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Finite-dimensional even and odd nonlinear pair coherent states and their some nonclassical properties*

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In this paper a new class of finite-dimensional even and odd nonlinear pair coherent states (EONLPCSs), which can be realized via operating the superposed evolution operators $D_{\pm}(\tau)$ on the state $|q,0\rangle$, is constructed, then their orthonormalized property, completeness relations and some nonclassical properties are discussed. It is shown that the finite-dimensional EONLPCSs possess normalization and completeness relations. Moreover, the finite-dimensional EONLPCSs exhibit remarkably different sub-Poissonian distributions and phase probability distributions for different values of parameters q, η and ξ.

Keywords: finite-dimensional even and odd nonlinear pair coherent state, sub-Poissonian distribution, phase probability distribution

PACC: 4250, 0365

1. Introduction

Pair coherent state (PCS) is an important type of correlated two-mode state, which involves strong entanglement and exhibits prominent nonclassical properties such as various forms of squeezing, many-photon antibunching effect and phase properties,[1,2] as well as Wigner functions and tomogram functions.[3] In the Fock representation the PCS[1−4] is expressed as

$$|\alpha,q\rangle=\sum_{n=0}^{\infty}\frac{N(n,q)\alpha^n}{\sqrt{n!(n+q)!}}\,|n+q,n\rangle, \tag{1}$$

where the parameter $\alpha = r\exp(\mathrm{i}\varphi)$ is complex, q is an integer, and the normalization constant $N(n,q)$ is expressed as

$$N(n,q)=[(\mathrm{i}|\alpha|)^{-q}J_q(\mathrm{i}2|\alpha|)]^{-1/2} \tag{2}$$

with $J_q(x)$ being the q-order Bessel function. Theoretically, the PCSs are eigenstates of the pair annihilation operator (a_1a_2) and the number difference operator $(a_1^+a_1-a_2^+a_2)$, where a_1 and a_2 are the annihilation operators of the light field modes. The states obey the following eigenvalue equations

$$a_1a_2\,|\alpha,q\rangle=\alpha\,|\alpha,q\rangle\,,$$

and

$$(a_1^+a_1-a_2^+a_2)\,|\alpha,q\rangle=q\,|\alpha,q\rangle\,. \tag{3}$$

In experimental realization, the PCS can be generated via the competition between four-wave mixing and two-photon absorption in a nonlinear medium.[1] Also the vibrational PCS can be obtained via the motion of a trapped ion in a two-dimensional trap.[4] Recently, as an important extension of PCS in the finite-dimensional Hilbert space, Obada and Khalil[5] and Khalil[6] have introduced the finite-dimensional PCS and the finite-dimensional nonlinear PCS (NLPCS) respectively, and then new schemes for generating two new classes of the finite-dimensional two-mode states of the vibrational motion of an ion in a two-dimensional trap have been demonstrated. The experimental realization of such nonclassical states is of practical importance. Moreover, their some nonclassical properties have been investigated and the differences in phenomenon between two classes of finite-dimensional PCSs[5,6] and the standard PCSs[4] and NLPCSs[7] have also been found.

In the present paper, on the basis of the concept of superposition principle in quantum optics

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and the work in Ref.[5] we introduce a new kind of finite-dimensional even and odd pair coherent states (EONLPCSs) and investigate their orthonormalized property, completeness relations and some nonclassical properties such as sub-Poissonian distribution and phase probability distribution in detail. Motivation to pay attention to the new finite-dimensional EONLPCSs and their some nonclassical properties lies in the following aspects: 1) The superposed states possess stronger entanglement and more remarkable nonclassical properties. We know that the nonclassically correlated light field plays an important role in quantum information theory both fundamentally and practically, and quantum entanglement is regarded as a physical resource in quantum communication. Besides, the EONLPCSs can more control their behaviour by employing entangled property and interference property. So for some properties of the finite-dimensional EONLPCSs it is possible to have some important applications in the fields of quantum optics and quantum information. 2) The finite-dimensional EONLPCSs may also be generated in the vibrational motion of a trapped ion in a two-dimensional harmonic potential experimentally.[5] Therefore, in theory it is worth constructing such a new type of correlated two-mode states associated with PCS and studying their nonclassical properties.

2. Constructing the finite-dimensional EONLPCSs

2.1. The orthonormalized and completeness relations

Recently, Khalil[6] has constructed the finite-dimensional NLPCSs, which are eigenstates of the pair operators

$$\left(f_1(n_\mathrm{a})a^+ f_2(n_\mathrm{b})b^+ \frac{\xi^{q+1}\left(a\dfrac{1}{f_1(n_\mathrm{a})f_2(n_\mathrm{b})}b^+\right)^q}{(q!)^2} \right)$$

and the sum of the photon number operators for the two modes $(a^+a + b^+b)$, i.e.

$$\left(f_1(n_\mathrm{a})a^+ f_2(n_\mathrm{b})b^+ \frac{\xi^{q+1}\left(a\dfrac{1}{f_1(n_\mathrm{a})f_2(n_\mathrm{b})}b^+\right)^q}{(q!)^2} \right) \times |\xi, q; f\rangle = \xi\,|\xi, q; f\rangle\,, \tag{4}$$

and

$$(a^+a + b^+b)\,|\xi, q; f\rangle = q\,|\xi, q; f\rangle\,, \tag{5}$$

where $n_x = x^+x$ $(x = a,\ b)$ is the number operator. Then the state $|\xi,\ q;\ f\rangle$ is expressed as

$$|\xi, q; f\rangle = \sum_{n=0}^{q} N(n, q; f)\xi^n \sqrt{\frac{(q-n)!}{n!q!}}\,\frac{f_1(q-n)!}{f_1(q)!f_2(n)!} \times |q-n, n\rangle\,. \tag{6}$$

With the aid of the definition of the finite-dimensional NLPCS, the new type of finite-dimensional EONLPCSs is defined as the symmetric (or antisymmetric) superposition of the finite-dimensional NLPCS, which takes the following form:

$$\begin{aligned} |\xi, q; f\rangle_\pm &= |\xi, q; f\rangle \pm |-\xi, q; f\rangle \\ &= \sum_{n=0}^{q} N_\pm(n, q; f)\xi^n \sqrt{\frac{(q-n)!}{n!q!}}\,\frac{f_1(q-n)!}{f_1(q)!f_2(n)!} \\ &\quad \times [1 \pm (-1)^n]\,|q-n, n\rangle\,, \end{aligned} \tag{7}$$

where the subscripts "±" correspond to the finite-dimensional even and odd NLPCS according to the parity relation $|-\xi, q; f\rangle_\pm = \pm\,|\xi, q; f\rangle_\pm$, and the normalization factor $N_\pm(n, q; f)$ is

$$N_\pm(n, q; f) = \left\{ 2\sum_{n=0}^{q} \frac{|\xi|^{2n}\,(q-n)!}{n!q!} \left(\frac{f_1(q-n)!}{f_1(q)!f_2(n)!} \right)^2 [1 \pm (-1)^n] \right\}^{-1/2}. \tag{8}$$

Set $f(n) = 1$ in expression (7), then the new type of finite-dimensional EONLPCS will become the form of the usual even and odd PCS (EOPCS) in Ref.[8]. Using expressions (7) and (8), it is easy to prove that two different finite-dimensional EONLPCSs are overlapped into

$$
\begin{aligned}
{}_{\pm}\langle \xi, q; f \,|\xi', q; f\rangle_{\pm} =& \sum_{n,m=0}^{q} N_{\pm}(n,q;f)N_{\pm}(m,q;f)\xi^{*n}\xi'^{m}\sqrt{\frac{(q-n)!(q-m)!}{n!q!m!q!}} \\
& \times \frac{f_1(q-n)!}{f_1(q)!f_2(n)!}\frac{f_1(q-m)!}{f_1(q)!f_2(m)!}[1\pm(-1)^n][1\pm(-1)^m]\langle q-n, n\,|q-m, m\rangle \\
=& \sum_{n=0}^{q}\frac{N_{\pm}^2(n,q;f)\xi^{*n}\xi'^{n}(q-n)!}{n!q!}\left(\frac{f_1(q-n)!}{f_1(q)!f_2(n)!}\right)^2[1\pm(-1)^n]^2. \qquad (9)
\end{aligned}
$$

Let $\xi' = \xi$, in expression (9) then we shall have ${}_{\pm}\langle \xi, q; f\,|\xi', q; f\rangle_{\pm} = 1$, namely the finite-dimensional EONLPCSs are normalizable; let $\xi' \neq \xi$, then we shall obtain ${}_{\pm}\langle \xi, q\,; f\,|\xi', q; f\rangle_{\pm} \neq 0$, which implies that two different finite-dimensional EONLPCSs are not orthogonal to each other. Moreover, we can use the density matrix method to obtain the completeness relations of the finite-dimensional EONLPCSs. Using expression (7) we can have the probability distribution with one can find the state $|q-n, n\rangle$ in the finite-dimensional EONLPCS, i.e.

$$
\begin{aligned}
P_n(\xi, q; f) =& \left|\langle q-n, n|\left(|\xi, q; f\rangle_{+} + |\xi, q; f\rangle_{-}\right)\right|^2 \\
=& 4N_{+}^2(m,q;f)\left(\frac{f_1(q-2m)!}{f_1(q)!f_2(2m)!}\right)^2\frac{(q-2m)!}{(2m)!q!}r^{4m} \\
& + 4N_{-}^2(m,q;f)\left(\frac{f_1(q-2m-1)!}{f_1(q)!f_2(2m+1)!}\right)^2\frac{(q-2m-1)!}{(2m+1)!q!}r^{4m+2} \\
& + 8\frac{N_{+}(m,q;f)f_1(q-2m)!}{f_1(q)!f_2(2m)!}\frac{N_{-}(m,q;f)f_1(q-2m-1)!}{f_1(q)!f_2(2m+1)!}\sqrt{\frac{(q-2m)!}{(2m)!q!}\frac{(q-2m-1)!}{(2m+1)!q!}}r^{4m}\cos\theta. \qquad (10)
\end{aligned}
$$

If we define the density matrix operator ρ_n of the state $|q-n, n\rangle$ as

$$
\rho_n = \sum_{n=0}^{q} P_n(q; f)\,|q-n, n\rangle\langle q-n, n|\,, \qquad (11)
$$

where $P_n(q; f) = \iint P_n(\xi, q; f)\mathrm{d}^2\xi$, with $\mathrm{d}^2\xi = r\mathrm{d}r\mathrm{d}\theta$. So we have

$$
\begin{aligned}
\rho_n =& 8\pi N_{+}^2(m,q;f)\sum_{m=0}^{q}\left(\frac{f_1(q-2m)!}{f_1(q)!f_2(2m)!}\right)^2\frac{(q-2m)!}{(2m)!q!}\int_0^{\infty}\mathrm{d}r r^{4m+1}\,|q-2m, 2m\rangle\langle q-2m, 2m| \\
& + 8\pi N_{-}^2(m,q;f)\sum_{m=0}^{q}\left(\frac{f_1(q-2m-1)!}{f_1(q)!f_2(2m+1)!}\right)^2\frac{(q-2m-1)!}{(2m+1)!q!} \\
& \times\int_0^{\infty}\mathrm{d}r r^{4m+3}\,|q-2m-1, 2m+1\rangle\langle q-2m-1, 2m+1|\,. \qquad (12)
\end{aligned}
$$

Since

$$
\begin{aligned}
& \rho_n^{-1}\iint \mathrm{d}^2\xi\left(|\xi, q; f\rangle_{+\,+}\langle \xi, q; f| + |\xi, q; f\rangle_{-\,-}\langle \xi, q; f|\right) \\
=& \rho_n^{-1}\left[8\pi N_{+}^2(n,q;f)\sum_{m,n=0}^{q}\left(\frac{f_1(q-2m)!}{f_1(q)!f_2(2m)!}\right)^2\frac{(q-2m)!}{(2m)!q!}\int \mathrm{d}r r^{4m+1}\,|q-2n, 2n\rangle\langle q-2m, 2m|\right. \\
& \left. + 8\pi N_{-}^2(n,q;f)\sum_{m,n=0}^{q}\left(\frac{f_1(q-2m-1)!}{f_1(q)!f_2(2m)!}\right)^2\frac{(q-2m-1)!}{(2m+1)!q!}\int \mathrm{d}r r^{4m+3}\,|q-2n-1, 2n\rangle\langle q-2m-1, 2m|\right] \\
=& 1, \qquad (13)
\end{aligned}
$$

thus we can see that the completeness relations of the finite-dimensional EONLPCSs are satisfied. Therefore, these states are capable of constructing a complete finite-dimensional Hilbert space.

2.2. SU(2) realization of the finite-dimensional EONLPCSs

For this purpose, we construct the following operators J_x, J_y and J_z, i.e.,

$$J_x = \frac{\frac{1}{n_{\rm a}+1}a\frac{1}{f_1(n_{\rm a})}\frac{1}{f_2(n_{\rm b})}b^+ + bf_2(n_{\rm b})f_1(n_{\rm a})a^+(n_{\rm a}+1)}{2}, \tag{14}$$

$$J_y = \frac{\frac{1}{n_{\rm a}+1}a\frac{1}{f_1(n_{\rm a})}\frac{1}{f_2(n_{\rm b})}b^+ - bf_2(n_{\rm b})f_1(n_{\rm a})a^+(n_{\rm a}+1)}{2i}, \tag{15}$$

and

$$J_z = \frac{n_{\rm b}-n_{\rm a}}{2}, \tag{16}$$

where the operators satisfy the commutation relations

$$[J_x, J_y] = iJ_z,\ [J_y, J_z] = iJ_x,$$

and

$$[J_z, J_x] = iJ_y. \tag{17}$$

In terms of the operators J_i $(i = x, y, z)$, we choose the following Bose operator to realize the SU(2) generators,

$$J_+ = \frac{1}{n_{\rm a}+1}a\frac{1}{f_1(n_{\rm a})}\frac{1}{f_2(n_{\rm b})}b^+,$$

$$J_- = bf_2(n_{\rm b})f_1(n_{\rm a})a^+(n_{\rm a}+1),$$

and

$$J_z = \frac{1}{2}(b^+b - a^+a), \tag{18}$$

then the operators $J_\pm$ and J_z satisfy the SU(2) commutative relations

$$[J_+, J_-] = 2\ J_z, \quad \text{and} \quad [J_z, J_\pm] = \pm J_\pm. \tag{19}$$

Correspondingly, the Casimir operator is

$$C = J_z^2 + \frac{1}{2}(J_+J_- + J_-J_+), \tag{20}$$

which commutes with all the generators J_i of the SU(2) Lie algebra. Furthermore, from expressions (14)–(16) and (18) we notice that J_+ is not the Hermitian conjugate of J_- because the operators J_x and J_y are not Hermitian operators, which is different from the SU(2) angular generators introduced by Schwinger.[9] Then we construct a unitary evolution operator

$$\begin{aligned} D(\tau) =& \exp(\xi J_+ - \xi^* J_-) \\ =& \exp(\tau J_+)\exp[J_z \ln(1+|\tau|^2)]\exp(-\tau^* J_-) \end{aligned}$$

with

$$\tau = \frac{\xi}{|\xi|}\tanh|\xi|. \tag{21}$$

Operating the unitary operator $D(\tau)$ on the state $|q,0\rangle$, we have

$$\begin{aligned} D(\tau)|q,0\rangle =& (1+|\tau|^2)^{-q/2}\sum_{n=0}^{\infty}\frac{\tau^n}{n!}\left(\frac{1}{n_{\rm a}+1}a\frac{1}{f_1(n_{\rm a})}\frac{1}{f_2(n_{\rm b})}b^+\right)^n|q,0\rangle \\ =& (1+|\tau|^2)^{-q/2}\sum_{n=0}^{q}\tau^n\sqrt{\frac{(q-n)!}{q!n!}}\frac{f_1(q-n)!}{f_1(q)!f_2(n)!}|q-n,n\rangle, \end{aligned} \tag{22}$$

which is just the finite-dimensional PCS given in expression (6). Applying the superposed evolution operators $D_\pm(\tau) = D(\tau) \pm D(-\tau)$ on the state $|q,0\rangle$, we obtain the finite-dimensional EOPCSs expressed in the following form:

$$D_{\pm}(\tau)\,|q,0\rangle = (1+|\tau|^2)^{-q/2}\sum_{n=0}^{\infty}\frac{[1\pm(-1)^n]\tau^n}{n!}\left(\frac{1}{n_{\rm a}+1}a\frac{1}{f_1(n_{\rm a})}\frac{1}{f_2(n_{\rm b})}b^{+}\right)^n|q,0\rangle$$
$$= (1+|\tau|^2)^{-q/2}\sum_{n=0}^{q}[1\pm(-1)^n]\tau^n\sqrt{\frac{(q-n)!}{q!n!}}\frac{f_1(q-n)!}{f_1(q)!f_2(n)!}\,|q-n,n\rangle\,. \tag{23}$$

3. Some nonclassical properties of the finite-dimensional EONLPCSs

In this section, we shall investigate the influence of the parameters η, q and ξ on the nonclassical properties of the modes, in which the sub-Poissonian distribution and the phase distribution are discussed in detail. However, we find that the nonclassical properties of the finite-dimensional EONLPCSs are related to the form of the nonlinearity function $f(n)$. Therefore, we shall specify the form of $f(n)$ before the nonclassical properties are investigated. Here we continue to choose the nonlinearity function $f(n)$[10−12] given by

$$f(n) = L_n^1(\eta^2)[(n+1)L_n^0(\eta^2)]^{-1}, \tag{24}$$

where η is the Lamb–Dicke parameter and $L_n^m(\eta^2)$ is an associated Laguerre polynomials. Owing to the fact that the function $f(n)$ is usually used to describe the motion of a trapped ion, it is of very high experimental interest to study the sub-Poissonian distributions and the phase distributions of the states $|\xi,q;f\rangle_{\pm}$.

3.1. Sub-Poissonian distributions

In this subsection we shall investigate the sub-Poissonian distributions for the finite-dimensional EONLPCSs. The distribution can be measured by using photon detectors based on photoelectric effect.[5,6] For a two-mode light field, the normalized second-order correlation function for the mode i satisfies[12]

$$g_i^{(2)}(0) = \frac{\langle n_i(n_i-1)\rangle}{\langle n_i\rangle^2},\ (i=1,2). \tag{25}$$

If $g_i^{(2)}(0) < 1$ (> 1), then the light field exhibits sub(super)-Poissonian distribution in the mode i; if $g_i^{(2)}(0) = 1$ the distribution is called Poissonian; if $g_i^{(2)}(0) = 2$ (> 2) the distribution is referred to as thermal (super-thermal). Using the numerical computation method we have plotted the functions $g_i^{(2)}(0)$ $(i=1,2)$ for the states $|\xi,q;f\rangle_{\pm}$ against the parameter $|\xi|$, with η fixed and values of q being different, in Figs.1 and 2.

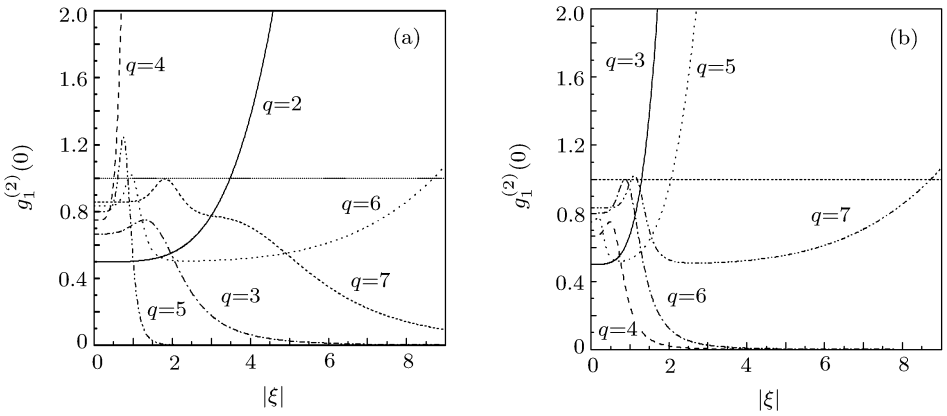


Fig.1. The second-order correlation function $g_1^{(2)}(0)$ against $|\xi|$ for $\eta^2 = 0.7$ and different values of q, where panel (a) is for the finite-dimensional even NLPCS and panel (b) is for the finite-dimensional odd NLPCS.

From Fig.1 it is clear that the variation of the function $g_1^{(2)}(0)$ of the finite-dimensional EONLPCS with parameter $|\xi|$ depends on the parity of the parameter q. The finite-dimensional even (or odd) NLPCS always exhibits sub-Poissonian distribution for mode 1 in the whole range of $|\xi|$ when the value of q is taken as a small odd (or even) number. As the odd (or even) number q increases, the function $g_1^{(2)}(0)$ of the finite-dimensional even (or odd) NLPCS has an increasing trend, and so for a sufficiently large value of q the states appear as the super-Poissonian distribution in a short interval of $|\xi|$. According to a comparison between Eq.1(a) and Fig.1(b), we also find that, when the value of q is taken as an even (or odd) number, the finite-dimensional even (or odd) NLPCS exhibits a sub-Poissonian distribution for a lower value of $|\xi|$ and shows super-Poissonian (thermal) behaviour for a larger value of $|\xi|$. And in principle we note that the sub-Poissonian distribution interval increases and super-Poissonian (thermal) distribution interval decreases with the value of q increasing.

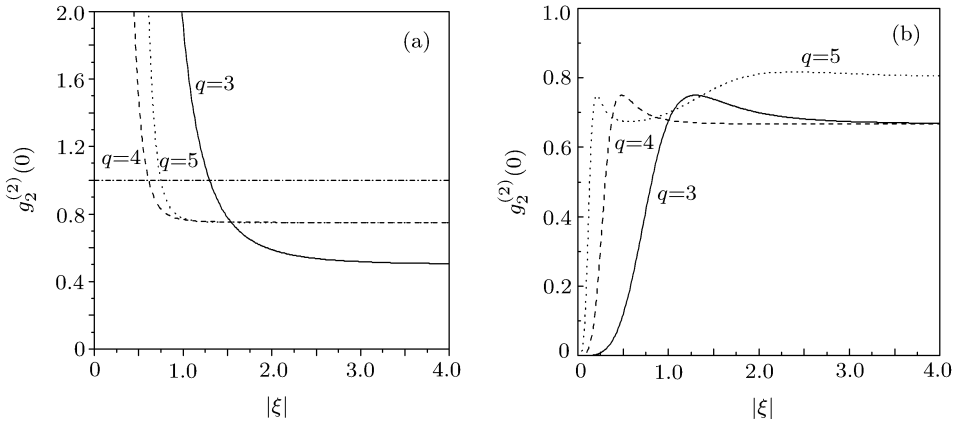


Fig.2. The second-order correlation function $g_2^{(2)}(0)$ against $|\xi|$ for $\eta^2 = 0.7$ and different values of q, where panel (a) is for the finite-dimensional even NLPCS and panel (b) is for the finite-dimensional odd NLPCS.

From Fig.2 it is seen that the variation of the function $g_2^{(2)}(0)$ of the finite-dimensional EONLPCS with parameter $|\xi|$ is not related to the parity of the parameter q. For different values of q, the finite-dimensional even NLPCS exhibits the sub-Poissonian distribution and the super-Poissonian (thermal) distribution in different ranges of $|\xi|$ in the two modes. The sub-Poissonian distribution interval decreases and subsequently increases with the value of q increasing. However, for small values of q the finite-dimensional odd NLPCS always exhibits the sub-Poissonian distribution in the whole range of $|\xi|$. As the value of q increases, we see that the function $g_2^{(2)}(0)$ of the finite-dimensional odd NLPCS has an increasing trend, and so for a sufficiently large value of q the states appear as the super-Poissonian distribution in a short interval of $|\xi|$.

On the other hand, the numerical computation results show that when $q = 1$ (or $q = 2$), the functions $g_i^{(2)}(0)(i = 1, 2)$ for the finite-dimensional even (or odd) NLPCS are zero. The result can be obtained from expression (7) due to the fact that the states present are either of vacuum or of one (or two) photon and $g_i^{(2)}(0) = 0$ for both.

3.2. Phase probability distributions

In terms of the Pegg–Barnett phase operator formalism in a finite-dimensional $(s + 1)$ Hilbert space, on the analogy of a set of complete orthonormal phase states $|\theta_{\rm m}\rangle$ and Hermite phase operator Φ_θ introduced by Pegg and Barnett,[13,14] the two-mode phase state and Hermite phase operator of two-mode state $|\varphi\rangle$ can also be determined in Refs.[5, 11]. Then for the states $|\xi, q; f\rangle_\pm$ the phase probability distributions $P_\pm(\theta_1, \theta_2)$ are drawn in the following eight figures.

From Eqs.(7) and (8) we can see that for a sufficiently small value of ξ, the finite-dimensional EONLPCS reduces to a two-mode Fock state, so the phase information of the state is lost. As the value of ξ increases, the phase begin to build up and then the phase information can be obtained. When the value of ξ is large enough, the phase information is lost once again. On the other hand, owing to the correlation between the two modes, the variation of the phase distribution depends on the difference in phase

between the modes. So for the state $|\xi, q; f\rangle_+$ we have plotted phase distributions $P_+(\theta_1, \theta_2)$ against angle $\theta = (\theta_2 - \theta_1)$ for different values of q and ξ in Fig.3.

For $\eta^2 = 0.2$ and $q = 3$, plotted in Fig.3(a), the phase distribution has a central peak at $\theta = 0$ and is symmetric around the single peak. It is also observed that the values of the central peak $P_-(0)$ are 0.0491, 0.0426, and 0.0373 when $\xi = 3$, 4 and 5 respectively. The maxima of the distribution at $\theta = 0$ decreases with the value of ξ increasing, namely, the peak structure becomes more and more obscure with q increasing. In Fig.3(b) when $\eta^2 = 0.2$ and $q = 8$, the peak structures of the distributions are not quite the same as those of the distributions in Fig.3(a). As q increases the peaks begin to develop slowly and for $q = 8$ there appear two well developed peaks around $\theta = \pm 1.572$. When the values of parameters q and η are kept fixed, with the value of ξ increasing the multi-peak structure of the distribution occurs. Therefore, for the states $|\xi, q; f\rangle_+$ the quantum interference effects become more prominent as the values of q and ξ increase. This result can be obtained via a comparison between Figs.3(c) and 3(d).

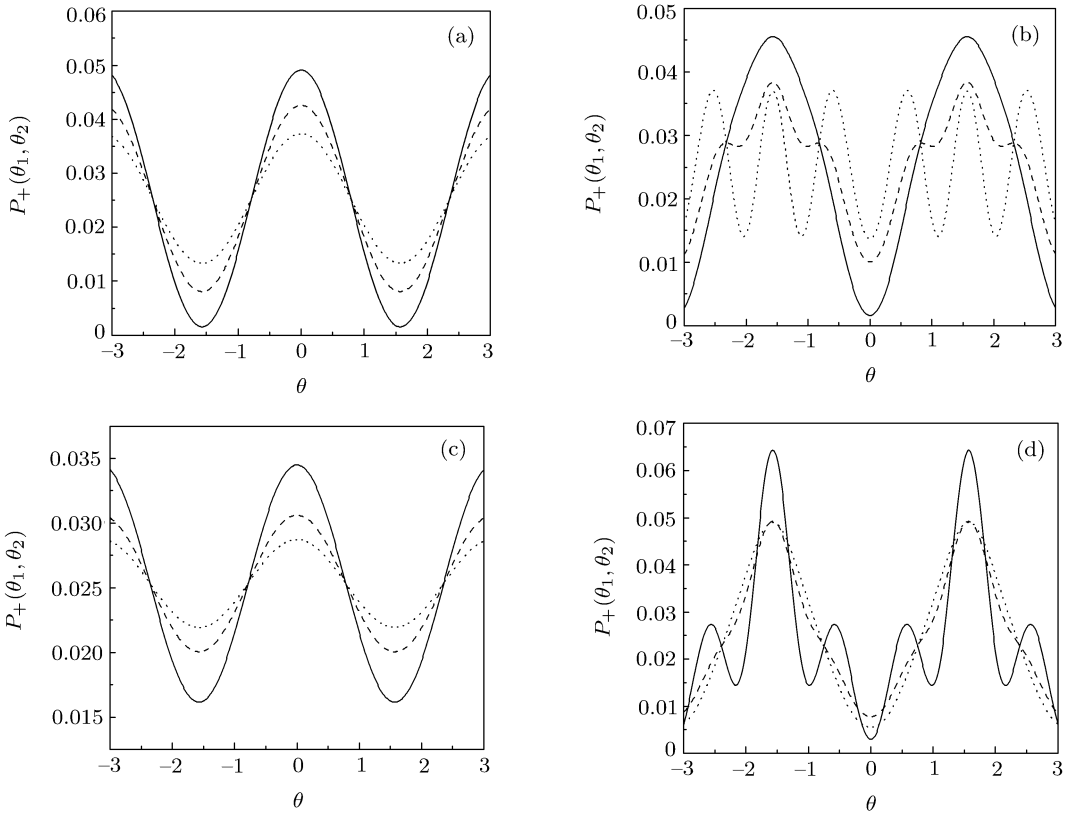


Fig.3. The phase probability distribution $P_+(\theta_1, \theta_2)$ against angle $\theta = (\theta_2 - \theta_1)$, where panel (a) is for $\eta^2 = 0.2$ and $q = 3$, panel (b) for $\eta^2 = 0.2$ and $q = 8$, panel (c) for $\eta^2 = 0.7$ and $q = 3$, and panel (d) for $\eta^2 = 0.7$ and $q = 8$, with the solid curve being for $\xi = 3$, the dashed curve for $\xi = 4$, and the dotted curve for $\xi = 5$.

From Figs.4(a) and 4(c) we find that the variations of the phase distribution of state $|\xi, q; f\rangle_-$ display the same forms as those of the states $|\xi, q; f\rangle_+$ for the same values of the parameters η and q, but for the same values of phase angle $\theta = (\theta_2 - \theta_1)$ the values of central peak are larger when $\eta^2 = 0.2$ and $q = 3$. In Fig.4(b) as the value of q increases the double-peak structure occurs and the values of the peaks increase. A comparison shows that the variation of the peak structure in Fig.4(b) is in contrast with that in Fig.4(a) as the value of phase angle θ increases. The appearance of the peaks is an indication of quantum interference. In Fig.4(b) with the value of ξ increasing the two peaks of the distribution begin to split into four peaks at $\theta = \pm 2.332$ and $\theta = \pm 1.582$. However, in Fig.4(d) the four peaks of the distribution become a double-peak structure with the value ξ increasing. Therefore, for the states $|\xi, q; f\rangle_-$, as the value of ξ increases the quantum interference effects become more prominent for small values of η and more obscure for large values of η. On the other hand, from Figs.3 and 4 we also see that the influ-

ence of the parameter η on the phase distribution of the finite-dimensional EONLPCSs is more apparent for relatively large values of η.

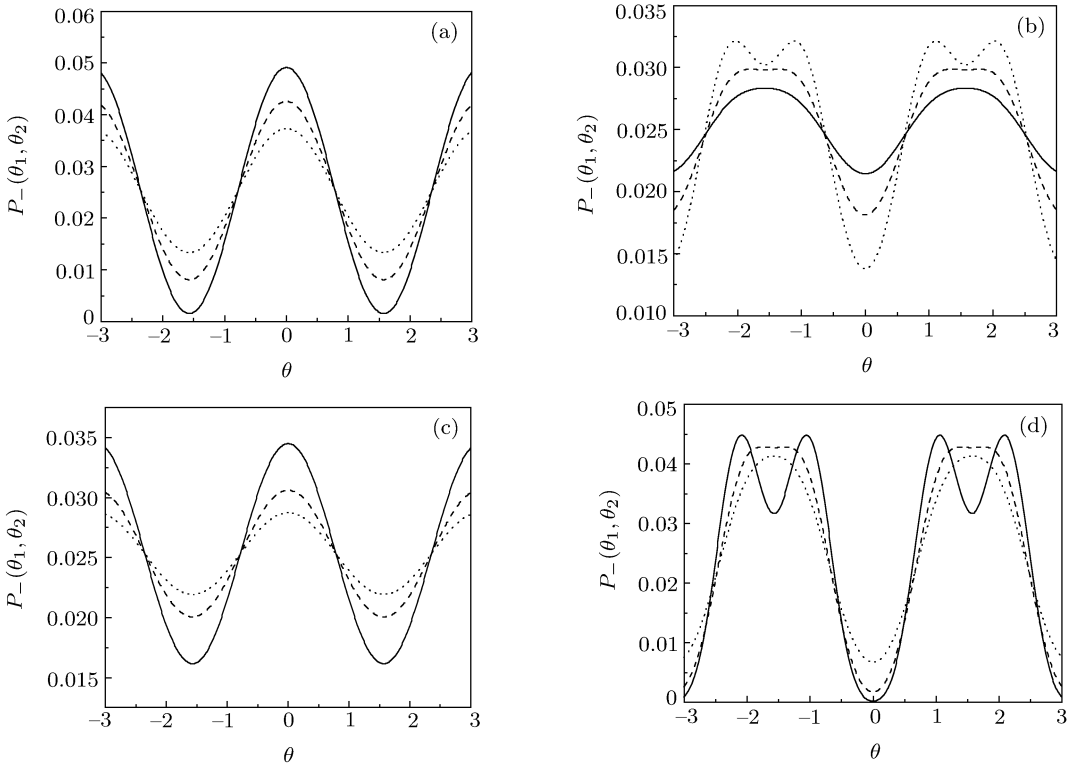


Fig.4. The phase probability distribution $P_-(\theta_1,\theta_2)$ against angle $\theta=(\theta_2-\theta_1)$, where panel (a) is for $\eta^2=0.2$ and $q=3$, panel (b) for $\eta^2=0.2$ and $q=8$, panel (c) for $\eta^2=0.7$ and $q=3$, and panel (d) for $\eta^2=0.7$ and $q=8$, with the solid curve being for $\xi=3$, the dashed curve for $\xi=4$, and the dotted curve for $\xi=5$.

4. Conclusions

In this paper we have obtained a new family of finite-dimensional EOLPCSs in the Hilbert space and discussed its orthonormalized property, completeness relations and some nonclassical properties. We find that the finite-dimensional EONLPCSs possess only normalized property and completeness relations. Moreover, the effects of the parameters q, η and ξ on sub-Poissonian distribution and phase probability distribution are shown. For different values of parameters q, η and ξ, the finite-dimensional EONLPCSs exhibit remarkably different sub-Poissonian distributions and phase probability distributions. And also the distributions completely distinct from those of the usual EOPCSs[7] and the finite-dimensional NLPCSs[6] and EOPCSs.[11] Therefore, the state described in Eq.(7) is a new kind of two-mode state associated with PCS, and it may have some applications in the fields of quantum optics and quantum information.

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