Vol 18 No 1, January 2009
1674-1056/2009/18(01)/0040-06

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Dynamical nonlocality of the entangled coherent state in the phase damping model*

Lu Huai-Xin(逯怀新)† and Li Ying-De(李英德)

Department of Physics and Electronic Science, Weifang University, Weifang 261061, *China*

(Received 16 May 2008; revised manuscript received 11 July 2008)

This paper studies the dynamics of nonlocality for a bosonic entangled coherent state in a phase damping model. The density operator of the system is solved by using a superoperator method. The dynamics of nonlocality for the bosonic entangled coherent state is uncovered by the Bell operator based on the pseudospin operator of a light field. The dynamics of the nonlocality for this state has also been studied by other Bell operators. The result of the numerical calculations of the Bell function shows that the quantum nonlocality heavily depends on the chosen Bell operator.

Keywords: entangled coherent state, nonlocality dynamics, Bell operators
PACC: 0367

The quantum entanglements and nonlocality are both of theoretical importance and of practical application in the quantum information processing[1] such as quantum teleportation,[2] quantum cryptography[3] and quantum computation,[4] etc.. Starting from local realism, Einstein, Podolsky, and Rosen (EPR)[5] argued that quantum mechanics can not be a complete theory. In 1964, based on Bohm's versions of EPR entangled state, Bell derived his famous inequality,[6] which enables a quantitative test of quantum mechanics versus local reality. Since then, various versions of Bell's inequality have been proposed for testing local reality.[7−10] With the help of these inequalities, quantum nonlocality for many quantum entangled systems with both discrete and continuous variables have been studied in detail.[10−13] It was also shown that the degree of quantum nonlocality crucially depends not only on the given quantum state but also on the 'Bell operator'.[14] Thus, the desirable Bell operators to uncover the quantum nonlocality for the entangled state play an essential role in the modern understanding of quantum phenomena.

It is well known that a quantum system loses its quantum characteristics if it is open to the environment.[15] In practice, the degree of an entangled state suffers by undesirable losses which strongly reduced amount of the entanglements. For such an open system, it is interesting to know the dynamics of quantum nonlocality and entanglement. It is obvious that the studies of dynamics of quantum nonlocality have become an issue of practical importance. In Ref.[13], the dynamics of quantum nonlocality have been studied in details for a two-mode squeezed vacuum state in lossy transmission lines. This result can be useful for security of continuous–variable quantum cryptography employing entangled states. In Ref.[16], we studied quantum nonlocality of a hybrid entangled state in the reservoir. Recently, it was shown that the periodical collapse and revival of the entanglement witness for 3-qubit cluster states in the common environment.[17] In this paper, we aim at studying the quantum nonlocality for bosonic entangled coherent state (ECS)[18,19] in the phase damping model.[13]

The entanglement of ECS which can be produced by using a 50/50 beam splitter has been shown in Ref.[20]. In the following, we first solve the master equation of the density operator with the superoperator method. Then, by constructing the Bell operator with the help of pseudospin operator we study the dynamics of the quantum nonlocality for the ECS in the phase damping.

Let us consider the following master equation for the phase damping model in the interaction picture[13]

$$\dot{\rho}(t) = (L_1 + L_2)\rho(t) \tag{1}$$

with

*Project supported by the National Natural Science Foundation of China (Grant No 60878001) and the Natural Science Foundation of Shandong Province, China (Grant No Y2006 A24).
†E-mail: luhuaixin@wfu.edu.cn

$$L_i\rho = \frac{\gamma}{2}(2a_i^+ a_i \rho a_i^+ a_i - (a_i^+ a_i)^2\rho - \rho(a_i^+ a_i)^2). \quad (2)$$

We define the following superoperators

$$\begin{aligned} J_0^{(i)}\rho &= a_i^+ a_i \rho a_i^+ a_i, \\ K_0^{(i)}\rho &= -\frac{1}{2}((a_i^+ a_i)^2\rho + (a_i^+ a_i)^2\rho). \end{aligned} \quad (3)$$

It is easy to verify the following commutation relations

$$[J_0^{(i)}, K_0^{(i)}]\rho = 0. \quad (4)$$

Using the method of solving the master equation presented in Ref.[19], we easily get the solution of Eq.(1) as follows:

$$\rho(t) = \mathrm{e}^{\gamma t J_0^{(1)}}\mathrm{e}^{\gamma t J_0^{(2)}}\mathrm{e}^{\gamma t K_0^{(1)}}\mathrm{e}^{\gamma t K_0^{(2)}}\rho(0). \quad (5)$$

Now, we use the solution in Eq.(5) to study the dynamics of quantum nonlocality for the ECS when the initial state is entangled pure ECS which is produced by using a 50/50 beam splitter. In Ref.[10], by introducing the pseudospin operator, the quantum nonlocality of the two-mode squeezed vacuum states has been uncovered. This work also establishes an interesting link between quantum information protocols of the discrete and continuous variables. This provides us with very useful information to study the quantum nonlocality for the ECS.

The pure ECS studied in Ref.[18] is

$$\begin{aligned} |\alpha,\alpha\rangle &= \frac{1}{\sqrt{2(1-\mathrm{e}^{-4|\alpha|^2})}}(|\alpha\rangle|\alpha\rangle - |-\alpha\rangle|-\alpha\rangle \\ &= \frac{1}{\sqrt{2}}(|\alpha\rangle_+|\alpha\rangle_- + |\alpha\rangle_-|\alpha\rangle_+), \end{aligned} \quad (6)$$

where $|\alpha\rangle$ is the bosonic coherent state, and $|\alpha\rangle_+$ ($|\alpha\rangle_-$) is the even (odd) coherent state defined as

$$\begin{aligned} |\alpha\rangle_+ &= \frac{1}{\sqrt{2(1+\mathrm{e}^{-2|\alpha|^2})}}(|\alpha\rangle + |-\alpha\rangle) \\ &= \frac{1}{\sqrt{\mathrm{ch}\,|\alpha|^2}}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle, \\ |\alpha\rangle_- &= \frac{1}{\sqrt{2(1-\mathrm{e}^{-2|\alpha|^2})}}(|\alpha\rangle - |-\alpha\rangle) \\ &= \frac{1}{\sqrt{\mathrm{sh}\,|\alpha|^2}}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle. \end{aligned} \quad (7)$$

Choosing Eq.(6) as the initial state $\rho(0) = |\alpha,\alpha\rangle\langle\alpha,\alpha|$, by a straightforward calculation we obtain:

$$\begin{aligned} &\mathrm{e}^{\gamma t J_0^{(1)}}\mathrm{e}^{\gamma t J_0^{(2)}}\mathrm{e}^{\gamma t K_0^{(1)}}\mathrm{e}^{\gamma t K_0^{(2)}}|\alpha_1\rangle_+|\alpha_2\rangle_-\,{}_-\langle\alpha_2|_+\langle\alpha_1| \\ &= \frac{1}{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_2|^2}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1}(\alpha_1^*)^{2n_1'}\alpha_2^{2n_2+1}(\alpha_2^*)^{2n_2'+1}}{\sqrt{(2n_1)!(2n_1')!(2n_2+1)!(2n_2'+1)!}} \\ &\times\mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]}|2n_1\rangle|2n_2+1\rangle\langle 2n_2'+1|\langle 2n_1'|, \end{aligned} \quad (8)$$

$$\begin{aligned} &\mathrm{e}^{\gamma t J_0^{(1)}}\mathrm{e}^{\gamma t J_0^{(2)}}\mathrm{e}^{\gamma t K_0^{(1)}}\mathrm{e}^{\gamma t K_0^{(2)}}|\alpha_1\rangle_-|\alpha_2\rangle_+\,{}_+\langle\alpha_2|_-\langle\alpha_1| \\ &= \frac{1}{\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_1|^2}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1+1}(\alpha_1^*)^{2n_1'+1}\alpha_2^{2n_2}(\alpha_2^*)^{2n_2'}}{\sqrt{(2n_1+1)!(2n_1'+1)!(2n_2)!(2n_2')!}} \\ &\times\mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]}|2n_1+1\rangle|2n_2\rangle\langle 2n_2'|\langle 2n_1'+1|, \end{aligned} \quad (9)$$

$$\begin{aligned} &\mathrm{e}^{\gamma t J_0^{(1)}}\mathrm{e}^{\gamma t J_0^{(2)}}\mathrm{e}^{\gamma t K_0^{(1)}}\mathrm{e}^{\gamma t K_0^{(2)}}|\alpha_1\rangle_+|\alpha_2\rangle_-\,{}_+\langle\alpha_2|_-\langle\alpha_1| \\ &= \frac{\mathrm{e}^{-\gamma t}}{\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1}(\alpha_1^*)^{2n_1'+1}\alpha_2^{2n_2+1}(\alpha_2^*)^{2n_2'}}{\sqrt{(2n_1)!(2n_1'+1)!(2n_2+1)!(2n_2')!}} \\ &\times\mathrm{e}^{-2\gamma t[(n_1-n_1')^2-(n_1-n_1')]}\mathrm{e}^{-2\gamma t[(n_2-n_2')^2-(n_2'-n_2)]}|2n_1\rangle|2n_2+1\rangle\langle 2n_2'|\langle 2n_1'+1|, \end{aligned} \quad (10)$$

$$\begin{aligned} &\mathrm{e}^{\gamma t J_0^{(1)}}\mathrm{e}^{\gamma t J_0^{(2)}}\mathrm{e}^{\gamma t K_0^{(1)}}\mathrm{e}^{\gamma t K_0^{(2)}}|\alpha_1\rangle_-|\alpha_2\rangle_+\,{}_-\langle\alpha_2|_+\langle\alpha_1| \\ &= \frac{\mathrm{e}^{-\gamma t}}{\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1+1}(\alpha_1^*)^{2n_1'}\alpha_2^{2n_2}(\alpha_2^*)^{2n_2'+1}}{\sqrt{(2n_1+1)!(2n_1')!(2n_2)!(2n_2'+1)!}} \\ &\times\mathrm{e}^{-2\gamma t[(n_1-n_1')^2-(n_1'-n_1)]}\mathrm{e}^{-2\gamma t[(n_2-n_2')^2-(n_2-n_2')]}|2n_1+1\rangle|2n_2\rangle\langle 2n_2'+1|\langle 2n_1'|. \end{aligned} \quad (11)$$

Substituting Eqs.(6) and (8–10) into Eq.(5), we obtain the mixed ECS as follows:

$$\begin{aligned}\rho(t) = \frac{1}{2}\Bigg\{ & \frac{1}{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_2|^2}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1}(\alpha_1^*)^{2n_1'}\alpha_2^{2n_2+1}(\alpha_2^*)^{2n_2'+1}}{\sqrt{(2n_1)!(2n_1')!(2n_2+1)!(2n_2'+1)!}} \\ & \times \mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]}\,|2n_1\rangle\,|2n_2+1\rangle\,\langle 2n_2'+1|\,\langle 2n_1'| \\ & + \frac{1}{\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_1|^2}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1+1}(\alpha_1^*)^{2n_1'+1}\alpha_2^{2n_2}(\alpha_2^*)^{2n_2'}}{\sqrt{(2n_1+1)!(2n_1'+1)!(2n_2)!(2n_2')!}} \\ & \times \mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]}\,|2n_1+1\rangle\,|2n_2\rangle\,\langle 2n_2'|\,\langle 2n_1'+1| \\ & + \frac{\mathrm{e}^{-\gamma t}}{\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1}(\alpha_1^*)^{2n_1'+1}\alpha_2^{2n_2+1}(\alpha_2^*)^{2n_2'}}{\sqrt{(2n_1)!(2n_1'+1)!(2n_2+1)!(2n_2')!}} \\ & \times \mathrm{e}^{-2\gamma t[(n_1-n_1')^2-(n_1-n_1')]}\mathrm{e}^{-2\gamma t[(n_2-n_2')^2-(n_2'-n_2)]}\,|2n_1\rangle\,|2n_2+1\rangle\,\langle 2n_2'|\,\langle 2n_1'+1| \\ & + \frac{\mathrm{e}^{-\gamma t}}{\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}}\sum_{n_1,n_1',n_2,n_2'}^{\infty}\frac{\alpha_1^{2n_1+1}(\alpha_1^*)^{2n_1'}\alpha_2^{2n_2}(\alpha_2^*)^{2n_2'+1}}{\sqrt{(2n_1+1)!(2n_1')!(2n_2)!(2n_2'+1)!}} \\ & \times \mathrm{e}^{-2\gamma t[(n_1-n_1')^2-(n_1'-n_1)]}\mathrm{e}^{-2\gamma t[(n_2-n_2')^2-(n_2-n_2')]}\,|2n_1+1\rangle\,|2n_2\rangle\,\langle 2n_2'+1|\,\langle 2n_1'|\Bigg\}. \end{aligned} \tag{12}$$

In order to study the quantum nonlocality for the state in Eq.(12), we introduce the following pseudospin operator:[10]

$$\begin{aligned} s_z &= \sum_{n=0}^{\infty}[|2n\rangle\,\langle 2n| - |2n+1\rangle\,\langle 2n+1|], \\ s_+ &= \sum_{n=0}^{\infty}|2n\rangle\,\langle 2n+1| = (s_-)^+, \end{aligned} \tag{13}$$

where $|n\rangle$ are the usual single-particle Fock states. By definition these operators obey the same commutation relations as the Pauli spin operators, i.e.

$$\begin{aligned}[s_z, s_\pm] &= \pm 2 s_\pm, \\ [s_+, s_-] &= s_z. \end{aligned} \tag{14}$$

The Bell operator is defined as[21]

$$\begin{aligned}\hat{B} = & (\boldsymbol{a}\cdot\boldsymbol{s}_1)(\boldsymbol{b}\cdot\boldsymbol{s}_2) + (\boldsymbol{a}'\cdot\boldsymbol{s}_1)(\boldsymbol{b}\cdot\boldsymbol{s}_2) \\ & + (\boldsymbol{a}\cdot\boldsymbol{s}_1)(\boldsymbol{b}'\cdot\boldsymbol{s}_2) - (\boldsymbol{a}'\cdot\boldsymbol{s}_1)(\boldsymbol{b}'\cdot\boldsymbol{s}_2). \end{aligned} \tag{15}$$

Where $\boldsymbol{a}(\boldsymbol{a}')$ and $\boldsymbol{b}(\boldsymbol{b}')$ are four unit vectors, e.g.,

$$\boldsymbol{a} = (\sin\theta_a, 0, \cos\theta_a), \tag{16}$$

which satisfy the relation $(\boldsymbol{a}\cdot\boldsymbol{s})^2 = I$, where I is the identity operator in the Hilbert space for the continuous-variable states.

Using Eqs.(12), (13) and (15), we derive the Bell function as follows:

$$\begin{aligned} <\hat{B}> &= \mathrm{Tr}\hat{B}\rho(t) \\ &= E(\theta_a,\theta_b) + E(\theta_a,\theta_b') \\ &\quad + E(\theta_a',\theta_b) - E(\theta_a',\theta_b'). \end{aligned} \tag{17}$$

Where $E(\theta_a,\theta_b)$ is the correlation function. Making use of the formulas (12), (13) , (15) and (17), after some algebra one finds

$$E(\theta_a,\theta_b) = -\cos\theta_a\cos\theta_b + f(t)\sin\theta_a\sin\theta_b, \tag{18}$$

where

$$\begin{aligned} f(t) = & \frac{\mathrm{e}^{-\gamma t}(\alpha_1\alpha_2^* + \alpha_2\alpha_1^*)}{2\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}} \\ & \times \sum_{n_1,n_2=0}^{\infty}\frac{(|\alpha_1|^2)^{2n_1}(|\alpha_2|^2)^{2n_2}}{\sqrt{(2n_1+1)(2n_2+1)}(2n_1)!(2n_2)!}. \end{aligned} \tag{19}$$

If we set $\theta_a = \pi, \theta_a' = \dfrac{\pi}{2}, \theta_b = -\theta_b'$, then we obtain the time-dependent Bell function from Eqs.(17) and (18) as follows:

$$\left\langle \hat{B} \right\rangle = 2(\cos\theta_b + f(t)\sin\theta_b). \tag{20}$$

Furthermore, choosing $\theta_b = \mathrm{tg}^{-1}f(t)$, we obtain the maximally time-dependent Bell function

$$\left\langle \hat{B} \right\rangle_{\max} = 2\sqrt{1+f(t)^2}. \tag{21}$$

It is clear from Eq.(21) that the state in Eq.(12) is always entangled for finite γt. Therefore Eq.(21) can be

considered as the equation describing the dynamics of nonlocality. One can analyse the quantum nonlocality for the ECS by using Eq.(21). For example, if we let $t = 0$, we have

$$f(0) = \frac{(\alpha_1\alpha_2^* + \alpha_2\alpha_1^*)}{2\sqrt{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2}} \times \sum_{n_1,n_2=0}^{\infty} \frac{(|\alpha_1|^2)^{2n_1}(|\alpha_2|^2)^{2n_2}}{\sqrt{(2n_1+1)(2n_2+1)}(2n_1)!(2n_2)!}. \tag{22}$$

Therefore, in this case we obtain the maximal Bell function as

$$\left\langle \hat{B} \right\rangle_{\max} = 2\sqrt{1 + f(0)^2}. \tag{23}$$

In the case of $t \to \infty$, we have

$$f(t \to \infty) = 0, \quad \left\langle \hat{B} \right\rangle_{\max} = 2 \tag{24}$$

in which the density operator becomes a separable state.

It was shown that the dynamics of nonlocality for the time-dependent mixed states are also related to the corresponding Bell operator. This can be seen from the chosen measurement operator for the given state. To illustrate this, we consider the following measurement operators for the state in Eq.(12)

$$\begin{aligned} s'_z &= |\alpha\rangle_{+\,+}\langle\alpha| - |\alpha\rangle_{-\,-}\langle\alpha|, \\ s'_+ &= |\alpha\rangle_{+\,-}\langle\alpha| = (s'_-)^+. \end{aligned} \tag{25}$$

It is easily seen that these operators obey the same commutation relations as the Pauli spin operators. Using the measurement operators in Eq.(25) to replace the pseudospin operator in Eq.(13), we obtain the corresponding Bell function as follows:

$$\left\langle \hat{B}' \right\rangle = 2\sqrt{g(t)^2 + h(t)^2}\cos(\theta_b - \varphi), \tag{26}$$

where

$$\tan\varphi = \frac{h(t)}{g(t)}, \tag{27}$$

$$g(t) = \frac{1}{2}\left\{ \frac{|\alpha_2|^4}{\mathrm{ch}^2\,|\alpha_1|^2\,\mathrm{sh}^2\,|\alpha_2|^2} \sum_{n_1,n_1',n_2,n_2'}^{\infty} \frac{(|\alpha_1|^2)^{2n_1+2n_1'}(|\alpha_2|^2)^{2n_2+2n_2'}}{(2n_1)!(2n_1')!(2n_2+1)!(2n_2'+1)!} \mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]} + \frac{|\alpha_1|^4}{\mathrm{ch}^2\,|\alpha_2|^2\,\mathrm{sh}^2\,|\alpha_1|^2} \sum_{n_1,n_1',n_2,n_2'}^{\infty} \frac{(|\alpha_1|^2)^{2n_1+2n_1'}(|\alpha_2|^2)^{2n_2+2n_2'}}{(2n_1+1)!(2n_1'+1)!(2n_2)!(2n_2')!} \mathrm{e}^{-2\gamma t[(n_1-n_1')^2+(n_2-n_2')^2]} \right\}, \tag{28}$$

$$h(t) = \frac{\mathrm{e}^{-\gamma t}}{\mathrm{ch}\,|\alpha_1|^2\,\mathrm{sh}\,|\alpha_1|^2\,\mathrm{ch}\,|\alpha_2|^2\,\mathrm{sh}\,|\alpha_2|^2} \sum_{n_1,n_1',n_2,n_2'}^{\infty} \frac{(|\alpha_1|^2)^{2n_1+2n_1'+1}(|\alpha_2|^2)^{2n_2+2n_2'+1}}{(2n_1)!(2n_1'+1)!(2n_2+1)!(2n_2')!} \times \mathrm{e}^{-2\gamma t[(n_1-n_1')^2-(n_1-n_1')]}\mathrm{e}^{-2\gamma t[(n_2-n_2')^2-(n_2'-n_2)]}. \tag{29}$$

Furthermore, choosing $\theta_b = \varphi$, we can write the maximal time-dependent Bell function as

$$\left\langle \hat{B}' \right\rangle_{\max} = 2\sqrt{g(t)^2 + h(t)^2}. \tag{30}$$

Obviously, from Eqs.(28–30) one can easily get the following results

$$\left\langle \hat{B}' \right\rangle_{\max} = 2\sqrt{2}, \quad \text{for} \quad t = 0, \tag{31}$$

$$\left\langle \hat{B}' \right\rangle_{\max} < 2, \quad \text{for} \quad t \to \infty. \tag{32}$$

To understand that the quantum nonlocality heavily depends on the chosen Bell operator, we give the numerical calculations of the value of the Bell functions in Eqs.(21) and (30). On the one hand, we analyse the time evolution by choosing the same parameters ($\alpha_1 = 0.2$, $\alpha_2 = 0.5$ and $\gamma = 1$) in Eqs.(21) and (30). It is clear from Fig.1 that the nonlocality for a given state varies according to the Bell operators for testing quantum nonlocality of the given entangled state.

It can be seen that the nonlocality persists longer when using the Bell operators in Eq.(13) than using the Bell operator based on the observables in Eq.(25). This indicates that the quantum nonlocality heavily

depends on the chosen Bell operator. On the other hand, we reveal the dependence of the nonlocality on the parameter α of the coherent state. The result shows that the value of the maximal Bell's factor $B_{\max}$ in Eq.(21) rapidly decreases with the increase of the value of parameter α, as shown in Fig.2.

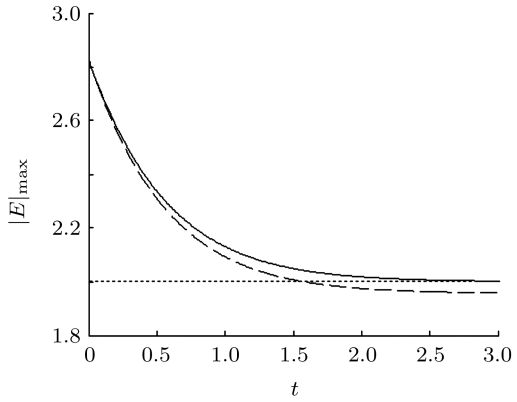


Fig.1. Nonlocality as a function of rescaled time t ($\alpha_1 = 0.2, \alpha_2 = 0.5, \gamma = 1$). We choose Bell operators based on $\hat{s}\,(-)$ and $\hat{s}'\,(- - -)$.

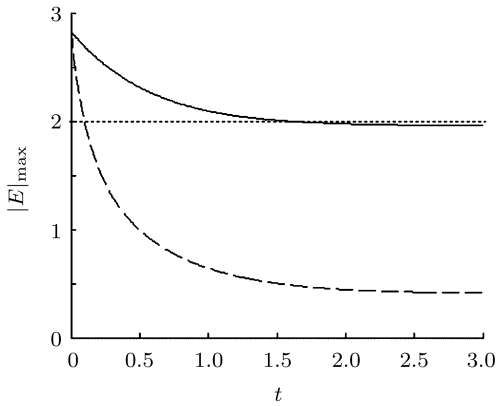


Fig.2. Nonlocality as a function of rescaled time t in dependence on the parameter α for Eq.(30). Solid line represents the dynamical nonlocality when choosing $\alpha_1 = 0.2, \alpha_2 = 0.5$ and $\gamma = 1$. Dotted line represents the dynamical nonlocality when choosing $\alpha_1 = 0.8, \alpha_2 = 2$ and $\gamma = 1$.

However, it is obvious from Fig.3 that the value of the maximal Bell factor $B_{\max}$ in Eq.(30) very slowly decreases with the increase of the value of parameter α.

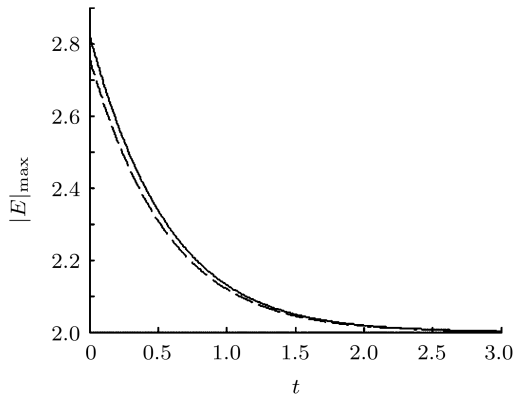


Fig.3. Nonlocality as a function of rescaled time t in dependence on the parameter α for Eq.(21). Solid line represents the dynamical nonlocality when choosing $\alpha_1 = 0.2, \alpha_2 = 0.5$ and $\gamma = 1$. Dotted line represents the dynamical nonlocality when choosing $\alpha_1 = 0.8, \alpha_2 = 2$ and $\gamma = 1$

These analyses of the numerical calculations show that the dynamical nonlocality for the given state in Eq.(12) can be more easily uncovered by the Bell operator in Eq.(13) than by the Bell operator in Eq.(25).

To summarize, we have studied the dynamics of the nonlocality for the ECS in the phase damping model. The time-dependent density operator of the studied system can be solved by the superoperator method. The dynamics of nonlocality for the ECS has also been studied by using different Bell operators. The results of the numerical calculations of the Bell function show that the degree of quantum nonlocality crucially depends not only on the given quantum state but also on the 'Bell operator'. These can be seen from the introduced Figs.1, 2 and 3. It is once again shown that the pseudospin operator provides us with very useful information to study the quantum nonlocality for the quantum entangled systems with continuous variables. The result shows that the quantum nonlocality heavily depends on the chosen Bell operator.

We thank Liu Jin-Zhong and Zhao Bo for useful discussions.

References

[1] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

[2] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[3] Ekert A 1991 *Phys. Rev. Lett.* **67** 91

[4] Knill E, Laflamme R and Milburn G J 2001 *Nature* (London) **409** 46

[5] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[6] Bell J S 1964 *Phys.* **1** 195

[7] Clauser J F, Horne M A, Shimony A and Holt R A 1969 *Phys. Rev. Lett.* **23** 880

[8] Bell J S 1987 *Speaking and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press) p29–38
[9] Reid M D 1989 *Phys. Rev.* A **40** 913
[10] Chen Z B, Pan J W, Hou G and Zhang Y D 2002 *Phys. Rev. Lett.* **88** 040406
[11] Peres A 1992 *Phys. Rev.* A **46** 4413
[12] Banaszek K and Wódkiewicz K 1998 *Phys. Rev.* A **58** 4345
Banaszek K and Wódkiewicz K 1999 *Phys. Rev. Lett.* **82** 2009
[13] Filip R and Mišta L Jr 2002 *Phys. Rev.* A **66** 044309
Gisin N and Peres A 1992 *Phys. Lett.* A **162** 15
[14] Braunstein S L, Mann A, and Revzen M 1992 *Phys. Rev. Lett.* **68** 3259
[15] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer) p91–111
[16] Lu H X, Zhao B, Zhao M S, Guo J, Chen Z B and Zhang Y D 2002 *J. Opt.* B*: Quantum Semiclass Opt.* **4** 411
[17] Lu H X 2007 *Chin. Phys.* **16** 1878
[18] Shanders B C 1992 *Phys. Rev.* A **45** 6811
[19] Wang X G 2002 *J. Phys.* A*: Math: Gen.* **35** 165
[20] Van S J and Hirota 2001 *Phys. Rev.* A **64** 022313
[21] Chen Z B, Hou G and Zhang Y D 2002 *Phys. Rev.* A **65** 032317