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One new fractional-order chaos system and its circuit simulation by electronic workbench*

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This paper proposes a new chaotic system and its fractional-order chaotic system. The necessary condition for the existence of chaotic attractors in this new fractional-order system is obtained. It finds that this new fractional-order system is chaotic for $q > 0.783$ if the system parameter m=6. The chaotic attractors for q=0.8, and q=0.9 are obtained. A circuit is designed to realize its fractional-order chaos system for q=0.9 by electronic workbench.

Keywords: fractional-order chaotic systems, necessary condition, electronic workbench
PACC: 0545

1. Introduction

Although, the notion of fractional derivative dates back to the 17th century, its application to physics and engineering has attracted lots of attention only recently. Many systems[1,2] are known to display fractional order dynamics, such as viscoelastic systems, dielectric polarization, electrode–electrolyte polarization and electromagnetic waves. Many researchers surprisingly found that many nonlinear fractional differential systems also display complex bifurcation and chaotic phenomena.[2−6] For example,[5,7,8] the fractional Chua circuit has a double scroll chaotic attractor, the fractional Rossler system, fractional Duffing system, the fractional Chen system also display chaotic or hyperchaotic phenomena.

In this paper, we construct a new chaotic system, and find its fractional-order chaotic system. According to stability theory of fractional-order systems, we obtain the necessary condition for the existence of chaotic attractors in this new fractional-order system. Based on the electronic workbench (EWB), the corresponding circuit is designed.

2. A new chaotic system

A new chaotic system is described by (1)

$$\begin{aligned} \dot{x} &= 10(y - x), \\ \dot{y} &= 40x - 10xz + w, \\ \dot{z} &= -2.5z + 40x^2, \\ \dot{w} &= -my - w, \end{aligned} \tag{1}$$

where $0 < m < 40$ is the parameter of system (1).

From Eq.(1), we can obtain

$$\nabla V = \sum_{i=1}^{4} \partial \dot{x}_i / \partial x_i = -13.5.$$

Therefore, system (1) is dissipative. Hence, a volume element V_0 is contracted by the flow into a volume element $V_0 \mathrm{e}^{t\nabla V}$ in time t. That is, each volume containing the system orbit shrinks to zero as $t \to +\infty$

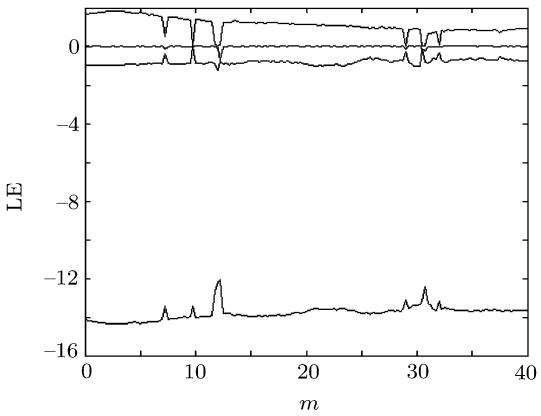


Fig.1. Lyapunov exponents spectrum of system (1).

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at an exponential rate ∇V, which is independent of x_1, x_2, x_3, x_4. Consequently, all system orbits will ultimately be confined to a specific subset of zero volume and the asymptotic motion settles onto an attractor. Then, the existence of an attractor is proved.

Figure 1 shows the Lyapunov exponents spectrum of system (1) with the parameter m varied.

According to Fig.1, the system (1) is a chaotic system.

Let m=6, We obtain the Lyapunov exponents of the system (1) as follows, $\lambda_1 = 1.6602$, $\lambda_2 = 0.0$, $\lambda_3 = -0.8995$, $\lambda_4 = -14.261$.

The chaotic attractor of the system (1) for m=6 is shown in Fig.2.

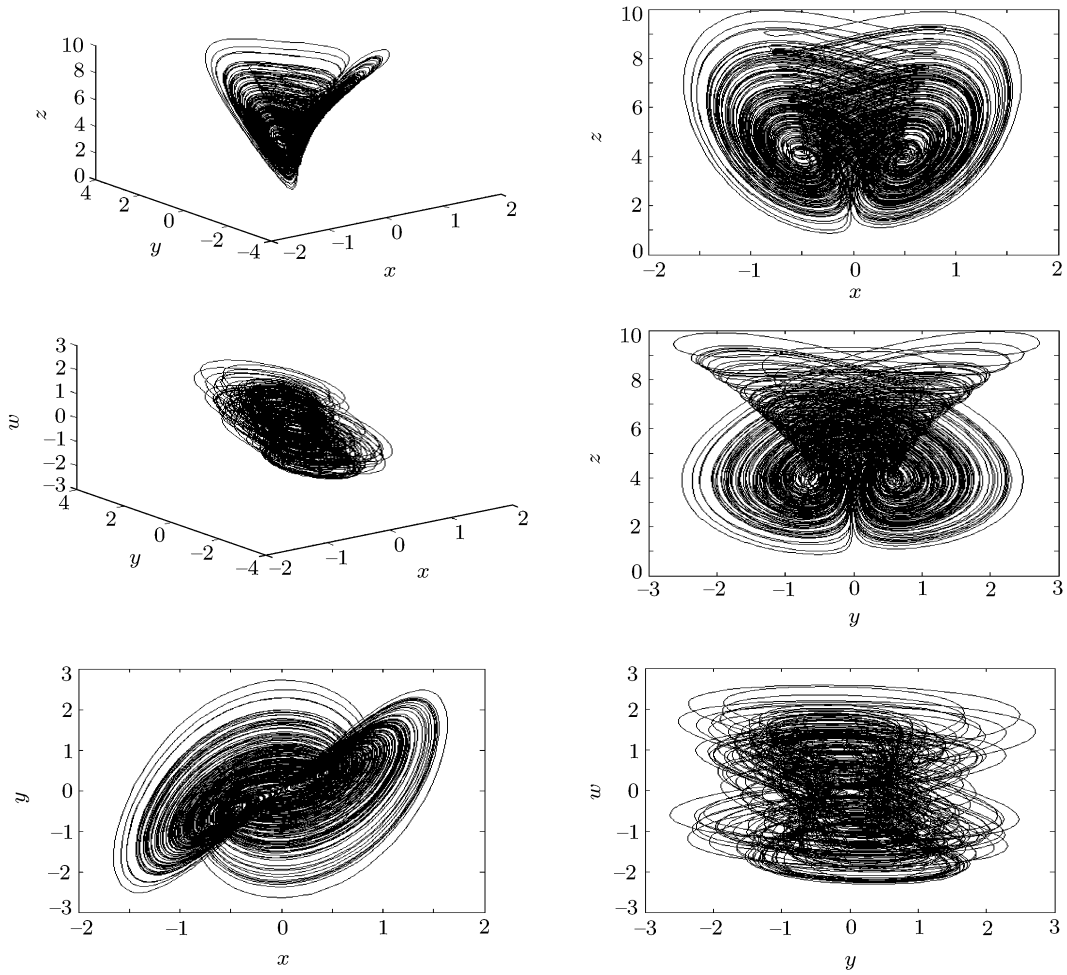


Fig.2. The chaos attractor of system (1) for m=6.

3. The fractional-order system of chaotic system (1)

According to the system (1), we can obtain the following fractional system (2):

$$\begin{aligned} \mathrm{d}^q x/\mathrm{d}t^q &= 10(y-x),\\ \mathrm{d}^q y/\mathrm{d}t^q &= 40x - 10xz + w,\\ \mathrm{d}^q z/\mathrm{d}t^q &= -2.5z + 40x^2,\\ \mathrm{d}^q w/\mathrm{d}t^q &= -my - w, \end{aligned} \tag{2}$$

where $0 < q < 1$. The fractional-order q will be determined later.

Now, we discuss the necessary condition for the existence of chaotic attractors in this new fractional-order system for $m = 6$ (the same discussion follows for other parameter m).

First, we can obtain the three equilibrium points of the system (2) as follows:

$$(0,0,0,0), (\pm\sqrt{3.4}/4, \pm\sqrt{3.4}/4, 3.4, \mp 3\sqrt{3.4}/2).$$

The three equilibrium points corresponding to the

eigenvalues are,

$$-25.5232, -2.5, -0.8656, 15.388$$
$$-18.6794, -1.0487, 3.1141 \pm 8.7797\mathrm{j}$$

According to Ref.[7], we know that equilibrium point (0,0,0,0) is saddle point of index 1 and $(\pm\sqrt{3.4}/4, \pm\sqrt{3.4}/4, 3.4, \mp 3\sqrt{3.4}/2)$ are saddle points of index 2. In chaotic systems, it is proved that scrolls are generated only around the saddle points of index 2. Moreover, saddle points of index 1 are responsible only for connecting scrolls.[7]

Suppose λ is the unstable eigenvalue of the saddle points of index 2, the necessary condition for the fractional-order system (2) to remain chaotic is keeping the eigenvalue λ in the unstable region.[7] According to Refs.[9, 10], if the eigenvalue λ is in the unstable region, then the following condition is satisfied:

$$|\arg(\lambda)| < 0.5\pi q,$$

where $\arg(\lambda)$ denotes the argument of the eigenvalue λ. That is

$$q > \frac{2}{\pi} |\arg(\lambda)| .$$

Therefore, the necessary condition for the existence of chaotic attractors in the fractional-order system (2) for $m = 6$ is,

$$q > \frac{2}{\pi} |\arg(\lambda)| = \frac{2}{\pi} \arctan \frac{8.7797}{3.1141} = 0.783.$$

Now, we take some case for numerical simulation. All the numerical simulation of fractional-order system in this paper is based on Ref.[11]. It is a direct time domain approximation numerical simulation. Reference [7] has shown that using frequency domain approximation in the numerical simulations of fractional systems may result in wrong consequences. This mistake has occurred in the recent literature that found the lowest-order chaotic systems among fractional-order systems.[7] Now, we introduce the numerical solution of fractional differential equations in Ref.[11]. Consider the following fractional-order system:

$$\begin{cases} \mathrm{d}^{q_1} x/\mathrm{d}t^{q_1} = f(x, y), \ 0 < q_1, \\ \mathrm{d}^{q_2} y/\mathrm{d}t^{q_2} = g(x, y), \ q_2 < 1 \end{cases}$$

with initial condition $(x_0,\ y_0)$. Now, set $h = T/N$ $t_n = nh (n = 0, 1, 2, \ldots, N)$. The above system can be discretized as follows:

$$\begin{cases} x_{n+1} = x_0 + \dfrac{h^{q_1}}{\Gamma(q_1+2)} \left[f(x^p_{n+1}, y^p_{n+1}) + \displaystyle\sum_{j=0}^{n} \alpha_{1,j,n+1} f(x_j, y_j) \right], \\ y_{n+1} = y_0 + \dfrac{h^{q_2}}{\Gamma(q_2+2)} \left[g(x^p_{n+1}, y^p_{n+1}) + \displaystyle\sum_{j=0}^{n} \alpha_{2,j,n+1} g(x_j, y_j) \right], \end{cases}$$

where

$$\begin{cases} x^p_{n+1} = x_0 + \dfrac{1}{\Gamma(q_1)} \displaystyle\sum_{j=0}^{n} \beta_{1,j,n+1} f(x_j, y_j), \\ y^p_{n+1} = y_0 + \dfrac{1}{\Gamma(q_2)} \displaystyle\sum_{j=0}^{n} \beta_{2,j,n+1} g(x_j, y_j), \end{cases}$$

$$\alpha_{i,j,n+1} = \begin{cases} n^{q_i+1} - (n - q_i)(n+1)^{q_i}, & j = 0, \\ (n-j+2)^{q_i+1} + (n-j)^{q_i+1} - 2(n-j+1)^{q_i+1}, & 1 \le j \le n, \\ 1 & j = n+1, \end{cases}$$

$$\beta_{i,j,n+1} = \frac{h^{q_i}}{q_i}[(n-j+1)^{q_i} - (n-j)^{q_i}], \qquad 0 \le j \le n, \quad i = 1, 2.$$

The error of this approximation is described as follows,

$$|x(t_n) - x_n| = o(h^{p_1}) \quad p_1 = \min(2, 1 + q_1),$$
$$|y(t_n) - y_n| = o(h^{p_2}) \quad p_2 = \min(2, 1 + q_2).$$

Now, we show some simulation results based on the time domain approximation. Figures 3 and 4 show the numerical simulation results for q= 0.9 and q= 0.8, respectively.

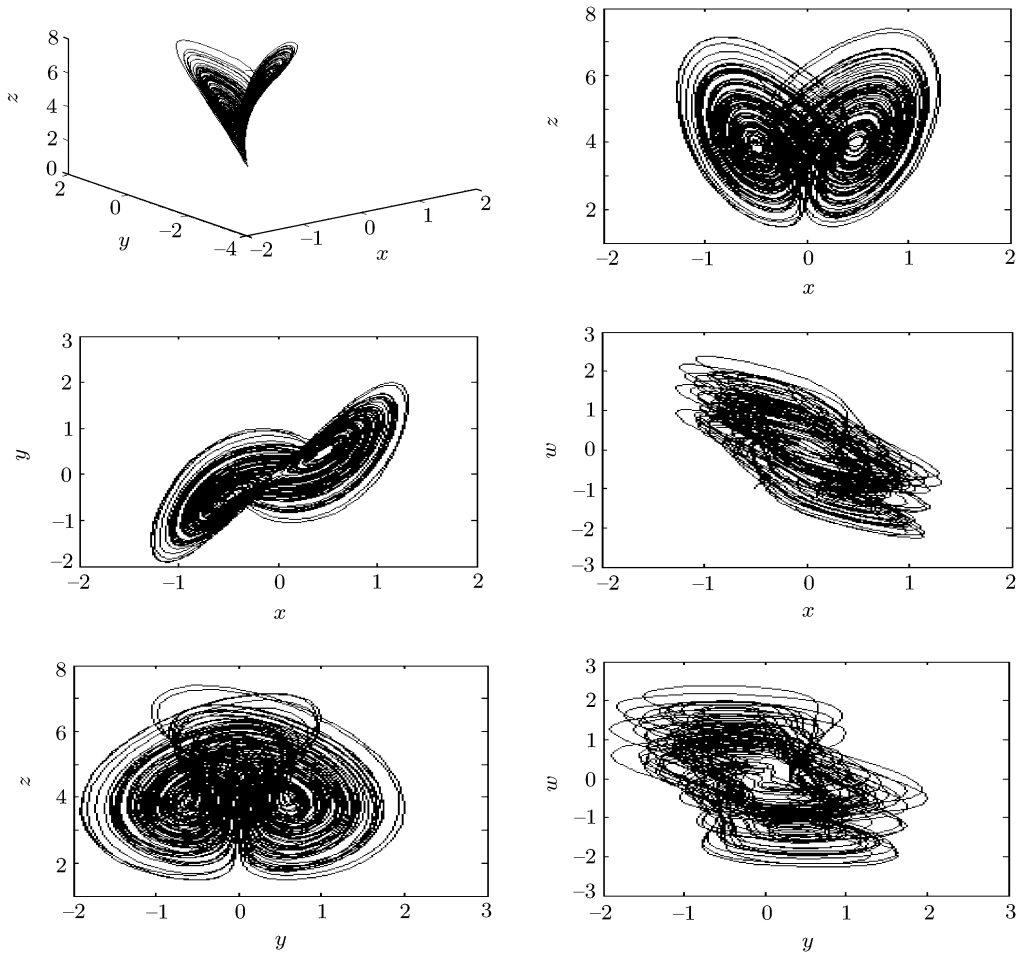


Fig.3. The chaotic attractor of system (2) for m=6 and q=0.9.

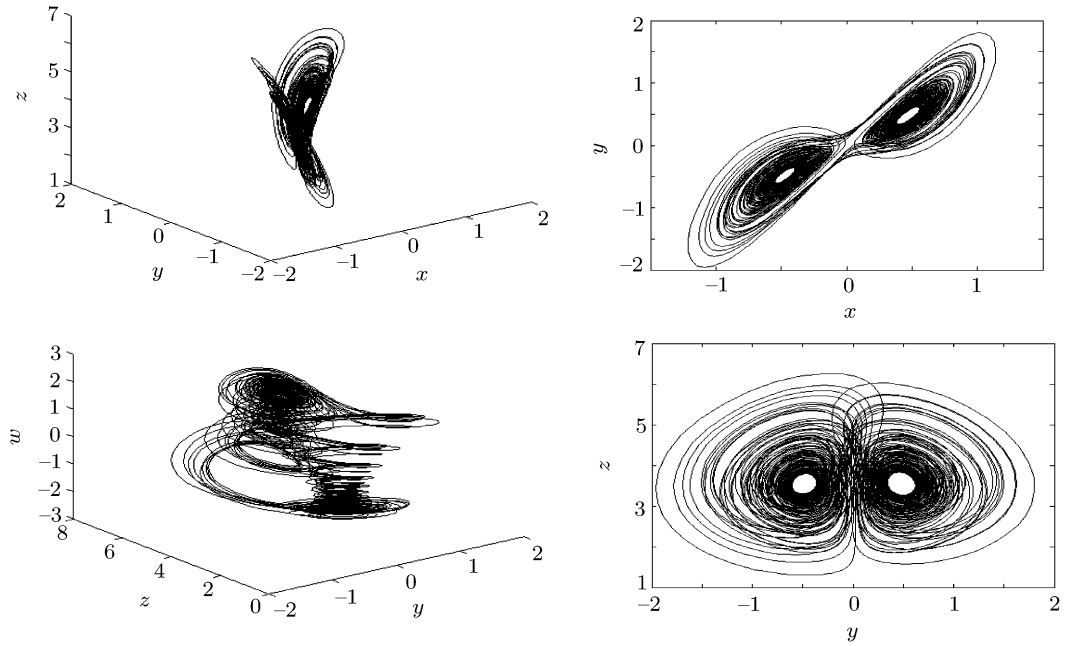


Fig.4. The chaotic attractor of system (2) for m=6 and q=0.8.

If we choose q=0.78 and m=6, so the fractional-order system (2) is nonchaotic. Based on Ref.[9, 10], the saddle points of index 2 become stable. Some simulation results are shown in the following two figures.

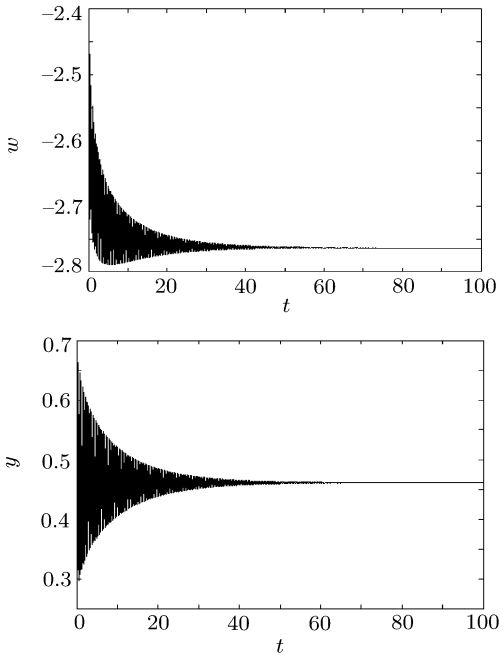


Fig.5. The fractional-order system (2) is no-chaotic for q=0.78 and m=6.

4. The electronic workbench (EWB) for fractional-order chaotic system (2)

A circuit is designed to realize the fractional-order chaotic system (2) as shown in Fig.6 for m=6 and q=0.9. In Fig.6(a) $R_1 = R_2 = R_4 = R_7 = R_{12} = R_{14} = R_{15} = R_{17} = R_{21} = R_{22} = R_{23} = R_{24}$=10kΩ, R_{25}=60kΩ, R_5= R_6=20kΩ, $R_3 = R_{10} = R_{11} = R_{13} = R_{19}$=100kΩ, R_{20}=400kΩ, $R_9 = R_{16}$=40kΩ, R_8=80kΩ. Where R_{18} is a variable resistor. If R_{18}=6kΩ, then m=6. The multiplier is AD633, and operational amplifier is LF353N, the power supply is ±15 V.

Box F is shown in Fig.6(b),[12] where R_1 = 62.84MΩ, R_2 = 0.25MΩ, R_3 = 0.0025MΩ, C_1 = 1.232μF, C_2 = 1.835μF, C_1 = 1.1μF.

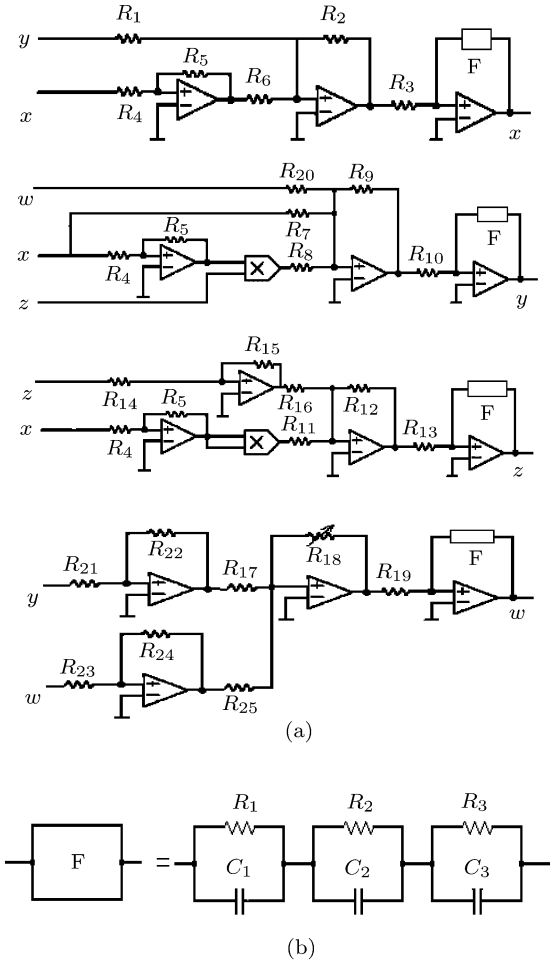
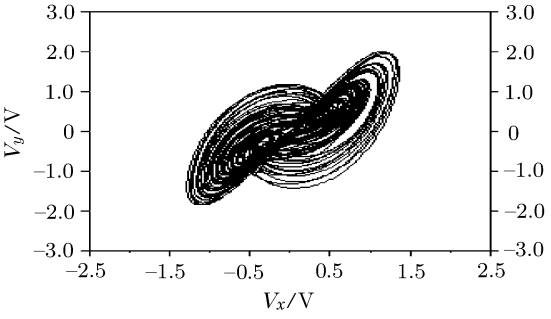
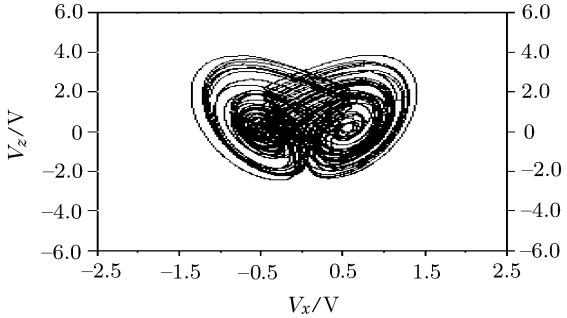


Fig.6. Circuit to realize the fractional-order chaotic system (2) for m=6 and q=0.9.

Utilizing EWB to simulate system (2), we obtain its phase portraits as shown in Fig.7.





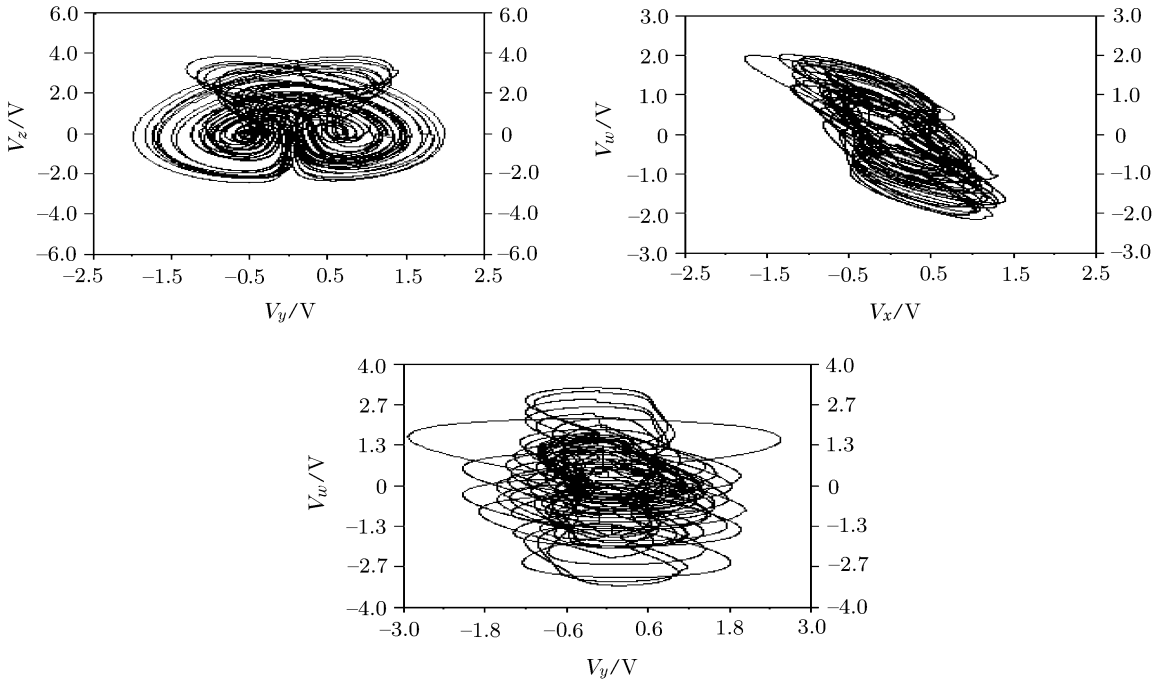


Fig.7. The chaotic attractor of system (2) for m=6 and q= 0.9 simulated by EWB.

5. Conclusions

We construct one new chaos system and its fractional-order chaotic system. The Lyapunov exponents spectrum of this new chaotic system is obtained. The necessary condition for the existence of chaotic attractors in this new fractional-order system is found to be $q > 0.783$. The chaotic attractor of this new fractional-order chaotic system for q=0.8, and q=0.9 are obtained. A circuit is designed to realize the fractional-order chaotic system for q=0.9 by EWB.

Further work on this fractional-order system (2) is an ongoing research topic in our group.

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