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The dispersive properties of an excited-doublet four-level atomic system*

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We have investigated the dispersive properties of excited-doublet four-level atoms interacting with a weak probe field and an intense coupling laser field. We have derived an analytical expression of the dispersion relation for a general excited-doublet four-level atomic system subject to a one-photon detuning. The numerical results demonstrate that for a typical rubidium D1 line configuration, due to the unequal dipole moments for the transitions of each ground state to double excited states, generally there exists no exact dark state in the system. Close to the two-photon resonance, the probe light can be absorbed or gained and propagate in the so-called superluminal form. This system may be used as an optical switch.

Keywords: photon-atom interaction, susceptibility, quantum interference phenomena
PACC: 4250, 3280, 3200

1. Introduction

Quantum interference phenomena in the field of atom–photon interaction have been investigated extensively in the past few years. Electromagnetically induced transparency (EIT),[1−3] which is based on quantum interference to prevent the probe light from being absorbed, is one of the phenomena. It has been extensively studied recently due to its very attractive properties[4−10] and its potential wide applications.[4,11,12] But in a realistic atomic system it is hard to find a real three-level Λ configuration system because there exist many hyperfine and Zeeman states. Therefore, for some cases the atomic system should be treated as a system with more than three levels. The interaction between excited-doublet four-level atoms and laser fields has been investigated for resonant fluorescence spectra,[13] coherent population transfer,[14] superposition of atomic states and phase-dependent nonlinear optics.[15,16] The earlier studies have shown that the dark state exists only when the probe(coupling) Rabi frequencies for the transitions from ground states to two excited states are proportional to a constant.[15] But for rubidium D1 line transitions $|F = 2,\ m_{\rm F} = 2\rangle \leftrightarrow |F' = 1, m_{\rm F'} = 1\rangle$ and $|F = 2,\ m_{\rm F} = 1\rangle \leftrightarrow |F' = 1, m_{\rm F'} = 1\rangle$, this condition cannot be satisfied. The Ref.[17] also considered the influence of the upper excited state on the EIT, but they dealt with the case in which the laser light beams were applied resonant to the lower excited states. In experiment one-photon detuning may exist. Here we investigate the dispersive properties of an excited-doublet four-level atomic system that has a rubidium D1 line and been used in EIT experiments.[7] We found that there occurred a little absorption of the probe light field when the two laser fields were applied near-resonant to the lower excited states. When the two laser fields are tuned in between the two excited states, the absorption and gain are evident. When the two laser fields are tuned far-off resonance to the lower excited states, the absorption is less. Usually this far-off resonant case is so-called Raman process[18,19] and is contrary to the resonant EIT case.

2. Theory

We consider an excited-doublet four-state rubidium atomic system as shown in Fig.1. $|a\rangle$ with an energy $\hbar\omega_a$ and $|b\rangle$ with an energy $\hbar\omega_b$ are the lower level states, which can be the hyperfine states or Zeeman states. $|c\rangle$ with an energy $\hbar\omega_c$ and $|d\rangle$ with an energy $\hbar\omega_d$ are the excited states, which are the hyperfine states. These states can be realized for ^{87}Rb

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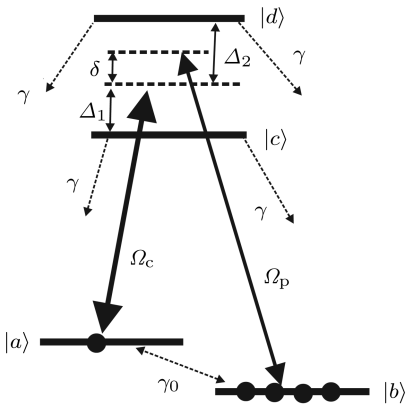


Fig.1. Four-level atomic structure with the upper states $|c\rangle$ and $|d\rangle$ decaying to $|a\rangle$ and $|b\rangle$ each with a decay rate γ and the lower states $|a\rangle$ and $|b\rangle$ relaxing at the same rate γ_0.

D1 line. The weak probe laser field is coupled to the transitions of $|b\rangle \leftrightarrow |c\rangle(|d\rangle)$ while the strong coupling laser field is coupled to the transitions of $|a\rangle \leftrightarrow |c\rangle(|d\rangle)$. The frequencies of the probe field and the coupling field are $\omega_{\rm p}$ and $\omega_{\rm co}$ respectively. We assume that the two excited states decay at identical rates and the relaxation rate of ground states is γ_0. The hamiltonian of the system can be written as

$$\begin{aligned} H = {} & \hbar\Delta_1|c\rangle\langle c| + \hbar\Delta_2|d\rangle\langle d| - \hbar\delta|b\rangle\langle b| \\ & -\hbar(\Omega_{\rm p1}|c\rangle\langle b| + \Omega_{\rm p2}|d\rangle\langle b| + \Omega_{\rm c1}|c\rangle\langle a| \\ & +\Omega_{\rm c2}|d\rangle\langle a|) + {\rm h.c.}, \end{aligned} \tag{1}$$

where $\Omega_{\rm p1} = \wp_{cb}E_{\rm p}/\hbar$, $\Omega_{\rm p2} = \wp_{db}E_{\rm p}/\hbar$, $\Omega_{\rm c1} = \wp_{ca}E_{\rm c}/\hbar$, and $\Omega_{\rm c2} = \wp_{da}E_{\rm c}/\hbar$ are the Rabi frequencies of the probe field and the coupling field. $\wp_{ij}$ are the dipole moments. $E_{\rm p}$ and $E_{\rm c}$ are the amplitudes of probe field and coupling laser field respectively. The detunings are defined as $\Delta_1 = \omega_{ca} - \omega_{\rm co}$, $\Delta_2 = \omega_{da} - \omega_{\rm co}$, and $\delta = \omega_{cb} - \Delta_1 - \omega_{\rm p}$, where ω_{ca}, ω_{da} and ω_{cb} are the frequencies of atomic states transitions $|c\rangle \leftrightarrow |a\rangle, |d\rangle \leftrightarrow |a\rangle$ and $|c\rangle \leftrightarrow |b\rangle$. The equations of motion for the density matrix elements in a rotating frame can be obtained by solving the Liouville equation:

$$\dot\rho_{ij} = -\frac{\gamma_i + \gamma_j}{2}\rho_{ij} - \frac{\rm i}{\hbar}[H,\rho]_{ij}, \tag{2}$$

$$\begin{aligned} \dot\rho_{dd} = {} & -2\gamma\rho_{dd} - {\rm i}\Omega_{\rm p2}\rho_{db} - {\rm i}\Omega_{\rm c2}\rho_{da} \\ & +{\rm i}\Omega^*_{\rm p2}\rho_{bd} + {\rm i}\Omega^*_{\rm c2}\rho_{ad}, \end{aligned} \tag{3}$$

$$\begin{aligned} \dot\rho_{cc} = {} & -2\gamma\rho_{cc} - {\rm i}\Omega_{\rm p1}\rho_{cb} - {\rm i}\Omega_{\rm c1}\rho_{ca} \\ & +{\rm i}\Omega^*_{\rm p1}\rho_{bc} + {\rm i}\Omega^*_{\rm c1}\rho_{ac}, \end{aligned} \tag{4}$$

$$\begin{aligned} \dot\rho_{bb} = {} & \gamma_0(\rho_{aa} - \rho_{bb}) + \gamma(\rho_{cc} + \rho_{dd}) \\ & +{\rm i}\Omega_{\rm p1}\rho_{cb} + {\rm i}\Omega_{\rm p2}\rho_{db} \\ & -{\rm i}\Omega^*_{\rm p1}\rho_{bc} - {\rm i}\Omega^*_{\rm p2}\rho_{bd}, \end{aligned} \tag{5}$$

$$\begin{aligned} \dot\rho_{aa} = {} & \gamma_0(\rho_{bb} - \rho_{aa}) + \gamma(\rho_{cc} + \rho_{dd}) \\ & +{\rm i}\Omega_{\rm c1}\rho_{ca} + {\rm i}\Omega_{\rm c2}\rho_{da} \\ & -{\rm i}\Omega^*_{\rm c1}\rho_{ac} - {\rm i}\Omega^*_{\rm c2}\rho_{ad}, \end{aligned} \tag{6}$$

$$\begin{aligned} \dot\rho_{dc} = {} & -\Gamma_{dc}\rho_{dc} - {\rm i}\Omega_{\rm p1}\rho_{db} - {\rm i}\Omega_{\rm c1}\rho_{da} \\ & +{\rm i}\Omega^*_{\rm p2}\rho_{bc} + {\rm i}\Omega^*_{\rm c2}\rho_{ac}, \end{aligned} \tag{7}$$

$$\begin{aligned} \dot\rho_{da} = {} & -\Gamma_{da}\rho_{da} - {\rm i}\Omega^*_{\rm c2}(\rho_{dd} - \rho_{aa}) \\ & +{\rm i}\Omega^*_{\rm p2}\rho_{ba} - {\rm i}\Omega^*_{\rm c1}\rho_{dc}, \end{aligned} \tag{8}$$

$$\begin{aligned} \dot\rho_{db} = {} & -\Gamma_{db}\rho_{db} - {\rm i}\Omega^*_{\rm p2}(\rho_{dd} - \rho_{bb}) \\ & +{\rm i}\Omega^*_{\rm c2}\rho_{ab} - {\rm i}\Omega^*_{\rm p1}\rho_{dc}, \end{aligned} \tag{9}$$

$$\begin{aligned} \dot\rho_{ca} = {} & -\Gamma_{ca}\rho_{ca} - {\rm i}\Omega^*_{\rm c1}(\rho_{cc} - \rho_{aa}) \\ & +{\rm i}\Omega^*_{\rm p1}\rho_{ba} - {\rm i}\Omega^*_{\rm c2}\rho_{cd}, \end{aligned} \tag{10}$$

$$\begin{aligned} \dot\rho_{cb} = {} & -\Gamma_{cb}\rho_{cb} - {\rm i}\Omega^*_{\rm p1}(\rho_{cc} - \rho_{bb}) \\ & +{\rm i}\Omega^*_{\rm c1}\rho_{ab} - {\rm i}\Omega^*_{\rm p2}\rho_{cd}, \end{aligned} \tag{11}$$

and

$$\begin{aligned} \dot\rho_{ab} = {} & -\Gamma_{ab}\rho_{ab} + {\rm i}\Omega_{\rm c1}\rho_{cb} + {\rm i}\Omega_{\rm c2}\rho_{db} \\ & -{\rm i}\Omega^*_{\rm p1}\rho_{ac} - {\rm i}\Omega^*_{\rm p2}\rho_{ad}. \end{aligned} \tag{12}$$

The generalized decay rates are defined as

$$\Gamma_{dc} = 2\gamma - {\rm i}(\Delta_1 - \Delta_2), \tag{13}$$

$$\Gamma_{da} = (\gamma + \gamma_0/2) + {\rm i}\Delta_2, \tag{14}$$

$$\Gamma_{db} = (\gamma + \gamma_0/2) + {\rm i}(\Delta_2 + \delta), \tag{15}$$

$$\Gamma_{ca} = (\gamma + \gamma_0/2) + {\rm i}\Delta_1, \tag{16}$$

$$\Gamma_{cb} = (\gamma + \gamma_0/2) + {\rm i}(\Delta_1 + \delta), \tag{17}$$

and

$$\Gamma_{ab} = \gamma_0 + {\rm i}\delta. \tag{18}$$

For a stead state, under the condition of first-order approximation of the probe field, we can obtain

$$\rho_{dc} = -\frac{\Omega_{\rm c1}\Omega^*_{\rm c2}}{A\Gamma_{dc}}\left(\frac{\rho^0_{cc} - \rho^0_{aa}}{\Gamma_{ac}} + \frac{\rho^0_{dd} - \rho^0_{aa}}{\Gamma_{da}}\right), \tag{19}$$

$$\rho_{cb} = \frac{C - D}{B}, \tag{20}$$

and

$$\rho_{db} = \frac{-\Gamma_{ab}[{\rm i}\Omega^*_{\rm p2}(\rho^0_{dd} - \rho^0_{bb}) + {\rm i}\Omega^*_{\rm p1}\rho_{dc}] - \Omega_{\rm c1}\Omega_{\rm c2}(C - D)/B}{\Gamma_{ab}\Gamma_{db} + |\Omega_{\rm c2}|^2}, \tag{21}$$

where

$$B = \left(\Gamma_{ab}\Gamma_{db} + |\Omega_{c2}|^2\right) \times \left(\Gamma_{ab}\Gamma_{cb} + |\Omega_{c1}|^2\right) - |\Omega_{c1}|^2|\Omega_{c2}|^2, \quad (22)$$

$$C = -\mathrm{i}\Gamma_{ab}\left(\Gamma_{ab}\Gamma_{db} + |\Omega_{c2}|^2\right) \times \left[\Omega_{p1}^*\left(\rho_{cc}^0 - \rho_{bb}^0\right) + \Omega_{p2}^*\rho_{cd}\right], \quad (23)$$

and

$$D = \mathrm{i}\Gamma_{ab}\Omega_{c1}^*\Omega_{c2}\left[\Omega_{p2}^*\left(\rho_{dd}^0 - \rho_{bb}^0\right) + \Omega_{p1}^*\rho_{dc}\right] \quad (24)$$

with ρ_{ii}^0 being the populations in the zero-order approximation. The susceptibility is given as

$$\chi = \kappa\left(\frac{\rho_{db}}{\Omega_{p2}} + \frac{\rho_{cb}}{\Omega_{p1}}\right), \quad (25)$$

where κ is given by $\kappa \equiv (3/8\pi)N\gamma_r\lambda^3$ for the atomic number density N and the wavelength λ of the probe field.

If initially all the populations of atoms are pumped into the state $|b\rangle$, then we have $\rho_{aa}^0 = \rho_{cc}^0 = \rho_{dd}^0 = 0$, $\rho_{bb}^0 = 1$ and $\rho_{dc} = 0$. Set $\dfrac{\Omega_{p2}}{\Omega_{p1}} = \alpha$ and $\dfrac{\Omega_{c2}}{\Omega_{c1}} = \beta$, and assume all the Rabi frequencies to be real, then susceptibility will be able to be written as

$$\chi = \mathrm{i}\kappa\left\{\frac{\Gamma_{ab}}{\Gamma_{ab}\Gamma_{db} + \beta^2\Omega_{c1}^2} + \frac{\left(\Gamma_{ab}\Gamma_{db} + \beta^2\Omega_{c1}^2 + \alpha\beta\Omega_{c1}^2\right)}{\Omega_{c1}^2(\Gamma_{db} + \beta^2\Gamma_{cb}) + \Gamma_{db}\Gamma_{cb}}\left[1 - \frac{\beta\Omega_{c1}^2}{\alpha(\Gamma_{ab}\Gamma_{db} + \beta^2\Omega_{c1}^2)}\right]\right\}, \quad (26)$$

which clearly shows that the dispersion relation is closely related to the dipole moments of the transitions. For example, when $\alpha = \beta$, the absorption of the system vanishes, which has been investigated already.[15] But for a typical EIT experimental system,[7] the condition $\alpha = \beta$ cannot be satisfied. In that experiment the coupling field is coupled to the transitions of $|F_g = 2, m_F = 0\rangle \leftrightarrow |F_e = 1, m_F = 1\rangle$ and $|F_g = 2, m_F = 0\rangle \leftrightarrow |F_e = 2, m_F = 1\rangle$. The probe field is coupled to the transitions of $|F_g = 2, m_F = 2\rangle \leftrightarrow |F_e = 1, m_F = 1\rangle$ and $|F_g = 2, m_F = 2\rangle \leftrightarrow |F_e = 2, m_F = 1\rangle$. The Clebsch–Gordon coefficients are $\sqrt{1/4}$, $\sqrt{1/12}$, $\sqrt{1/2}$ and $-\sqrt{1/6}$ corresponding to the transitions $|F_g = 2, m_F = 0\rangle \leftrightarrow |F_e = 1, m_F = 1\rangle$, $|F_g = 2, m_F = 0\rangle \leftrightarrow |F_e = 2, m_F = 1\rangle$, $|F_g = 2, m_F = 2\rangle \leftrightarrow |F_e = 1, m_F = 1\rangle$ and $|F_g = 2, m_F = 2\rangle \leftrightarrow |F_e = 2, m_F = 1\rangle$. So according to the dipole moments the condition of Eq.(10) is not satisfied. Generally speaking, in this system there exists no exact dark state. But when all the optical fields are applied far-off resonant to the state $|F_e = 2, m_F = 1\rangle$ or $|F_e = 1, m_F = 1\rangle$, the excited states can be eliminated and there remains a dark state in the system. Otherwise atomic system will absorb the probe light.

3. Numerical results

With $\alpha = -\sqrt{1/3}$, $\beta = \sqrt{3}$, and different one-photon detunings, we numerically calculate the susceptibility and the absorption. The computed results are shown in Figs.2–7. All the parameters are scaled with the decay rates of the excited states which are assumed to be equal. From Fig.2 we see that due to the excited state $|d\rangle$ present, the absorption is not zero when the two laser fields are applied resonant to the state $|c\rangle$, but the absorption in this case is very small.

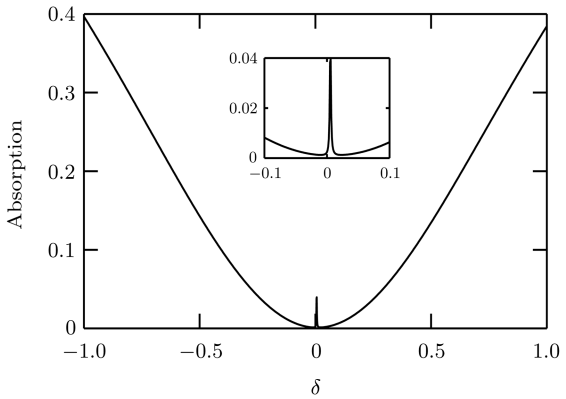


Fig.2. Absorption of the probe field when the probe field is resonant to the state $|c\rangle$, with $\Omega_{c1} = 0.5$, $\Delta_1 = 0$, $\Delta_2 = 136.0$ and $\gamma_0 = 0.001$.

When the two laser fields are tuned at the centre of the two excited states, the absorption is very large and this case can be considered as a double Λ state configuration which has two transparent windows as seen from Fig.3. Figures 4 and 5 clearly show that the susceptibility changes rapidly close to the two-photon resonant point. That means near the two-photon resonant point the index of refraction will be enhanced

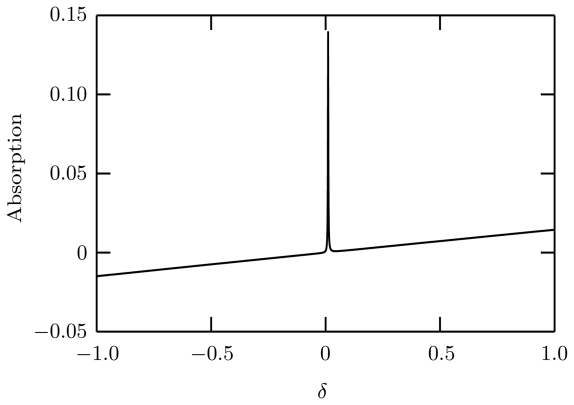


Fig.3. Absorption of the probe field when the two laser fields are tuned at the centre of the two excited states, with $\Omega_{c1} = 0.5$, $\Delta_1 = 68.0$, $\Delta_2 = 68.0$ and $\gamma_0 = 0.001$.

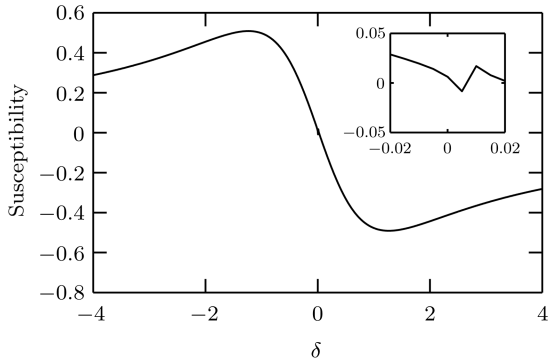


Fig.4. Susceptibility of the system when the probe field is resonant to the state $|c\rangle$, with $\Omega_{c1} = 0.5$, $\Delta_1 = 0$, $\Delta_2 = 136.0$ and $\gamma_0 = 0.001$.

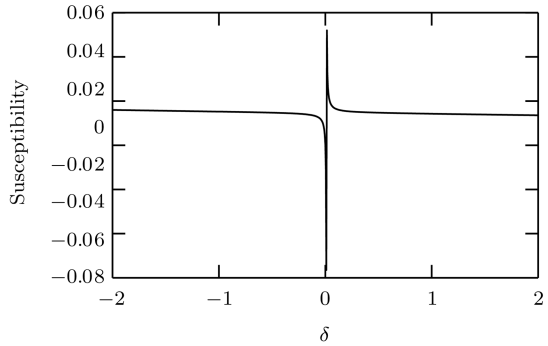


Fig.5. Susceptibility of the system when the two laser fields are tuned at the centre of the two excited states, with $\Omega_{c1} = 0.5$, $\Delta_1 = 68.0$, $\Delta_2 = 68.0$ and $\gamma_0 = 0.001$.

greatly, but the sign of the index of refraction is negative, which shows that the probe light propagation in the so-called superluminal form. Figures 6 and 7 show that the absorption and the susceptibility vary with the one-photon detuning when the two-photon transition is resonant. When the two laser fields are applied close to the state $|c\rangle$ or $|d\rangle$, there are the absorptions and gains of the probe field. This kind of gain phenomenon was also found in Ref.[15]. According to Fig.7 when the two-photon transition is resonant and close to one-photon resonance, there occur subluminal and so-called superluminal phenomena, that is, there are slow light and fast light near the one-photon resonance. Also the so-called superluminal propagation is companied with absorption. However due to the transition dipole moments unequal, so the absorption and gain as well as the susceptibility are not equal. Therefore in the previous experimental researches on EIT by exploiting rubidium D1 line, usually the probe field and the coupling laser field are used with red detuning far off resonance to the lower excited states. It can be seen from the numerical results that when we change the frequencies of the lasers the atomic system can be transparent or absorptive to the probe laser light. Therefore this system can be used as an optical switch and can be realized in experiment by using a modulator to change the frequencies of laser lights.

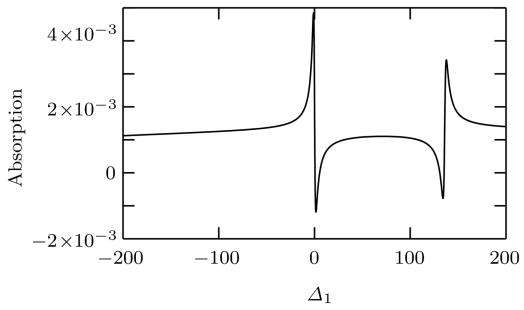


Fig.6. Absorption of the probe field when the one-photon detuning is varying and two-photon transition is resonant, with $\Omega_{c1} = 0.5$, $\Delta_2 = 136.0 - \Delta_1$ and $\gamma_0 = 0.001$.

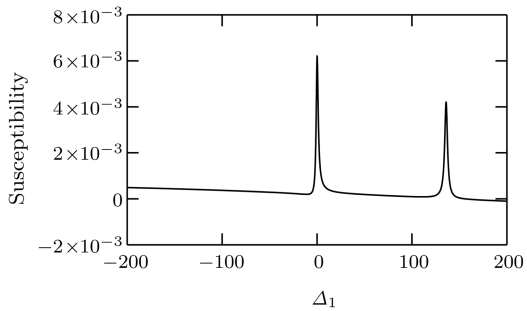


Fig.7. Susceptibility of the system when the one-photon detuning is varying and two-photon transition is resonant, with $\Omega_{c1} = 0.5$, $\Delta_2 = 136.0 - \Delta_1$ and $\gamma_0 = 0.001$.

4. Conclusions

We have closely investigate the spectra of a realistic excited-doublet four-level atomic system interacting with a probe field and a coupling field. Especially we check the rubidium atomic D1 line tran-

sition between two excited states $|F = 1, m_{\rm F} = 1\rangle$, $|F = 2, m_{\rm F} = 1\rangle$ and lower states $|F = 2, m_{\rm F} = 0\rangle$, $|F = 2, m_{\rm F} = 2\rangle$. The analytical and numerical results clearly show there is no exact dark state due to the unequal Rabi frequencies for the probe field and the coupling laser field interacting with the two excited states. Close to the two-photon resonant point, the probe field can be absorbed or amplified and the so-called superluminal propagation of the probe light is companied with an absorption. When the two laser fields are tuned far-off resonance to the excited state, dark state will appear and the system can have the transparent properties, but the process is called Raman process. By using a modulator to change the frequencies of the laser fields, the system can be used as an optical switch.

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