$

According to the SNR theory presented in Section 2, we can get the following expressions:

$$\begin{aligned} E[y_i(t)\,|x\,] &= \int_{-\infty}^{+\infty} h(x+u)f_\eta(u)\mathrm{d}u \\ &= c(x+m), \\ E[y_i^2(t)\,|x\,] &= \int_{-\infty}^{+\infty} h^2(x+u)f_\eta(u)\mathrm{d}u \\ &= c^2\left\{x^2 + 2mx + \frac{2}{3}ab^2 + 2m^2\right\}, \end{aligned}$$

and

$$\begin{aligned} E[y(t)] &= \int_{-\infty}^{+\infty} E[y(t)\,|x\,]f_\xi(x - s(t))\mathrm{d}x \\ &= c\left\{s(t) + m\right\}, \end{aligned} \tag{16}$$

$$\begin{aligned} E[y^2(t)] &= \int_{-\infty}^{+\infty} E[y^2(t)\,|x\,]f_\xi(x - s(t))\mathrm{d}x \\ &= \mu_1 A(t) + \mu_2 B(t), \end{aligned} \tag{17}$$

$$\begin{aligned} \mathrm{var}[y(t)] &= E[y^2(t)] - [E[y(t)]]^2 \\ &= \mu_1 A(t) + \mu_2 B(t) - C(t) \end{aligned} \tag{18}$$

with $\mu_1 = \dfrac{c^2}{4abN}$, $\mu_2 = \dfrac{(N-1)c^2}{2aN}$, $A(t) = 4abs^2(t) + \dfrac{4a^3b}{3} + 8abms(t) + 4abm^2 + \dfrac{4ab^3}{3}$, $B(t) = 2as^2(t) + \dfrac{2a^3}{3} + 4ams(t) + 2am^2$, $C(t) = c^2[s^2(t) + m^2 + 2ms(t)]$. And Eqs.(16) and (18) are the mean and variance of the steady-state response $y(t)$ of the parallel array linear system (1) and (13).

Then substituting Eqs.(16), (17), and (18) into Eqs.(3), (4), and (5), we will get the expression of output SNR of the parallel array linear systems (1) and (13) as:

$$\mathrm{SNR} = \frac{\left|\overline{Y_1}\right|^2}{\langle \mathrm{var}[y(t)]\rangle\,(\Delta t \Delta B)} = \frac{\left|\dfrac{1}{T_s}\displaystyle\int_0^{T_s} c\left\{s(t)+m\right\}\cos\left(\frac{2\pi}{T_s}t\right)\mathrm{d}t\right|^2 + \left|\dfrac{1}{T_s}\displaystyle\int_0^{T_s} c\left\{s(t)+m\right\}\sin\left(\frac{2\pi}{T_s}t\right)\mathrm{d}t\right|^2}{(\Delta t \Delta B)\dfrac{1}{T_s}\displaystyle\int_0^{T_s}\{\mu_1 A(t) + \mu_2 B(t) - C(t)\}\mathrm{d}t}. \tag{19}$$

According to Eq.(19), we will apply numerical integration method to study the effects on the output SNR of additive noise rms amplitude $a/\sqrt{3}$ with signal-independent noise.

Figure 1 gives the curve of the output SNR as a function of additive noise rms amplitude $a/\sqrt{3}$ with various N and signal-independent noise, the figure in the graph is the value of N. From Fig.1 we can see that the curve of SNR$-a/\sqrt{3}$ is monotone, this result is the same conclusion as that in Ref.[24], i.e., in a linear parallel array static device with signal-independent noise, SR phenomenon cannot arise, in spite of its high order. At the same time, Fig.1 tells us that the output SNR will enhance rapidly with the increase of parameter N, this result provides us that we can get great output SNR by adding the number of the elements in systems (1) and (13).

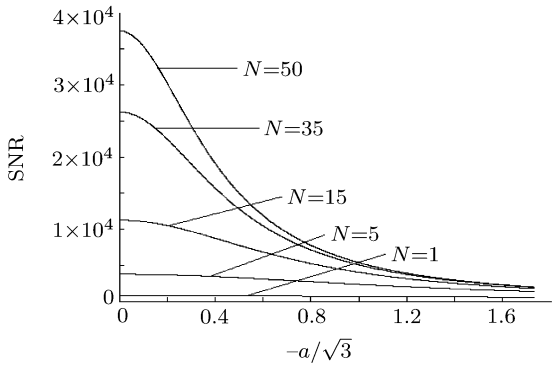


Fig.1. The curve of the output SNR as a function of additive noise rms amplitude $a/\sqrt{3}$ with various N with signal-independent noise ($A = 5,\ c = 0.1,\ b = 5,\ T_s = 1,\ m = 0$).

4. The output SNR with signal-dependent noise

Based on Section 3, in this section, our interest focuses on the case with signal-dependent noise. So we consider the following system with signal-dependent noise to realize the input–output transformation:

$$y_i(t) = h[s(t)+s(t)\xi(t)+\eta_i(t)],\ \ i = 1, 2, \ldots, N. \quad (20)$$

$$h(u) = cu,\ \ c \neq 0 \ \text{ is a constant.} \quad (13)$$

In order to compare with the results in Section 3, in Eqs.(20) and (13) we will let $h(.)$, $s(t)$, $\xi(t)$, and $\eta_i(t)(i = 1, 2, ..., N)$ to employ the same status as in Eqs.(1) and (13). And we can obtain the following expressions by using the same method as that in Section 3:

$$\begin{aligned} E[y_i(t)\,|x\,] &= \int_{-\infty}^{+\infty} h(x+u) f_\eta(u)\mathrm{d}u \\ &= c(x+m), \\ E[y_i^2(t)\,|x\,] &= \int_{-\infty}^{+\infty} h^2(x+u) f_\eta(u)\mathrm{d}u \\ &= c^2\left\{x^2 + 2mx + m^2 + \frac{b^2}{3}\right\}, \end{aligned}$$

then we will get the mean and variance of Eqs.(20) and (13)

$$\begin{aligned} E[y(t)] &= \int_{-\infty}^{+\infty} \frac{1}{s(t)} E[y(t)\,|x\,] f_\xi\left(\frac{x-s(t)}{s(t)}\right)\mathrm{d}x \\ &= c\left\{s(t)+m\right\}, \quad (21) \\ E[y^2(t)] &= \int_{-\infty}^{+\infty} \frac{1}{s(t)} E[y^2(t)\,|x\,] f_\xi\left(\frac{x-s(t)}{s(t)}\right)\mathrm{d}x \\ &= \mu_{11}A_1(t) + \mu_{12}B_1(t), \quad (22) \\ \mathrm{var}[y(t)] &= E[y^2(t)] - [E[y(t)]]^2 \\ &= \mu_{11}A_1(t) + \mu_{12}B_1(t) - C(t), \quad (23) \end{aligned}$$

where $\mu_{11} = \dfrac{c^2}{4abNs(t)}$, $\mu_{12} = \dfrac{(N-1)c^2}{8ab^2Ns(t)}$, $C(t) = c^2[s^2(t) + m^2 + 2ms(t)]$,

$$\begin{aligned} A_1(t) =& 4abs^3(t) + \frac{4a^3bs^3(t)}{3} + 8a^2b^2ms^3(t) \\ &+ 4abm^2s(t) + \frac{4ab^3s(t)}{3}, \\ B_1(t) =& 8ab^2s^3(t) + \frac{8a^3b^2s^3(t)}{3} \\ &+ 16a^2b^3ms^3(t) + 8ab^2m^2s(t). \end{aligned}$$

Applying Eqs.(21), (4), (5) and (23) to Eq.(3),we will educe the expression of the output SNR of the parallel array linear systems (20) and (13) as:

$$\mathrm{SNR} = \frac{\left|\overline{Y_1}\right|^2}{\langle \mathrm{var}[y(t)]\rangle\,(\Delta t \Delta B)} = \frac{\left|\dfrac{1}{T_s}\displaystyle\int_0^{T_s} c\left\{s(t)+m\right\}\cos\left(\frac{2\pi}{T_s}t\right)\mathrm{d}t\right|^2 + \left|\dfrac{1}{T_s}\displaystyle\int_0^{T_s} c\left\{s(t)+m\right\}\sin\left(\frac{2\pi}{T_s}t\right)\mathrm{d}t\right|^2}{(\Delta t \Delta B)\dfrac{1}{T_s}\displaystyle\int_0^{T_s}\left\{\mu_{11}A_1(t) + \mu_{12}B_1(t) - C(t)\right\}\mathrm{d}t}. \quad (24)$$

According to Eq.(24), we will apply numerical integration method to study the effects on the output SNR of additive noise rms amplitude $b/\sqrt{3}$ with signal-dependent noise.

In Fig.2, we plot the curve of the output SNR as a function of additive noise rms amplitude $b/\sqrt{3}$ with various N and $m \neq 0$, the figure in the graph is the value of N. From Fig.2 we can see that there is a peak in the curve of SNR $- b/\sqrt{3}$ when $N \geq 2$, in others words, SR phenomenon arises. Moreover, Fig.2 tells us that the value of N does alter the qualitative form of the curve of SNR $- b/\sqrt{3}$, when $N = 1$, the curve of SNR $- b/\sqrt{3}$ is monotone, while when $N \geq 2$, there is a peak in the curve of SNR $- b/\sqrt{3}$, i.e., SR phenomenon arises only when $N \geq 2$. And also we can see from Fig.2 that when additive noise rms amplitude $b/\sqrt{3} > 0.25$, while the parameter N increase and other parameters are fixed, the output SNR will enhance rapidly, and the breadth of the peak will broaden. This results tell us that we can obtain gigantic output SNR by adding the number of the elements in systems (20) and (13) with $N \geq 2$ and additive noise rms amplitude $b/\sqrt{3} > 0.25$.

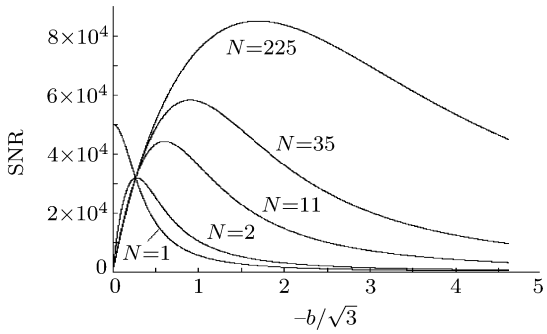


Fig.2. The curve of the output SNR as a function of additive noise rms amplitude $b/\sqrt{3}$ with signal-dependent noise and $m \neq 0$ ($A = 5,\ c = 0.3,\ a = 0.1,\ T_s = 1,\ m = 0.3$).

Figure 3 plots the curve of the output SNR as a function of additive noise rms amplitude $b/\sqrt{3}$ with various N and $m = 0$. From Fig.3 we can see that the curve of SNR $- b/\sqrt{3}$ is monotone, and now in a linear parallel array static device with signal-dependent noise, SR phenomenon cannot arise, in spite of its high order with $m = 0$. At the same time, Fig.3 tells us that the output SNR will enhance with the increase of parameter N.

How do we explain the phenomenon in Fig.1 to Fig.3. At first, we compare Fig.1 with Fig.2, from the condition of Eq.(1) and Eq.(20) we can find that the causation which brings difference between Fig.1 and Fig.2 is the fashion of noise with signal. In Eq.(1), the noise is signal-independent; while in Eq.(20), the noise is signal-dependent, so we can get the conclusion that signal-dependent noise can cause SR in parallel array linear system, but signal-independent noise cannot cause SR. Then, we compare Fig.2 and Fig.3, from the condition of Fig.2 and Fig.3, we can see the difference between them is the different value of m. In Fig.2 $m \neq 0$, while in Fig.3 $m = 0$, accordingly we can get another conclusion that noise mean m can cause different effects on the output SNR. When $m \neq 0$, SR appears in parallel array linear system with signal-dependent noise, while $m = 0$, SR cannot appear. In this parallel array linear system, we can give a "possible" explanation of the SR effect in terms of a one dimensional dynamic system with a parabolic potential wherein there is a large information offset that skews the potential and then there is a noise term superimposed on a periodic signal in addition to two additional noise terms. It is easy to imagine the potential developing transient stable steady states among which the system switches, then SR appears.

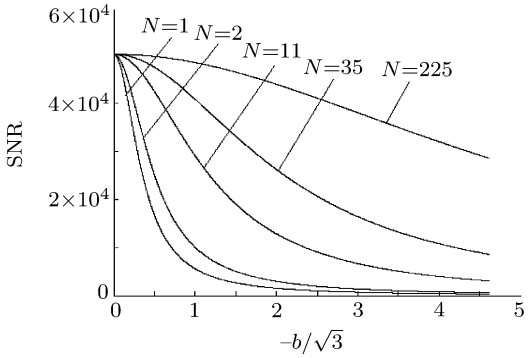


Fig.3. The curve of the output SNR as a function of additive noise rms amplitude $b/\sqrt{3}$ with signal-dependent noise and $m = 0$ ($A = 5,\ c = 0.3,\ a = 0.1,\ T_s = 1,\ m = 0.0$).

5. Conclusion and discussion

There are plenty of papers discussing SR in various fields, but few papers deal with SR in a parallel array of linear elements. This paper investigated the SR phenomenon in a parallel array of linear elements with noise. And we discussed the effects on the output SNR of the parallel array of linear elements with signal-independent noise and signal-dependent noise respectively. Numerical results show that SR can arise in this parallel array linear system with signal-dependent noise, under the case where $N \geq 2$ and the noise mean $m \neq 0$. Furthermore, the output SNR

enhances rapidly with the increase of N. These results may constitute another instance of the linear phenomenon of SR, while the presence of noise enhances the signal.

Acknowledgments

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