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# Chaotic ferroresonance in a non-autonomous circuit*

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Accurate description of magnetization curve has important effect on ferroresonance. In most of earlier ferroresonance studies the magnetization curve is modelled as a 3rd or 5th order polynomial. However, it is not comprehensive. This paper investigates the chaotic ferroresonance behaviour exhibited by a non-autonomous circuit which contains a nonlinear flux-controlled inductance. The ferromagnetic characteristic of this nonlinear inductance represented by a magnetization curve could be expressed as an $n$th order two-term polynomial. By varying the value of exponent $n$, the circuit can assume a diverse range of steady-state regimes including fundamental and subharmonic ferroresonance, quasi-periodic oscillations, and chaos. A detailed analysis of some simulations demonstrates that the probability of chaos increases as the exponent of the magnetization curve rises. The effect of varying the magnitude of the source voltage on the chaotic behaviour of the circuit is also studied.

**Keywords:** ferroresonance, chaotic behaviour, magnetization curve
**PACC:** 0545, 0547

## 1. Introduction

Since the first chaotic attractor (Lorenz attractor[1]) was found in 1963, there has been much attention paid to the study of chaos.[2−5] Recently, the trend of using Chaos theory to analyse the nonlinear phenomena such as ferroresonance in RLC circuit[6−8] and power system has attracted increasing attention.[9−14]

Ferroresonance is a complicated nonlinear phenomenon, which is caused by magnetic saturation. It is harmful for electric circuit and power system because of the overvoltage and overcurrent it generates. Many researchers have paid great attention to this problem. Feng and Shen [6] discussed the chaos in a RCL-shunted Josephson junction by delayed feedback. Edoardo *et al* [7] investigated the ferroresonance in a RCL series electric circuit in order to deepen the effects of the magnetization phenomena in the ferromagnetic core. Liu *et al* [8] analysed a sort of typical chaotic behaviour of a non autonomous circuit containing a nonlinear ferromagnetic core. Dick plotted the instantaneous magnetization curve for transient study[9] and this curve has been used by many authors in their studies.[10−14] This paper investigates the general chaotic behaviour of the circuit in Ref.[8] based on the magnetization curve presented in Refs.[9,11] with the aim of finding out the effects of the magnetization phenomena on the ferroresonance behaviour of the circuit.

## 2. System modelling

The circuit given in Fig.1 consists of one ferromagnetic core L2 which is a kind of flux controlled inductance. For very high currents the ferromagnetic core might be driven into saturation where the flux-current characteristic is modelled as a polynomial[9,11]

$$i_{\mathrm{L}} = a\varphi + b\varphi^n, \tag{1}$$

where $\varphi$ denotes the flux in the nonlinear inductance L2 and $n = 3, 5, 7, 9, 11$.

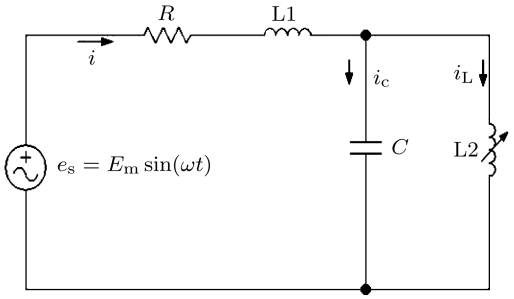



**Fig.1.** Ferroresonance circuit.

The differential equations for this circuit are

$$C\frac{\mathrm{d}u_{\mathrm{c}}}{\mathrm{d}t} = i - i_{\mathrm{L}}, \tag{2}$$

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$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = u_\mathrm{c}, \tag{3}$$

$$L_1\frac{\mathrm{d}i}{\mathrm{d}t} = E_\mathrm{m}\sin(\omega t) - u_\mathrm{c} - Ri. \tag{4}$$

where $a = 2.8\times10^{-3}, b = 7.2\times10^{-3}$, $n = 3, 5, 7, 9, 11$, $\varphi$ is flux in inductance L2 in per unit (p.u.) value, $\omega$ is the power frequency of 1.0 p.u. value. Since it has been studied in Ref.[8] for the exponent $n$ equal to 3, we do not analyse it here. All simulation results are discussed for the exponent $n = 5, 7, 9, 11$ in Eq.(1).

The performance of the mathematical model shown in Eqs.(1)–(4) for describing magnetization phenomena in ferromagnetic core depends on the accurate description of the magnetization curve. In the low current range for steady-state, the relationship between flux $\phi$ and current $i$ can be approximated linearly by the first term of Eq.(1), where the coefficient $a$ corresponds to the reciprocal of the inductance ($a = 1/L$). Nevertheless, for high current range, the inductance might be driven into saturation where the flux current ($\varphi - i$) characteristic becomes highly nonlinear. Using a two-term polynomial of order less than eleven to represent the magnetization curve might be adequate for accurate digital simulation, but it does not bend sharply enough at the knee point to satisfy the dynamic characteristic of the ferromagnetic inductance. Figure 2 shows the details of the saturation region for different exponents of the nonlinear term in Eq.(1).[11]

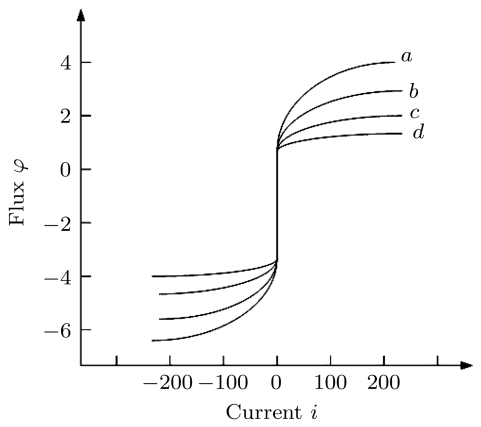



**Fig.2.** Curves of $\varphi$ versus $i$ for different exponents curves $a$ $i_\mathrm{L} = a\varphi + b\varphi^5$; curves $b$ $i_\mathrm{L} = a\varphi + b\varphi^7$; curves $c$ $i_\mathrm{L} = a\varphi + b\varphi^9$; curves $d$ $i_\mathrm{L} = a\varphi + b\varphi^{11}$.

For the convenience of simulation, the nonlinear ordinary differential equations Eqs.(2)–(4) can now be simplified through introducing the dummy variable $\tau = \omega t$, and setting $x = \frac{u_\mathrm{c}}{\omega}$, $y = \varphi$, $z = i$, $F = \frac{E_\mathrm{m}\sin\tau}{L_1\omega}$, $\alpha = \frac{1}{\omega^2 C}$, $\beta = \frac{R}{\omega L_1}$, $\zeta = \frac{1}{L_1}$. Substituting this parameters into Eqs.(2), (3) and (4) yields

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \alpha[z - (ay + by^n)], \tag{5}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = x, \tag{6}$$

$$\frac{\mathrm{d}z}{\mathrm{d}\tau} = F\sin(\tau) - \zeta x - \beta z. \tag{7}$$

Equations (5), (6) and (7) are simple and clear for simulation and discussion.

## 3. Simulation results and discussion

This paper presents the results of an investigation of the effect of magnetization phenomena on the behaviour of the ferroresonance circuit. Bifurcation diagrams, Poincaré maps and phase-plane diagrams are three main means to distinguish between various modes of behaviour. Furthermore, the Lyapunov number is used to distinguish chaos from steady state and Lyapunov dimension is used to distinguish strange attractor from non-strange attractor. The system differential equations are solved numerically in Matlab environment using an embedded fourth-order Runge–Kutta method.

The initial conditions are kept constant: $\omega = 1$, $E_\mathrm{m} = 1$, $y = 0$, $y' = \sqrt{2}$ in p.u value, $\alpha = 1$, $\beta = 1$, $\zeta = 2.5$ through all simulations. The varying parameters are the magnitude of source voltage and the exponent $n$ equals odd number which can vary from 5 to 11. Several cases and combinations have been investigated, however, only the representative ones are presented here.

To classify the effects of varying exponent $n$ of Eq.(5) on the behaviour of the system shown in Fig.1, extensive simulations have been carried out. To predict which kind of ferroresonance may occur for a wide range of magnitude of source voltage change, the bifurcation diagram obtained by varying the exponent $n$ in Eq.(5) has been used. Figure 3 shows the bifurcation diagrams for $n = 5, 7, 9, 11$; and the varying parameter is the magnitude of source voltage. In Fig.3 the parameter given on $x$-axis represents the parameter $F$ in Eq.(7).

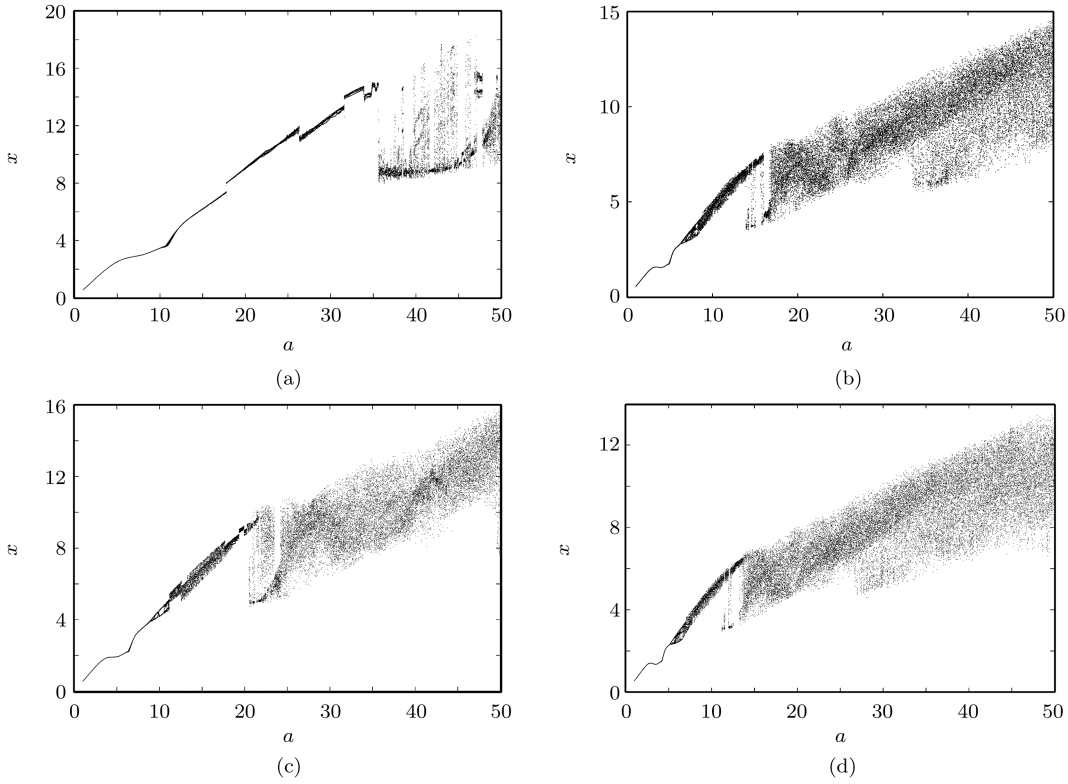



**Fig.3.** Bifurcation diagram obtained by varying $F$ for (a) $n = 5$, (b) $n = 7$, (c) $n = 9$, (d) $n = 11$.

These bifurcation diagrams reveal what modes of ferroresonance are possible for a given value of the source voltage. The single-value area of the bifurcation map indicates one period, dual-value indicates two periods, etc. In fact, subharmonics play an important role in non-chaotic vibrations. One characteristic precursor to chaotic motion is the appearance of sub-harmonics. It is a transitional region from periodic road to chaos. In this region the waveform of $u_c$ is still periodic but not a sinusoidal one as the source voltage and the trajectory in phase-plane is changing from limit cycle to strange attractors. Likewise, the region with many scattered points that looks blurred in Fig.3 indicates chaotic ferroresonance.

Bifurcation diagrams Figs.3(b)–3(d) have three region, i.e. period-one region, transitional region and chaos region. But the period-one region and transitional region are shorter than Fig.3(a) for $n = 5$. This reveals that the chaotic region begins much sooner for high exponent $n$. For $n = 11$, the chaotic region begins at about $F = 11$ p.u.; nevertheless for $n = 5$, the chaotic region begins at about $F = 36$ p.u. Therefore, it can be concluded that the higher the order $n$, the lower the magnitude of source voltage at which the chaos starts.

Moreover, Fig.4 shows the $u_c$ waveform, phase plane diagram and Poincaré map for $n = 5$, $F = 5$. The waveform of $u_c$ (Fig.4(a)) is also a periodic sinusoidal one despite its non-periodic oscillation at first. Figure 4(b) indicates that the trajectory projected on the $x - y$ plane is a limit cycle eventually. Figure 4(c) shows that after the initial oscillation the trajectory becomes periodic, and only six stable points are shown at Poincaré map. In addition, the Lyapunov exponents are calculated to be $\sigma_1 = -0.3208$, $\sigma_2 = -0.3321$, $\sigma_3 = -0.3470$. There is no positive Lyapunov exponent so the circuit is stable.

Unlike Fig.4, we set $n = 5$, $F = 16$ in Fig.5, and the $u_c$ waveform has distortion compared with the source voltage but it is also a periodic waveform eventually. Figure 5(b) indicates that the trajectory is a metamorphic limit cycle which is similar to the attractor. The Lyapunov exponents are calculated to be $\sigma_1 = -0.1527$, $\sigma_2 = -0.3946$, $\sigma_3 = -0.4526$. The circuit is also stable. Figures 6 and 7 show the trajectories projected on the $x - y$ plane and their Poincaré maps for $n = 7$ and 9. When the exponent $n = 7$, the Lyapunov exponents are calculated to be $\sigma_1 = 0.1174$,

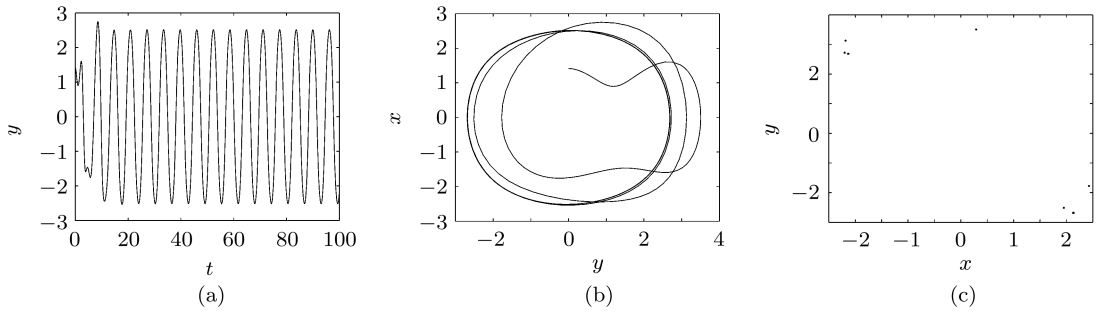



**Fig.4.** (a) Waveform, (b) Trajectory and (c) Poincaré map for $F = 5$, $n = 5$.

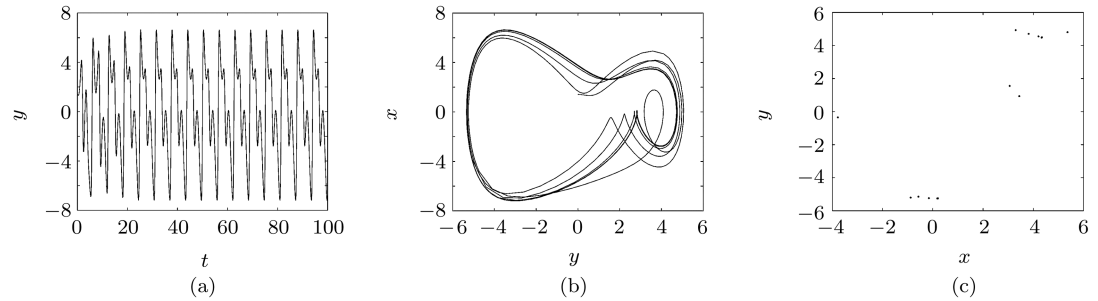



**Fig.5.** (a) Waveform, (b) Trajectory and (c) Poincaré map for $F = 16$, $n = 5$.

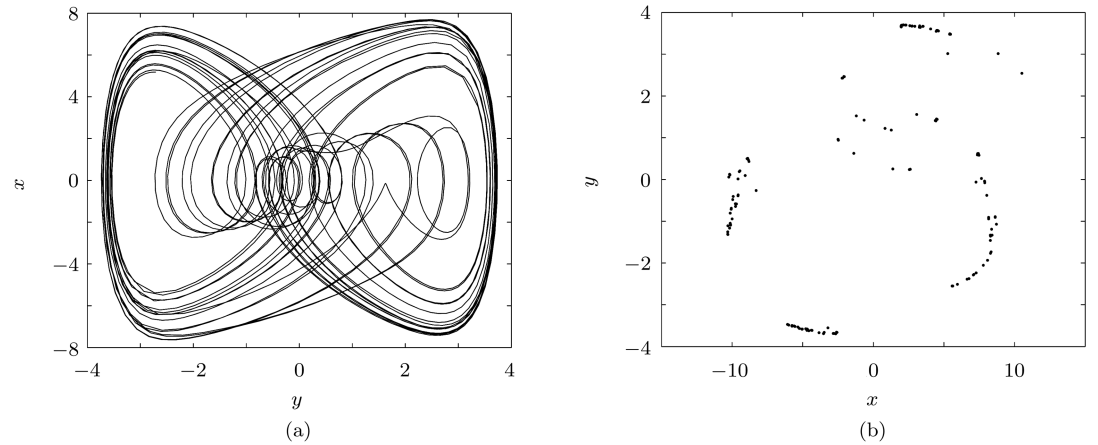



**Fig.6.** (a) Trajectory and (b) Poincaré map of $F = 16$, $n = 7$.

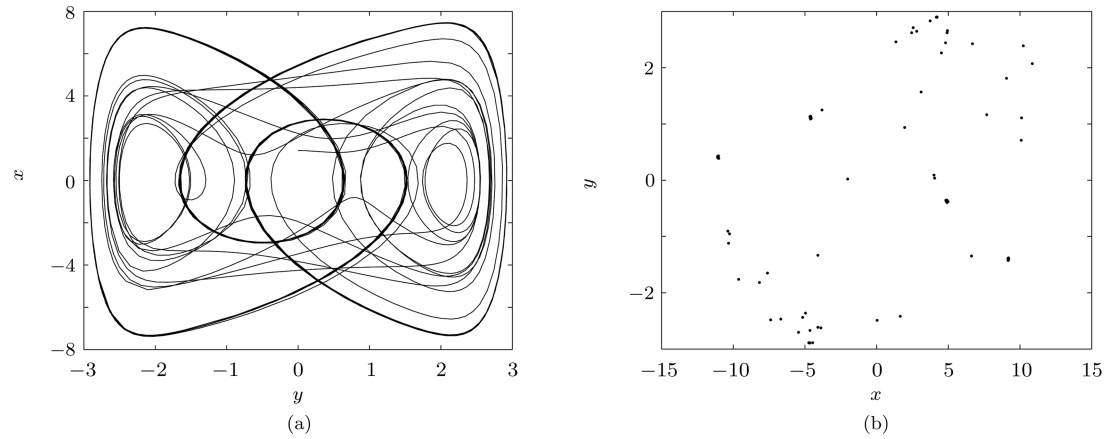



**Fig.7.** (a) Trajectory and (b) Poincaré map for $F = 16$, $n = 9$.

$\sigma_2 = -0.5038$, $\sigma_3 = -0.6136$. In the same way, when the exponent $n = 9$, the Lyapunov exponents are calculated to be $\sigma_1 = 0.0760$, $\sigma_2 = -0.3760$, $\sigma_3 = -0.7000$. Both of them have positive Lyapunov exponents and the trajectory has changed from limit cycle to strange attractors. The points shown in the Poincaré maps are randomly distributed and unsystematic. This indicates that the system has changed through periodic to chaos when the exponent value increases.

In Fig.8, the source voltage is sinusoidal, but the waveform shows serious distortion and becomes non-periodic. The trajectory projected on $x - y$ plane becomes a strange attractor. Furthermore, the Lyapunov exponents are calculated to be $\sigma_1 = 0.2508$, $\sigma_2 = -0.4374$, $\sigma_3 = -0.8134$ and the Lyapunov dimension $d = 2.201$. The positive Lyapunov exponent indicates that the trajectory of Fig.8(b) is chaotic, which generally shows the sensitive nature of the nonlinear dynamical system to initial conditions. This feature distinguishes a chaotic behaviour from other types of steady-state behaviours. The non-integer Lyapunov dimension $d$ indicates that this attractor is a strange attractor.

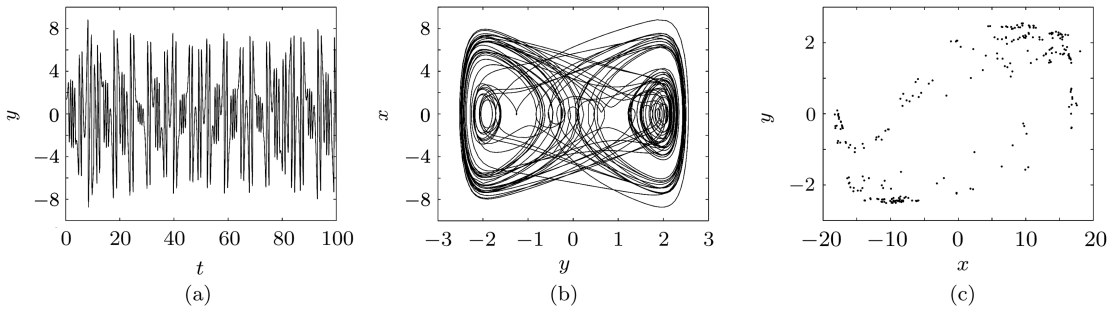



**Fig.8.** (a) Waveform, (b) Trajectory and (c) Poincaré map for $F = 25$, $n = 11$.

Besides the positive Lyapunov exponent and fractional Lyapunov dimension, another visible characteristic of a chaotic signal is its frequency spectrum. The power spectrum of $u_c$ signal contains not only discrete frequency components due to the driving function and its harmonics, but also distributed frequency components. Hence the appearance of a broad spectrum of frequencies in the output, when the input is a single-frequency harmonic motion, is a foreshowing of chaotic vibration. Figure 9 is the spectrum for chaotic vibration when $F = 16$, $n = 7$. It is a broad spectrum which indicates the system is a chaotic system.

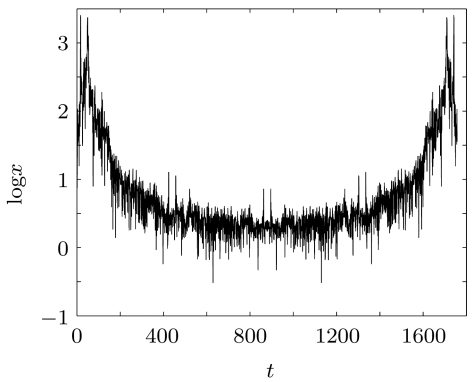



**Fig.9.** Spectrum for $F = 16$, $n = 7$.

# 4. Conclusions

This paper shows some results obtained of investigating the influence of different magnetization characteristics on the behaviour of a non-autonomous circuit when it is being driven into ferroresonance. It can be seen that variations in the exponent of the nonlinearity or the applied voltage can result in different modes of behaviour which may be periodic or chaotic. A more accurate representation of the $\varphi - i$ curve characteristic of ferromagnetic inductance results in chaos occurring at a much lower magnitude of the source voltage. Since the ferroresonance modes under different orders are known, the ferroresonance could be defended or mitigated.

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