Vol 17 No 9, September 2008
1674-1056/2008/17(09)/3289-05

Chinese Physics B

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$^3PF_2$ neutron superfluidity in neutron stars and three-body force effect*

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(Received 7 November 2007; revised manuscript received 2 January 2008)

We investigate the $^3PF_2$ neutron superfluidity in β-stable neutron star matter and neutron stars by using the BCS theory and the Brueckner–Hartree–Fock approach. We adopt the Argonne V_{18} potential supplemented with a microscopic three-body force as the realistic nucleon–nucleon interaction. We have concentrated on studying the three-body force effect on the $^3PF_2$ neutron pairing gap. It is found that the three-body force effect is to enhance remarkably the $^3PF_2$ neutron superfluidity in neutron star matter and neutron stars.

Keywords: $^3PF_2$ superfluidity, neutron star, three-body force, Brueckner–Hartree–Fock approach
PACC: 2165, 2130, 9760J, 6750F

1. Introduction

Superfluidity plays an important role in understanding a number of astrophysical phenomena in neutron stars.[1−9] In the inner crust of a neutron star where the total baryon density is low, neutrons are thought to be superfluid in the $^1S_0$ channel. In the nuclear core part (i.e, the outer core part), protons may form a superfluid in the $^1S_0$ partial wave state due to the small proton fraction, and neutron superfluidity is expected in the $^3P_2-^3F_2$ coupled partial wave channel since the nucleon–nucleon (NN) scattering data indicate that the $^3P_2-^3F_2$ component of NN interaction is attractive at relatively high energies. It is generally expected that the cooling processes via neutrino emission,[3−6] the properties of rotating dynamics, the post-glitch timing observations,[7,8] the possible vertex pinning[9] of a neutron star are rather sensitive to the presence of neutron and proton superfluid phases as well as to their pairing strength.

The $^3PF_2$ nucleon superfluidity (i.e., the nucleon superfluidity in the $^3PF_2$ channel, where the $^3PF_2$ channel means the partial wave channel with total angular momentum $J = 2$; total spin $S = 1$; total isospin $T = 1$ and orbital angular momentum $L, L' = 1, 3$, see Ref.[2]) in pure neutron matter has been investigated by several authors using various theoretical approaches.[10−14] There also exist a few investigations on the neutron $^3PF_2$ superfluidity in β-stable neutron star matter.[15−17] However, in these studies, the calculations were done by adopting two-body interactions and the effects of three-body forces (TBF) were not considered. Since the $^3PF_2$ neutron superfluid phase exists in the high density region of neutron stars, three-body forces may play a crucial role in determining the strength of the $^3PF_2$ pairing gap. Only recently, the effects of three-body forces on the $^1S_0$ nucleon superfluidity[18] and on the $^3PF_2$ neutron superfluidity[19] in the interior of neutron stars were explored. However, in the above investigation, the phenomenological Urbana three-body force[20] was adopted, and it was shown that the Urbana TBF may induce an extremely strong $^3PF_2$ neutron superfluidity in the interior of a neutron star. The Urbana phenomenological TBF[20] has two adjustable parameters that are determined by reproducing the empirical saturation properties of nuclear matter,[21] therefore its applicability away from the saturation density, especially at high densities, is not completely guaranteed. In Ref.[22], we explored the effect of a microscopic TBF on the $^3PF_2$ pairing gap in pure neutron matter. It was found that the effect of the microscopic TBF is to enhance the $^3PF_2$ superfluidity in neutron matter and inclusion of the TBF increases the predicted maximum gap value from ~ 0.2 MeV to ~ 0.5 MeV. As is

*Project supported in part by the National Natural Science Foundation of China (Grant Nos 10575119 and 10775061), the Knowledge Innovation Project (Grant No KJCX3-SYW-N2) of Chinese Academy of Sciences, the Major State Basic Research Developing Program of China (Grant No 2007CB815004), CAS/SAFEA International Partnership Program for Creative Research Teams (Grant No CXTD-J2005-1), and the Asia-Link project (Grant No CN/ASIA-LINK/008(94791)) of the European Commission.
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well known, the neutron star matter at β equilibrium contains generally not only neutrons, but also protons. In the interior of neutron stars the proton fraction might reach $0.15-0.4$ depending on the adopted high-density behaviour of symmetry energy.[18] Therefore the isospin dependence of both the single particle (s.p.) energy and the equivalent TBF contribution in asymmetric nuclear medium may affect the predicted superfluidity in neutron stars.

In this paper, we shall extend our previous work for pure neutron matter and explore the microscopic TBF effect on the $^3PF_2$ neutron superfluidity in neutron star matter and neutron stars by adopting a microscopic TBF model based on the meson-exchange current approach.[23,24]

2. Theoretical approaches

Since our purpose is to investigate the TBF effect, we shall not go beyond the BCS framework. In this case, the pairing gap in the $^3PF_2$ coupled channel, which characterizes the $^3PF_2$ superfluidity in a homogeneous Fermi system, is determined by the following coupled-channel gap equations:[12,25]

$$\begin{pmatrix} \Delta_L(k) \\ \Delta_{L+2}(k) \end{pmatrix} = -\frac{1}{\pi}\int_0^\infty \mathrm{d}k' k'^2 \frac{1}{E_{k'}} \times \begin{pmatrix} V_{L,L}(k,k') & V_{L,L+2}(k,k') \\ V_{L+2,L}(k,k') & V_{L+2,L+2}(k,k') \end{pmatrix} \times \begin{pmatrix} \Delta_L(k') \\ \Delta_{L+2}(k') \end{pmatrix}, \quad (1)$$

$$\rho = 2\sum_k \frac{1}{2}\left[1 - \frac{\epsilon(k)-\epsilon(k_\mathrm{F})}{E_k}\right], \quad (2)$$

where $E_k^2 = [\epsilon(k)-\mu]^2 + \Delta_L(k)^2 + \Delta_{L+2}(k)^2$; $\epsilon(k) = \hbar^2k^2/(2m) + U(k)$ is the neutron single particle (s.p.) energy in neutron star matter, and $U(k)$ is the neutron s. p. potential; $V_{L,\,L'}(k,\,k')$ are the matrix elements of the realistic NN interaction in the coupled $^3PF_2$ channel. In the BCS gap equation, the most important ingredients are the realistic NN interaction $V(k,k')$ and the s. p. energies $\epsilon(k)$. For the NN interaction, we adopt the Argonne V18 (AV_{18}) two-body interaction[26] plus the microscopic TBF in Refs.[18, 23] based on the meson-exchange current approach. The TBF includes the contributions from different intermediate virtual processes such as virtual nucleon-antinucleon pair excitations and nucleon resonances. Its parameters, i.e., the coupling constants and the form factors, are determined from the one-boson-exchange potential model to meet the self-consistent requirement with the adopted AV_{18} two-body force. The TBF adopted here is a microscopic one in the sense that it is determined completely by the adopted two-body force and there is no free parameter in the TBF model. A more detailed description of the TBF model and the related approximations can be found in Ref.[23].

The proton and neutron s.p. energies in neutron star matter are calculated by using the Brueckner–Hartree–Fock (BHF) approach for isospin asymmetric nuclear matter.[27] The starting point of the BHF approach is the Brueckner G-matrix, which satisfies the Bethe–Goldstone (BG) equation[28]

$$\boldsymbol{G}(\rho,\beta;\omega) = V + V\sum_{k_1k_2}\frac{|k_1k_2\rangle Q(k_1,k_2)\langle k_1k_2|}{\omega-\epsilon(k_1)-\epsilon(k_2)+\mathrm{i}\eta} \times \boldsymbol{G}(\rho,\beta;\omega), \quad (3)$$

where $k_i \equiv (\boldsymbol{k}_i,\ \sigma_i,\ \tau_i)$ denotes the momentum, the z-components of spin and isospin of a nucleon, respectively; ω is the starting energy and $Q(k_1,\ k_2)$ is the Pauli operator. The isospin asymmetry parameter is defined as $\beta = (\rho_\mathrm{n}-\rho_\mathrm{p})/\rho$, with ρ_n, ρ_p, and ρ being the neutron, proton and total nucleon number densities, respectively. The s.p. energy is given by $\epsilon(k) = \hbar^2k^2/(2m) + U(k)$. In solving the BG equation, we adopt the continuous choice[28] for the s.p. potential $U(k)$ since it has been proved to provide a much faster convergency than the gap choice.[29] In particular, under the continuous choice, the s.p. potential describes physically the nuclear mean field felt by a nucleon in nuclear medium[30] and is calculated from the real part of the on-shell G-matrix.

The s.p. energies in neutron star matter can be obtained according to the following three steps: 1) first, we calculate the equation of state (EOS) of asymmetric nuclear matter (i.e., the energy per nucleon as a function of density and isospin asymmetry) in the whole asymmetry range of $0 \le \beta \le 1$ by using the BHF approach; 2) second, by using the obtained EOS of the asymmetric nuclear matter, we can determine the proton fraction $Y_\mathrm{p} \equiv \rho_\mathrm{p}/\rho = (1-\beta)/2$ as a function of density in β-stable nuclear matter according the requirement of charge neutrality and the β-stable condition;[27] 3) for any given total density, we can determine the isospin asymmetry according to the above obtained relation between the proton fraction and density in β-stable neutron star matter, and

carry out the BHF calculation again to obtain the s.p. energies in β-stable neutron star matter.

The neutron and proton superfluidity in nuclear matter can be described by their pairing energy gaps at their respective Fermi surfaces. To solve the gap equation for the $^3PF_2$ superfluidity, we follow the scheme proposed in Ref.[12]. In our calculations, the microscopic TBF contribution has been included by reducing the TBF to an equivalently effective two-body interaction according to the standard scheme described in Ref.[23]. A detailed description and justification of the method are discussed in Refs.[23, 24].

3. Results and discussions

In Fig.1, we show the obtained $^3PF_2$ neutron pairing gap $\Delta_{\rm F}^{\rm n} = \Delta(k_{\rm F}^{\rm n})$ in neutron star matter as a function of the total baryon density ρ. The open squares are obtained by adopting the pure $\rm AV_{18}$ two-body interaction, while the filled squares are predicted by using the $\rm AV_{18}$ interaction plus the microscopic TBF. We notice that the neutron $^3PF_2$ superfluid phase exists in the high density region of neutron star matter in both cases with and without including the TBF. In the case of not including the TBF, the pairing gap increases first with density in the relatively low density region of $\rho \leq 0.24\,\rm fm^{-3}$ and then decreases as the density increases in the region of $\rho \geq 0.24\,\rm fm^{-3}$. The pairing gap is peaked around $\rho \sim 0.24\,\rm fm^{-3}$ and the peak value of the pairing gap is about 0.2 MeV, in agreement with the previous prediction.[15] Comparing the open squares and the filled squares, we find that inclusion of the TBF enhances remarkably the $^3PF_2$ superfluidity in β-stable neutron star matter, especially at high densities, and makes the $^3PF_2$ neutron pairing gap become an monotonically increasing function of density. The TBF effect on the $^3PF_2$ neutron superfluidity in neutron star matter turns out to be quite different from that in pure neutron matter obtained in Ref.[22]. It is clear from Fig.1 that the TBF effect on the $^3PF_2$ pairing gap in neutron star matter increases monotonically as a function of the total baryon density. Whereas, it was shown in Ref.[22] that in pure neutron matter, the TBF effect on the $^3PF_2$ superfluidity increases first with density in the relatively low density region of $\rho \leq 0.32\,\rm fm^{-3}$ and then reduces gradually as the density further increases. This difference can be explained by the isospin dependence of the TBF effect. The β-stable neutron star matter contains generally not only neutrons but also a fraction of protons. In addition, the proton fraction increases continuously with density and may reach quite high values at high densities as shown in Fig.2. It can be seen that the proton fraction at high densities in β-stable matter predicted by including the TBF is much larger than that without considering the TBF.

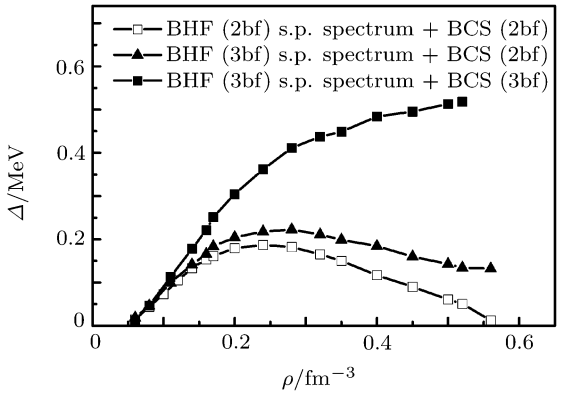


Fig.1. Neutron $^3PF_2$ energy gap as a function of density in β-stable neutron star matter, predicted for the two cases with and without including the TBF, where bf = body force.

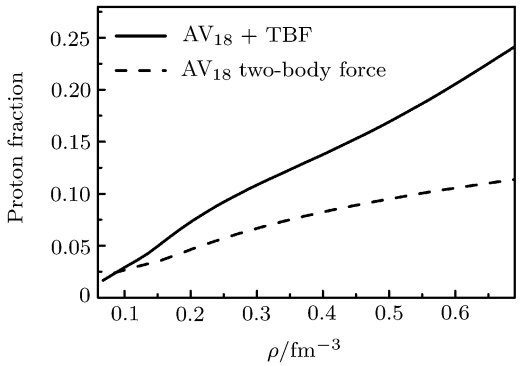


Fig.2. Proton fraction predicted for the two cases with including (solid curve) and without including (dashed curve) the TBF.

In order to see clearly the isospin dependence of the TBF effect, we compare in Fig.3 the TBF contribution to the neutron–neutron pairing interaction in the $^3PF_2$ channel in pure neutron matter with that in neutron star matter at several densities. In the figure, the solid (dashed) curves and filled (empty) squares are the $^3P_2$ ($^3F_2$) components of the TBF contributions in neutron star matter and in pure neutron matter, respectively. It can be seen that in the density region of $\rho \leq 0.32\rm fm^{-3}$ the $^3P_2$ ($^3F_2$) TBF components are almost the same in pure neutron matter and neutron star matter. Both the $^3P_2$ and $^3F_2$ components of the TBF are essentially attractive and their attraction enhances rapidly as the density increases up to

$0.32\,\mathrm{fm}^{-3}$. As a consequence, the TBF induces an enhancement of the ${}^3PF_2$ superfluidity and its effect increases as the density increases. Above $\rho = 0.32\,\mathrm{fm}^{-3}$, the ${}^3P_2$ (${}^3F_2$) components of the TBF become different in pure neutron matter and in neutron star matter. In pure neutron matter, the increase of the attraction of the ${}^3P_2$ and ${}^3F_2$ components of the TBF becomes slow and the repulsion of the two components (especially the ${}^3P_2$ component) around $R = 1$fm becomes stronger as the density increases in the density region above $\rho = 0.32\,\mathrm{fm}^{-3}$. However, in neutron star matter, due to the isospin effect, the attraction of the ${}^3P_2$ and ${}^3F_2$ components of the TBF at high densities above $\rho = 0.32\,\mathrm{fm}^{-3}$ turns out to be stronger than that in pure neutron matter. As a consequence, the TBF-induced enhancement of the ${}^3PF_2$ superfluidity at high densities in neutron star matter is stronger than that in pure neutron matter.

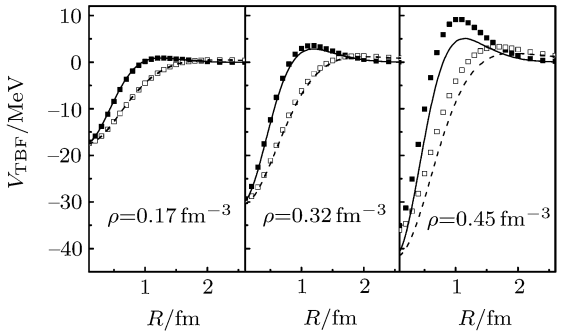


Fig.3. The ${}^3P_2$ and ${}^3F_2$ components of the microscopic TBF as a function of the distance R between the two interacting neutrons in neutron star matter (the solid and dashed curves) and in pure neutron matter (the filled and empty squares) for several densities.

In general, the TBF is expected to influence the superfluidity phase in β-stable matter by modifying the pair interaction $V_{L,L'}$ in the BCS gap equation and by correcting the s.p. energy spectrum. To see which is the most important mechanism responsible for the enhancement of the ${}^3PF_2$ neutron superfluidity, we plot in Fig.1 also the result obtained by including the TBF in the BHF calculations to obtain the s.p. energy, but adopting only the pure AV_{18} as the pairing interaction $V_{L,L'}$ in the gap equation (triangles). By comparing the triangles and the empty squares, it is clear that inclusion of the TBF correction in the s.p. energy spectrum plays a minor role and increases only slightly the pairing gaps at high densities. The strong enhancement is mainly due to the TBF modification of the pairing interaction in the ${}^3PF_2$ channel.

In Fig.4, we illustrate the distribution of the ${}^3PF_2$ neutron pairing gap in a typical neutron star with a mass of M=1.4M$_\odot$. In the figure, the empty squares are the results obtained by adopting the pure AV_{18} two-body interaction; the filled squares are obtained by adopting AV_{18} plus the microscopic TBF. It is shown that by adopting purely the AV_{18} two-body force, the predicted pairing gap increases from the outer part of the neutron star, reaches a maximum value of about 0.2 MeV located at around $r \simeq 8.4$ km, then starts to decrease and finally vanishes at high enough densities in the inner part of the neutron star. It is also seen that in the case without including the TBF, the ${}^3PF_2$ neutron superfluid phase is restricted within a relatively narrow region of neutron stars, in agreement with the previous result of Ref.[15] by using the AV_{14} interaction. Inclusion of the microscopic TBF turns out to enhance significantly the ${}^3PF_2$ neutron superfluidity in neutron stars. On the one hand, the TBF makes the superfluid phase spread throughout the whole neutron star; on the other hand, it enlarges the maximum strength of the pairing gap from ∼0.2 MeV to ∼0.5 MeV. For comparison, we also give the result of Ref.[19] obtained by using the phenomenological Urbana UIX TBF (empty triangles). We notice that in both cases by adopting the phenomenological Urbana UIX TBF and the microscopic TBF, the predicted pairing gaps increase continuously from the outer part to the inner part of neutron star. However, the phenomenological TBF leads to an extremely strong ${}^3PF_2$ neutron superfluidity in neutron stars. At variance, our microscopic TBF enhances the superfluidity moderately.

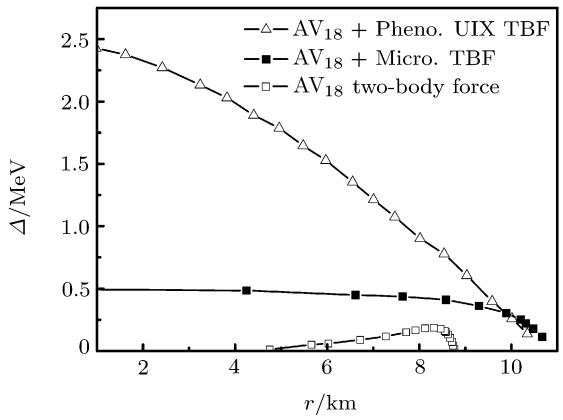


Fig.4. Distribution of neutron ${}^3PF_2$ pairing gap in a typical neutron star.

4. Summary

In summary, we have calculated the neutron $^3PF_2$ pairing gap in neutron star matter and neutron stars based on the BHF approach and the BCS theory. We have especially investigated the effect of the microscopic TBF on the neutron superfluidity in the $^3PF_2$ coupled channel. It is shown that inclusion of the microscopic TBF leads to a remarkable enhancement of the predicted pairing gap at high densities since the TBF turns out to be essentially attractive in the $^3PF_2$ channel. In neutron star matter at β-equilibrium, the TBF effect on the neutron $^3PF_2$ superfluidity is found to increase continuously with the total baryon density, mainly due to the isospin dependence of the $^3PF_2$ component of the TBF in asymmetric matter. The distribution of the $^3PF_2$ neutron pairing gap in a typical neutron star with a mass of $1.4M_\odot$ has also been calculated. It is shown that by adopting purely the AV_{18} two body force, the predicted pairing gap is restricted within a relatively narrow region of neutron stars and the maximum strength of the gap is about 0.2 MeV. Whereas, if the TBF is included, the obtained pairing gap increases continuously from the outer part to the inner part of the neutron star and the strength of the pairing gap is found to be much larger than the one calculated without including the TBF. On the one hand, the TBF enhances significantly the predicted neutron $^3PF_2$ pairing gap in neutron stars; on the other hand, it makes the superfluid phase spread throughout the neutron star. As compared to the result obtained by using the phenomenological Urbana TBF, which leads to an extremely strong enhancement of the $^3PF_2$ neutron superfluidity in the interior of neutron stars,[19] the enhancement predicted by the microscopic TBF turns out to be much less pronounced and to be moderate.

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