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Realization of Toffoli gate operation using four-level atoms in cavity QED system*

Tang Shi-Qing(唐世清)[a)b)†], Zhang Deng-Yu(张登玉)[b)], Xie Li-Jun(谢利军)[a)b)], Zhan Xiao-Gui(詹孝贵)[b)], and Gao Feng(高 峰)[b)]

[a)] *Faculty of Material and Photoelectronic Physics, Xiangtan University, Xiangtan* 411005, *China*

[b)] *Department of Physics and Electronic Information Science, Hengyang Normal University, Hengyang* 421008, *China*

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This paper proposes a scheme for realization of a three-qubit Toffoli gate operation using three four-level atoms by a selective atom–field interaction in a cavity quantum electrodynamics system. In the proposed protocol, the quantum information is encoded on the stable ground states of atoms, and atomic spontaneous emission is negligible as the large atom–cavity detuning effectively suppresses the spontaneous decay of the atoms. The influence of the dissipation on fidelity and success probability of the three-qubit Toffoli gate is also discussed. The scheme can also be applied to realize an N-qubit Toffoli gate and the interaction time required does not rise with increasing the number of qubits.

Keywords: Toffoli gate, four-level atom, cavity QED, fidelity
PACC: 0367, 4250, 3280

Quantum computing has attracted much interest since it is clear that the quantum computer is in principle able to solve hard computational problems much more efficiently than classical computer.[1,2] The operation of the quantum computer relies on quantum logic gates. Therefore, the core of quantum computing is the quantum logic gate. A number of physical systems were proposed as candidates for implementing quantum logic gates, and many of these have been demonstrated, such as cavity quantum electrodynamics (QED),[3,4] trapped ions,[5−7] nuclear magnetic resonance,[8] quantum dots,[9] and trapped atoms in superconducting quantum interference devices (SQUID).[10] Among these protocols, cavity QED is considered to be one of the most effective schemes to implement quantum information processing.[11]

It is well known that two-qubit quantum logic gates and one-qubit gates are universal for constructing a quantum computer, and any multi-qubit gate can be decomposed into these elementary gates.[12] But those important implementation of quantum information all need the multiqubit gates, such as constructing quantum computational networks,[13] realizing quantum error correction protocols[14,15] and implementing quantum algorithms,[16] and procedure of decomposing multiqubit gates into the elementary gates becomes complicated as the number of qubits increases. For example, we construct the N-qubit controlled phase gate using one- and two-qubit gates, however, this multiqubit gate will need $2N-3$ two-qubit controlled gates without auxiliary qubits, or $2N-19$ two-qubit controlled gates with $N-2$ additional qubits.[17] Therefore, it is important to find a more efficient way to implement multiqubit quantum logic gates. Lately, several important schemes for implementing multiqubit gates directly have been proposed. Chen *et al*[18] and Shao *et al*[19] have presented a protocol of realizing a Toffoli gate in a cavity QED system, respectively. However, the influence of the atomic spontaneous emission on their protocols is unavoidable.

Recently, Zou *et al*[20] realize a two-qubit conditional phase gate operation utilizing a selective atom–field interaction. Enlightened by the foregoing works, we construct a theoretical model of realizing a three-qubit Toffoli gate using three four-level atoms by a selective atom–field interaction in cavity QED system. Our proposal has the following advantages: (i) our theoretical model does not use the cavity mode as the quantum memory, and the ground states of atoms are encoded as qubits, which have the lower energy than

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†E-mail: tangshiqinghn@163.com

the excited state and can be relatively stable in experiment; (ii) during the operation, decoherence effect arising through atomic spontaneous emission can be effectively suppressed due to large atom–cavity detuning; (iii) the fidelity and success probability of the three-qubit Toffoli gate are very high in the existence of cavity dissipation; (iv) the scheme can also be applied to realize an N-qubit Toffoli gate and the interaction time between the atoms and the cavity-mode does not change with the increase of the number of qubits.

The system considered here consists of three four-level atoms simultaneously passing through a single-mode vacuum cavity–field. The level structure of the atomic configuration is shown in Fig.1(a). Each atom has three stable ground states $|l\rangle$, $|r\rangle$, $|k\rangle$, and one excited state $|e\rangle$. Here, we can take ^{87}Rb to meet the required level structures. The ground states $|l\rangle$, $|r\rangle$ and $|k\rangle$ correspond to Zeeman levels $|F=1,\ m=0\rangle$ of $5S_{1/2}$, $|F=2,\ m=2\rangle$ of $5S_{1/2}$ and $|F=2,\ m=3\rangle$ of $5S_{1/2}$ respectively, while the excited state $|e\rangle$ is denoted by the Zeeman level $|F=2,\ m=2\rangle$ of the $5P_{1/2}$. The coherence time of the atomic ground levels $|l\rangle$, $|r\rangle$ and $|k\rangle$ is so long that these states can be used to store quantum information. As shown in Fig.1(a), the atomic transition $|r\rangle \rightarrow |e\rangle$ is nonresonantly coupled to a single quantized cavity mode with the coupling constant g. The classical laser field dispersively drives the atomic transition $|k\rangle \rightarrow |e\rangle$ with the coupling constant Ω_L, while the atomic ground level $|l\rangle$ is not affected by the cavity-mode and classical field. δ represents the corresponding atom–cavity transition and atom–laser interaction one–photon detuning.

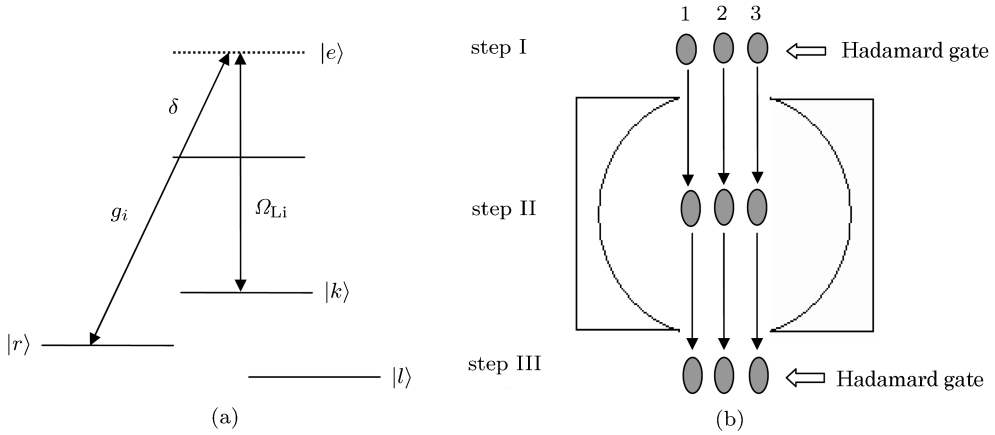


Fig.1. (a) Involved level structure of the four-level atoms and the corresponding atomic transition. (b) The physical device for implementing a three-qubit Toffoli gate in cavity QED.

Under the large-detuning limit conditions, $|\delta| \gg |\Omega_{Li}|$, $|\delta| \gg |g_i|$, we can eliminate adiabatically the level $|e\rangle$ by neglecting the effect of rapidly oscillating terms with high frequencies, when the laser field is taken into account. Then an effective Hamiltonian is given by[21]

$$H_{\mathrm{eff}} = \sum_{i=1}^{3} \frac{g_i^2}{\delta}\left[a^+ a|r_i\rangle\langle r_i| + \frac{\Omega_{Li}^2}{\delta}|k_i\rangle\langle k_i| + \frac{g_i\Omega_{Li}}{\delta}(a|k_i\rangle\langle r_i| + \mathrm{H.c.})\right], \quad (1)$$

where g_i represents coupling strength between the ith atom and the cavity, a^+ and a are the creation and annihilation operators associated with the quantized cavity mode, respectively.

We now begin to analyse how to realize a three-qubit Toffoli gate using nonidentical atom–field coupling strengths, resorting to the effective Hamiltonian. The quantum information of the two control qubits is encoded on the states $|r_1\rangle$, $|l_1\rangle$, $|r_2\rangle$ and $|l_2\rangle$, while the target qubit is encoded on the states $|r_3\rangle$ and $|k_3\rangle$. In order to realize a three-qubit Toffoli gate, the scheme consists of three-step operations, corresponding physical device is shown in Fig.1(b).

Step I Perform a Hadamard transformation on the third atom with the conversions $|r_3\rangle \rightarrow (|r_3\rangle + |k_3\rangle)/\sqrt{2}$, $|k_3\rangle \rightarrow (|r_3\rangle - |k_3\rangle)/\sqrt{2}$.

Step II Let the three atoms pass through the high-Q optical cavity, the selective atom–cavity interaction plays an important role in the time evolution of the basic states, which is the core of this paper.

Next let us give the detailed analysis of the second

stage. It is obvious that, if the atom 3 is initially in the state $|r\rangle$, and the other two atoms are in the states $|l\rangle$ or $|r\rangle$, the atom–cavity system does not experience any dynamical evolution since $H_{\rm eff}|\alpha_1\rangle|\alpha_2\rangle|r_3\rangle|0\rangle = 0$, where $|\alpha\rangle \in |r\rangle, |l\rangle$. In other words, we only need consider the temporal evolution of the initial system state $|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle$, $|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle$, $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$, $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$.

In the following part, we will analyse the time evolution of the states $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ and $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ in detail. If the initial state of system is $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$, there will be only two possible states in the temporal evolution, which are $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ and $|l_1\rangle|l_2\rangle|r_3\rangle|1\rangle$. The state $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ acquires a phase shift $\mathrm{e}^{\mathrm{i}\pi}$ through the selective atom–cavity interaction. For simplicity, we define the state $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ as $|\phi_{k_3,0}\rangle$, and $|l_1\rangle|l_2\rangle|r_3\rangle|1\rangle$ as $|\phi_{r_3,1}\rangle$, with the uniform subscripts omitted. According to the interaction in Eq.(1), the Hamiltonian in the subspace can be written as

$$\begin{aligned} H_{\rm i} =& \frac{\Omega_{L3}^2}{\delta}(|\phi_{k_3,0}\rangle\langle\phi_{k_3,0}|) + \frac{g_3^2}{\delta}(|\phi_{r_3,1}\rangle\langle\phi_{r_3,1}|) \\ &+ \frac{g_3\Omega_{L3}}{\delta}(|\phi_{k_3,0}\rangle\langle\phi_{r_3,1}| + \text{H.c.}). \end{aligned} \tag{2}$$

Here we will adopt another expression for a more convenient investigation in the subsystem as follows:

$$H_0 = \frac{\Omega_{L3}^2}{\delta}(|\phi_{k_3,0}\rangle\langle\phi_{k_3,0}|) + \frac{g_3^2}{\delta}(|\phi_{r_3,1}\rangle\langle\phi_{r_3,1}|) \tag{3}$$

$$H_{\rm I} = \frac{g_3\Omega_{L3}}{\delta}(|\phi_{k_3,0}\rangle\langle\phi_{r_3,1}| + \text{H.c.}). \tag{4}$$

For the sake of further simplifying the dynamics of the subsystem, we switch to the interaction picture with respect to the H_0,[22] then the interaction Hamiltonian of the subsystem becomes

$$\begin{aligned} H'_{\rm I} =& \frac{g_3\Omega_{L3}}{\delta}\Big\{|\phi_{k_3,0}\rangle\langle\phi_{r_3,1}| \\ &\times \exp\Big[-\mathrm{i}\Big(\frac{g_3^2-\Omega_{L3}^2}{\delta}\Big)t\Big] + \text{H.c.}\Big\}. \end{aligned} \tag{5}$$

If we choose the parameters to satisfy $\Omega_{L3} = g_3$, then Eq.(6) will result in the resonant transition between the states $|\phi_{k_3,0}\rangle$ and $|\phi_{r_3,1}\rangle$. Thus we can obtain transformation

$$|\phi_{k_3,0}\rangle \to \cos(\eta t)|\phi_{k_3,0}\rangle - \mathrm{i}\sin(\eta t)|\phi_{r_3,1}\rangle, \tag{6}$$

where $\eta = g_3^2/\delta$. Choosing the interaction time $t = \pi/\eta$, we can achieve a phase shift $\mathrm{e}^{\mathrm{i}\pi}$ for $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$.

For the state $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$, there will be some transitions among $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$, $|l_1\rangle|k_2\rangle|r_3\rangle|0\rangle$ and $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$. For the sake of simplicity, the states $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ and $|l_1\rangle|k_2\rangle|r_3\rangle|0\rangle$ have been denoted as $|\phi_{l_1,k_i,0}\rangle$, the state $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$ as $|\phi_{l_1,r_3,1}\rangle$, respectively. The Hamiltonian in this subspace in the interaction picture is

$$\begin{aligned} H'_{\rm i} =& \sum_{i=2}^{3}\frac{\Omega_{Li}^2}{\delta}|\phi_{l_1,k_i,0}\rangle\langle\phi_{l_1,k_i,0}| + \frac{g_i^2}{\delta}|\phi_{l_1,r_3,1}\rangle\langle\phi_{l_1,r_3,1}| \\ &+ \frac{g_i\Omega_{Li}}{\delta}(|\phi_{l_1,k_i,0}\rangle\langle\phi_{l_1,r_3,1}| + \text{H.c.}). \end{aligned} \tag{7}$$

In the same way, we switch to the interaction picture with respect to

$$\sum_{i=2}^{3}\frac{\Omega_{Li}^2}{\delta}|\phi_{l_1,k_i,0}\rangle\langle\phi_{l_1,k_i,0}| + \frac{g_i^2}{\delta}|\phi_{l_1,r_3,1}\rangle\langle\phi_{l_1,r_3,1}|,$$

then the interaction Hamiltonian of the subsystem becomes

$$\begin{aligned} H''_{\rm I} =& \sum_{i=2}^{3}\frac{g_i\Omega_{Li}}{\delta}\Big\{|\phi_{l_1,k_i,0}\rangle\langle\phi_{l_1,r_3,1}| \\ &\times \exp\Big[-\mathrm{i}\Big(\frac{g_2^2+g_3^2-\Omega_{L3}^2}{\delta}\Big)t\Big] + \text{H.c.}\Big\}. \end{aligned} \tag{8}$$

Then the state $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ only has one possible coupled state $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$, according to Eq(8). Choosing the parameters to satisfy $\Omega_{L2} = \Omega_{L3} = g_3$, $g_2 \gg g_3$, using the Schrödinger equation $\mathrm{i}\partial_t|\Psi(t)\rangle = H''_{\rm I}|\Psi(t)\rangle$, we have

$$\begin{aligned} |\Psi(t)\rangle =& \Big\{\frac{\beta-g_2^2}{2\beta}\exp\Big[-\frac{\mathrm{i}}{2}(g_2^2+\beta)t\Big] + \frac{\beta+g_2^2}{2\beta}\exp\Big[-\frac{\mathrm{i}}{2}(g_2^2-\beta)t\Big]\Big\}|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle \\ &+ \Big\{\frac{g_3^2}{\beta}\exp\Big[-\frac{\mathrm{i}}{2}(g_2^2+\beta)t\Big] - \frac{g_3^2}{\beta}\exp\Big[-\frac{\mathrm{i}}{2}(g_2^2-\beta)t\Big]\Big\}|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle, \end{aligned} \tag{9}$$

where we have adopted $\sqrt{g_2^2+4g_3^2}=\beta$.

To validate the feasibility of the above theoretical derivation, we choose the parameters $\Omega_{L2}=\Omega_{L3}=g_3$, $t=\pi/\eta$, and perform direct numerical calculation on the probability amplitude of the states $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ and $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$. As showed in Fig.2, the state $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ is almost unchanged, so the transition between the states $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$ and $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ can be neglected, when the large-detuning limit is satisfied with $g_2/g_3=6,7,8,\ldots$.

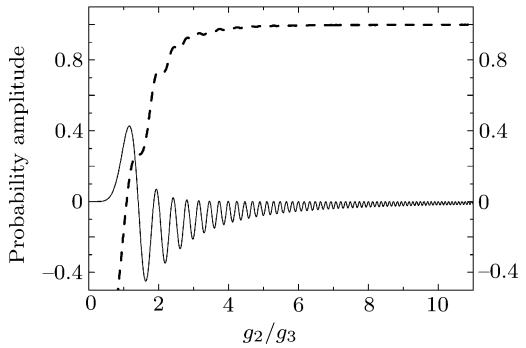


Fig.2. Real part of probability amplitude of the system states $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ (dashed line) and $|l_1\rangle|r_2\rangle|r_3\rangle|1\rangle$ (solid line) vs g_2/g_3. Other parameters : $t=\pi/\eta$, $\eta=g_3^2/\delta$, $\Omega_{L2}=\Omega_{L3}=g_3$.

By repeating the above-mentioned procedures, it is easy to find that the states $|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ and $|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ are similar to that of state $|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ in the process of the time evolution. The difference is that $|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ is coupled to the state $|r_1\rangle|l_2\rangle|r_3\rangle|1\rangle$ with the coupling strength $\frac{g_3\Omega_{L3}}{\delta}$ and the detuning $\frac{g_1^2}{\delta}$, where $|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ is coupled to the state $|r_1\rangle|r_2\rangle|r_3\rangle|1\rangle$ with the coupling strength $\frac{g_3\Omega_{L3}}{\delta}$ and the detuning $\frac{g_1^2+g_2^2}{\delta}$. In other words, when the experimental parameters satisfy the conditions $g_1=g_2=10g_3$, $\Omega_{L1}=\Omega_{L2}=\Omega_{L3}=g_3$, the states $|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ and $|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle$ do not experience any dynamical evolution at the time $t=\pi/\eta$, which can be ascribed to the largely detuned limit.

Step III Apply the Hadamard gate on the third atom with $|r_3\rangle\rightarrow(|r_3\rangle+|k_3\rangle)/\sqrt{2}$, $|k_3\rangle\rightarrow(|r_3\rangle-|k_3\rangle)/\sqrt{2}$.

In our protocol, the first and third steps are not affected by the atom–cavity interaction. Both can be achieved by putting the atom outside of the cavity since the atoms can be moved inside and outside the high-Q optical cavity.[23] The three-qubit Toffoli gate operation can be obtained after the three steps

$$\begin{aligned}
&|r_1\rangle|r_2\rangle|r_3\rangle|0\rangle\rightarrow|r_1\rangle|r_2\rangle|r_3\rangle|0\rangle,\\
&|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle\rightarrow|r_1\rangle|r_2\rangle|k_3\rangle|0\rangle,\\
&|r_1\rangle|l_2\rangle|r_3\rangle|0\rangle\rightarrow|r_1\rangle|l_2\rangle|r_3\rangle|0\rangle,\\
&|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle\rightarrow|r_1\rangle|l_2\rangle|k_3\rangle|0\rangle,\\
&|l_1\rangle|r_2\rangle|r_3\rangle|0\rangle\rightarrow|l_1\rangle|r_2\rangle|r_3\rangle|0\rangle,\\
&|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle\rightarrow|l_1\rangle|r_2\rangle|k_3\rangle|0\rangle,\\
&|l_1\rangle|l_2\rangle|r_3\rangle|0\rangle\rightarrow|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle,\\
&|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle\rightarrow|l_1\rangle|l_2\rangle|r_3\rangle|0\rangle. \qquad (10)
\end{aligned}$$

It is necessary to give a brief discussion on the influence of decoherence during the gate operation process to examine how much efficiency our method has. The decoherence mechanisms arise through two dominant channels: (i) cavity decay (at the rate κ) and (ii) atomic spontaneous emission (at the rate Γ). First, the atomic spontaneous emission can be neglected under the condition of large atom–cavity detuning, which makes the excited state adiabatically eliminated in our protocol. Second, to implement a Toffoli gate, the state $|l_1\rangle|l_2\rangle|k_3\rangle|0\rangle$ couples with the single photon state $|l_1\rangle|l_2\rangle|r_3\rangle|1\rangle$, this will cause the cavity decay to become the dominant noise source during the gate operation process. In order to confirm this issue, we can implement the numerical simulation for the fidelity and corresponding success probability of three-qubit Toffoli gate. The calculated relation of fidelity can be defined by $F\equiv|\langle\Psi_{\rm out}^{\rm ideal}|\Psi_{\rm out}^{\rm con}\rangle|^2$, where $|\Psi_{\rm out}^{\rm ideal}\rangle$ describes the ideal output $\left\{\frac{1}{2\sqrt{2}}(|r_1\rangle+|l_1\rangle)(|r_2\rangle+|l_2\rangle)(|r_3\rangle+|k_3\rangle)\right\}$, and $|\Psi_{\rm out}^{\rm con}\rangle$ represents the normalized actual output after a three-qubit Toffoli gate operation, here[18]

$$\begin{aligned}
|\Psi_{\rm out}^{\rm con}\rangle=&(\xi_{000}|r_1,r_2\rangle|r_3\rangle+\xi_{001}|r_1,r_2\rangle|k_3\rangle+\xi_{010}|r_1,l_2\rangle|r_3\rangle+\xi_{011}|r_1,l_2\rangle|k_3\rangle\\
&+\xi_{100}|l_1,r_2\rangle|r_3\rangle+\xi_{101}|l_1,r_2\rangle|k_3\rangle+\xi_{110}|l_1,l_2\rangle|k_3\rangle+\xi_{111}|l_1,l_2\rangle|r_3\rangle)/2\sqrt{2}.
\end{aligned}$$

The corresponding success probability is calculated by $P = \left(\sum_{i,j,k=0}^{1} |\xi_{i,j,k}|^2\right)/8$.[18] From Fig.3, it can be seen that cavity decay plays a dominant role in the dissipative channel during the gate implementation. With κ increasing from $0.001g_3$ to $0.005\ g_3$, the fidelity F and the success probability P rapidly decrease from 0.999 to 0.997, and from 0.964 to 0.864, respectively.

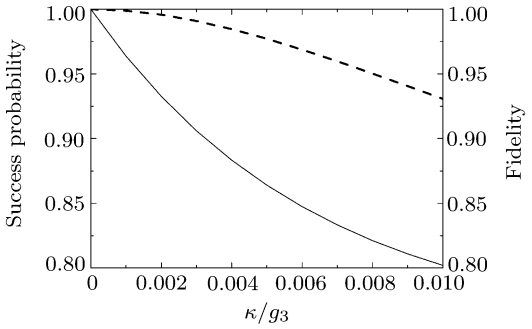


Fig.3. The fidelity (dashed line) and success probability (solid line) of the three-qubit Toffoli gate vs κ/g_3. Other parameters: $g_1 = g_2 = 10g_3$, $\Omega_{L1} = \Omega_{L2} = \Omega_{L3} = g_3$, $\delta = 10g_1$.

Now let us briefly discuss the experimental feasibility of the proposed scheme. The scheme requires three atoms in a vacuum cavity–field, which has different coupling strengths with the cavity mode. The coupling constants depend on the atomic positions $g_i = \Omega \mathrm{e}^{-r^2/\omega^2}$, where Ω is the coupling strength at the cavity centre, ω is the waist of the cavity mode, and r represents the distance between the atom and cavity centre.[24,25] The condition $g_1 = g_2 = 10g_3$ in our scheme can be satisfied by locating anterior two atoms at the centre of the cavity and locating the third atom at the position $r = \omega\sqrt{\ln 10}$. According to recent experiments with ^{87}Rb atoms trapped in an optical cavity, the parameters $(g,\ \kappa,\ \Gamma)/2\pi = (16\ , 1.4\ ,\ 3)$ MHz have been reported.[26] For the sake of a higher gate fidelity and the success probability of the quantum gate, we need a very small cavity decay, which implies that the cavity dissipation should be smaller than the atom–field coupling strength by two orders of magnitude at least. This still cannot match well with the current experimental technique. However, the experimental parameters in the near future may satisfy this scheme very well.

Finally, we will show that an N-qubit Toffoli gate can be realized via extending our scheme to the N atoms case. The same as the situation of three atoms, the scheme consists of three steps, and quantum information is stored in the states $|r_i\rangle, |l_i\rangle,\ |r_N\rangle$ and $|k_N\rangle$, and cavity field is the vacuum field, the difference is that the target atom is the Nth atom. The interaction time required is independent of the number of qubits all the same. It is easy to find that if the Nth atom is initially in the state $|r_N\rangle$, while the other atoms occupy the states $|r\rangle$ or $|l\rangle$, these states will keep unchanged, since $H_{\mathrm{eff}}|\alpha_1 \ldots \alpha_{N-1}\rangle|r_N\rangle|0\rangle = 0$, where $|\alpha_i\rangle = |r_i\rangle, |l_i\rangle$. Therefore, we can only consider the temporal evolution of the states of the form $|\alpha_1 \ldots \alpha_{N-1}\rangle|k_N\rangle|0\rangle$, where $|\alpha_i\rangle = |r_i\rangle,\ |l_i\rangle$. We first consider the time evolution of the state $|l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle$, then repeat the similar process from Eq.(2) to Eq.(6), and we can obtain the evolution

$$|l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle \rightarrow \cos(\eta t)|l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle - \mathrm{i}\sin(\eta t)|l_1 \ldots l_{N-1}\rangle|r_N\rangle|1\rangle. \quad (11)$$

Choosing $g_N\Omega_{LN}t/\delta = \eta t = \pi$, we have $-|l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle$. For other states of the form of $|\alpha_1 \ldots \alpha_{N-1}\rangle|k_N\rangle|0\rangle$, we assume that the coupling strengths could satisfy the conditions $g_1 = g_2 = g_3 = \ldots = 10g_N$, $\Omega_{L1} = \Omega_{L2} = \Omega_{L3} \ldots = g_N$, which will leave these states in a highly dispersive regime. After undergoing the homologous procedures from Eq.(7) to Eq.(9), the following transformation can be obtained:

$$|\alpha_1, \ldots \alpha_{N-1}\rangle|k_N\rangle|0\rangle \rightarrow |\alpha_1, \ldots \alpha_{N-1}\rangle|k_N\rangle|0\rangle, \quad (12)$$

where $|\alpha_i\rangle = |r_i\rangle,\ |l_i\rangle$ and at least one qubit initially occupies the ground state $|r\rangle$. If we utilize physical device as Fig.1(b), let atoms pass through the high-Q optical cavity via three steps alike, implementing Hadamard gate on the target atom with $|r_N\rangle \rightarrow (|r_N\rangle + |k_N\rangle)/\sqrt{2}$, $|k_N\rangle \rightarrow (|r_N\rangle - |k_N\rangle)/\sqrt{2}$, then

$$|l_1 \ldots l_{N-1}\rangle|r_N\rangle|0\rangle \rightarrow |l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle,$$
$$|l_1 \ldots l_{N-1}\rangle|k_N\rangle|0\rangle \rightarrow |l_1 \ldots l_{N-1}\rangle|r_N\rangle|0\rangle. \quad (13)$$

The above-mentioned transformation can be considered as an N-qubit Toffoli gate operation. This quantum gate implies that if and only if the control atoms are all in the state $|l\rangle$, the flip of target qubits happens to the states $|r_N\rangle$ and $|k_N\rangle$ ($|r_N\rangle \rightarrow |k_N\rangle$ and $|k_N\rangle \rightarrow |r_N\rangle$) and the others keep the same.

In summary, we have considered a cavity QED scheme to realize a three-qubit Toffoli gate by using three four-level atoms through a selective atom–field interaction in an optical cavity QED system. During the manipulation process, the atomic spontaneous decay can be strongly suppressed due to the large atom–field detuning. The fidelity and corresponding success

probability of the three-qubit Toffoli gate keep an ultrahigh level in the presence of cavity dissipation. The scheme can also be extended to realize an N-qubit Toffoli gate and the gating time does not rise with the increase of the number of qubits.

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