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The periodic death and anabiosis of the entanglement between two moving atoms*

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This paper investigates the periodic death and anabiosis of the entanglement between two moving atoms interacting with the mode field, and discusses the influences of the atomic motion and the parameter of the mode field. The results show that, the atomic motion leads to the periodic death and anabiosis of the entanglement between two moving atoms, the time of the death and the amplitude of the anabiosis of the entanglement between two moving atoms depend on the initial states of two moving atoms and the parameter of the mode field.

Keywords: entanglement, periodic death and anabiosis, moving atom
PACC: 0365, 4250

1. Introduction

Quantum entanglement (QE) is one of the most profound features of quantum mechanics. It is key problem in Einstein–Podolsky–Rosen (EPR) Paradox,[1] quantum cryptography,[2] quantum teleportation,[3] quantum computation[4] and so on, and has been viewed as an elementary resource for quantum information processing.

In open systems, entanglement can vanish completely in a finite time for certain initial states. This phenomenon is usually called entanglement sudden death (ESD).[5,6] While under the same condition, after the entanglement dies, it can revive completely in a finite time, we call this phenomenon as entanglement sudden anabiosis (ESA). ESD is an intriguing and potentially very important discovery. Since the first theory of ESD is demonstrated, further investigations of various systems have been made by different groups.[6−15] By using other entanglement measures,[16,17] the ESD has been observed for more complicated systems, and an attempt has also been made to give a geometric interpretation for the phenomena of ESD.[18] Recently, experimental researches have also been carried out to demonstrate ESD by using engineered interaction between systems and environments.[19,20]

In this paper, we investigate the entanglement of two moving atoms interacting with the mode field, and discuss the influences of the initial state of moving atoms and the parameter of the mode field. The results show that, this system may present not only the periodic sudden death of the entanglement, but also the periodic anabiosis of the entanglement, and the periodic sudden death and anabiosis of the entanglement are relative to the initial state of moving atoms and the parameter of the mode field.

2. The model and its solution

We consider two moving two-level atoms interacting with a single-mode field in the cavity. The effective Hamiltonian in the rotating-wave approximation[21] is ($\hbar = 1$)

$$H = \omega a^{+}a + +\omega_0 \sum_{i=1}^{2} S_z^i + g \sum_{i=1}^{2} f(z_i)(aS_+^i + a^{+}S_-^i), \quad (1)$$

where S_3^i and $S_\pm^i$ are the pseudo-spin operators of ith atom. $a^{+}(a)$ is the photon creation (annihilation) operator of the mode field. ω_0 and ω are the frequencies of the atomic transition and the mode field respectively; g is the atoms–field coupling constant. $f(z_i)$ is the shape function of the atomic motion along the z axis so that only the z-dependence of the mode field

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function would be needed to take into account. The atomic motion can be incorporated in an usual way

$$f(z_i) \to f(v_i t), \tag{2}$$

where v_i denotes the ith atomic motion velocity. In order to be specific, we will define the $\mathrm{TEM}_{\mathrm{mnp}}$ modes:

$$f(z_i) = \sin\left(\frac{\pi p v_i t}{L}\right), \tag{3}$$

where p represents the number of half-wave lengths of the field mode inside a cavity of the length L. For simplicity, we consider that two moving atoms have the same velocity so that $f(z_1) = f(z_2)$ under the resonant case ($\omega_0 = \omega$).

Here, we assume that two moving two-level atoms enter the cavity at time $t = 0$ in the superposition state

$$\begin{aligned} |\psi_{\mathrm{A}}(0)\rangle &= \cos(\gamma/2)|++\rangle \\ &\quad + \exp(-\mathrm{i}\varphi)\sin(\gamma/2)|--\rangle, \end{aligned} \tag{4}$$

and the coherent mode field is

$$|\psi_{\mathrm{F}}(0)\rangle = \sum_{n=0}^{\infty} f_n |n\rangle, \tag{5}$$

and $f_n = \exp\left(-\frac{N}{2}\right)\frac{N^{\frac{n}{2}}}{\sqrt{n!}}\exp(\mathrm{i}\psi)$, where N is the average photon number and ψ is the phase angle of the mode field (in this paper $\psi = 0$). The initial state of the system is

$$|\psi_{\mathrm{FA}}(0)\rangle = |\psi_{\mathrm{F}}(0)\rangle \otimes |\psi_{\mathrm{A}}(0)\rangle. \tag{6}$$

Using the standard techniques,[22] we can show that this Hamiltonian gives rise to the following time evolution operator in the interaction picture:[23]

$$U_{\mathrm{I}}(t) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned} a_{11} &= \frac{a^+a + 2 + (a^+a+1)\cos\left(g\Theta(t)\sqrt{2(2a^+a+3)}\right)}{2a^+a+3}, \\ a_{12} &= a_{13} = -\mathrm{i}\frac{\sin\left(g\Theta(t)\sqrt{2(2a^+a+3)}\right)}{\sqrt{2(2a^+a+3)}}a, \\ a_{14} &= \frac{-1+\cos\left(g\Theta(t)\sqrt{2(2a^+a+3)}\right)}{2a^+a+3}a^2, \\ a_{21} &= a_{31} = -\mathrm{i}\frac{\sin\left(g\Theta(t)\sqrt{2(2a^+a+1)}\right)}{\sqrt{2(2a^+a+1)}}a^+, \\ a_{22} &= a_{33} = \frac{1+\cos\left(g\Theta(t)\sqrt{2(2a^+a+1)}\right)}{2}, \\ a_{23} &= a_{32} = \frac{-1+\cos\left(g\Theta(t)\sqrt{2(2a^+a+1)}\right)}{2}, \\ a_{24} &= a_{34} = -\mathrm{i}\frac{\sin\left(g\Theta(t)\sqrt{2(2a^+a+1)}\right)}{\sqrt{2(2a^+a+1)}}a, \\ a_{41} &= \frac{-1+\cos\left(g\Theta(t)\sqrt{2(2a^+a-1)}\right)}{2a^+a-1}(a^+)^2, \\ a_{42} &= a_{43} = -\mathrm{i}\frac{\sin\left(g\Theta(t)\sqrt{2(2a^+a-1)}\right)}{\sqrt{2(2a^+a-1)}}a^+, \\ a_{44} &= \frac{a^+a - 1 + a^+a\cos\left(g\Theta(t)\sqrt{2(2a^+a-1)}\right)}{2a^+a-1} \end{aligned} \tag{8}$$

with

$$\Theta(t) = \int_0^t f(vt')\mathrm{d}t' = \frac{L}{\pi p v}\left[1 - \cos\frac{\pi p v t}{L}\right]. \tag{9}$$

For a particular choice of the atomic motion velocity $v = \frac{gl}{\pi}$, $\Theta(t)$ becomes

$$\Theta(t) = \frac{1}{pg}[1 - \cos(pgt)]. \tag{10}$$

At any time $t > 0$, the state vector of the system is given by

$$\begin{aligned} |\psi_{\mathrm{FA}}(t)\rangle &= U_{\mathrm{I}}(t)|\psi_{\mathrm{FA}}(0) \\ &= |a\rangle|++\rangle + |b\rangle|+-\rangle \\ &\quad + |c\rangle|-+\rangle + |d\rangle|--\rangle, \end{aligned} \tag{11}$$

with

$$\begin{aligned} |a\rangle &= \cos(\gamma/2)\sum_{n=0}^{\infty} a_{11} f_n |n\rangle \\ &\quad + \exp(-\mathrm{i}\varphi)\sin(\gamma/2)\sum_{n=0}^{\infty} a_{14} f_n |n\rangle, \\ |b\rangle &= \cos(\gamma/2)\sum_{n=0}^{\infty} a_{21} f_n |n\rangle \\ &\quad + \exp(-\mathrm{i}\varphi)\sin(\gamma/2)\sum_{n=0}^{\infty} a_{24} f_n |n\rangle, \\ |c\rangle &= \cos(\gamma/2)\sum_{n=0}^{\infty} a_{31} f_n |n\rangle \\ &\quad + \exp(-\mathrm{i}\varphi)\sin(\gamma/2)\sum_{n=0}^{\infty} a_{34} f_n |n\rangle, \\ |d\rangle &= \cos(\gamma/2)\sum_{n=0}^{\infty} a_{41} f_n |n\rangle \\ &\quad + \exp(-\mathrm{i}\varphi)\sin(\gamma/2)\sum_{n=0}^{\infty} a_{44} f_n |n\rangle. \end{aligned} \tag{12}$$

At any time $t > 0$, the density operator for the system is given by

$$\rho_{\mathrm{FA}}(t) = |\psi_{\mathrm{FA}}(t)\rangle\langle\psi_{\mathrm{FA}}(t)|. \tag{13}$$

The information about the entanglement between two moving atoms is contained in the reduced density matrix $\rho_{\rm A}$. In the system, $\rho_{\rm A}$ can be obtained from Eq.(13) by tracing out the photonic part of the total pure state. In the bases $|++\rangle$, $|+-\rangle$, $|-+\rangle$, and $|--\rangle$ $\rho_{\rm A}$ can be given by

$$\begin{aligned}\rho_{\rm A}(t) &= {\rm Tr_f}(\rho_{\rm FA}(t)) \\ &= \begin{pmatrix} \langle a|a\rangle & \langle b|a\rangle & \langle c|a\rangle & \langle d|a\rangle \\ \langle a|b\rangle & \langle b|b\rangle & \langle c|b\rangle & \langle d|b\rangle \\ \langle a|c\rangle & \langle b|c\rangle & \langle c|c\rangle & \langle d|c\rangle \\ \langle a|d\rangle & \langle b|d\rangle & \langle c|d\rangle & \langle d|d\rangle \end{pmatrix}. \end{aligned} \quad (14)$$

3. The periodic death and anabiosis of the entanglement between two moving atoms

To describe the dynamic evolution of quantum entanglement between two moving atoms, we need a concrete measure of the degree of entanglement contained in a quantum state. For any two-qubit case, Wootters concurrence[24] is particularly convenient. Any reliable measure of entanglement will yield the same conclusions. The concurrence varies from $C = 0$ for a separable state to $C = 1$ for a maximally entangled state. The concurrence is defined by

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (15)$$

where λ_i are the eigenvalues in decreasing order of the matrix $\tilde{\rho} = \rho_{\rm A}(\sigma_y \otimes \sigma_y)\rho_{\rm A}^*(\sigma_y \otimes \sigma_y)$, σ_y is the Pauli matrix, $\rho_{\rm A}$ is the reduced density matrix representing the quantum state given by Eq.(14) and the matrix elements are taken with respect to the bases $|++\rangle$, $|+-\rangle$, $|-+\rangle$,$|--\rangle$.

We present numerical results of the concurrence C given by Eq.(15) for different parameters of two moving atoms and the mode field in Figs.1 and 2. Figure 1 shows the influences of the parameters of the two atomic initial states on the concurrence C for the given average photon number N and the mode field parameter p, while Fig.2 illustrates the effects of the mode field parameter p on the concurrence C for some initial states of two moving atoms.

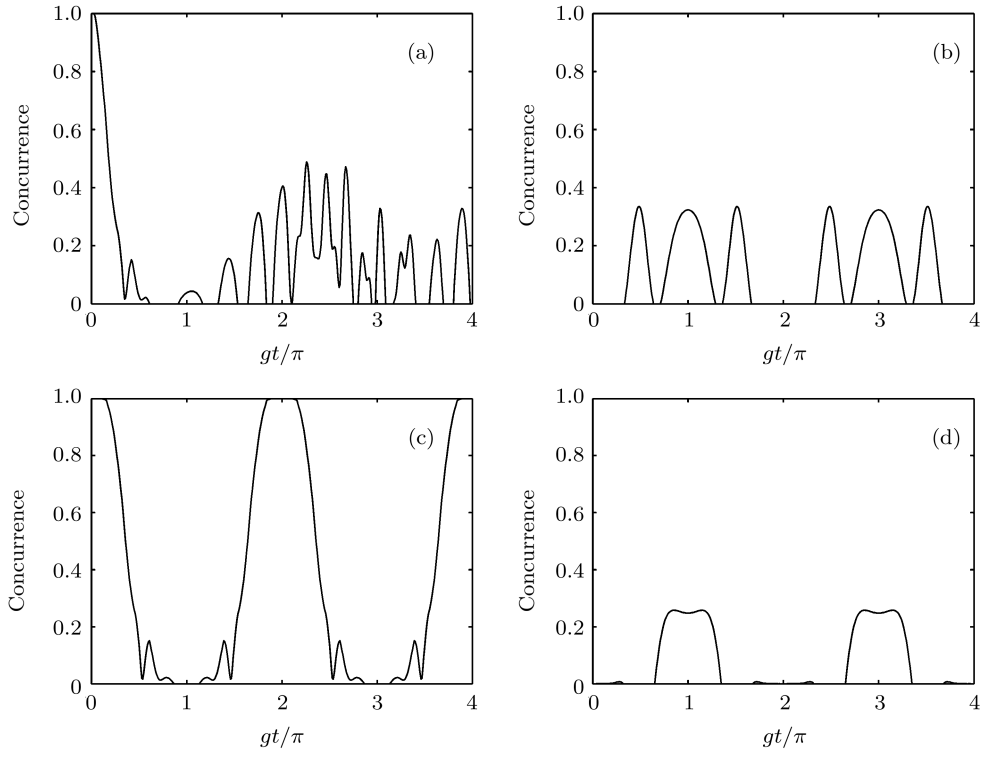


Fig.1. The time evolution of the concurrence C for the moving atom–atom entanglement with the atomic initial state $|\psi_{\rm atom}\rangle = \cos(\gamma/2)|++\rangle + \sin(\gamma/2)|--\rangle$ is shown with $N = 5$ and $p = 1$: (a) $\gamma = \pi/2$ (the motion of two atoms is not taken into account), (b) $\gamma = 0$, (c) $\gamma = \pi/2$ and (d) $\gamma = \pi$.

In Fig.1(a), the motion of two atoms is not taken into account for $N = 5$, $p = 1$ and $\gamma = \pi/2$ (namely the two atoms are initially in the maximal entangled state). It is obvious that concurrence C between the two atoms evolves without periodicity, and there are distinct death with long death time and anabiosis with short revival time of the entanglement. When the motion of two atoms is taken into account, it is evident that the concurrence C of two moving atoms evolves periodically with period 2π shown in Figs.1(b), 1(c), 1(d). In the time evolution process, we can see that the death and anabiosis of the entanglement between two moving atoms become periodic. This periodical dynamic behaviour of the concurrence C depends on the mode field parameter p, and it can be understood by Eq.(10). From Eq.(10), we have $g\Theta(t) = 1/p[1-\cos(pgt)]$. It is observed that $g\Theta(t)$ is a periodical function on the scaled time gt with period $2\pi/p$, and when $p = 1$, the entanglement of the atom–atom evolves in period 2π. The time of the death and the time of the anabiosis of the entanglement between two moving atoms rely on the initial state of two moving atoms. For example, when two moving atoms are initially in their excited states (direct product state), it presents the death of the entanglement with long time at $gt = 2k\pi$ $(k = 0, 1, 2, \ldots)$ near here and the anabiosis of the entanglement with short time at $gt = (2k + 1)\pi$ and $(2k + 1)\pi/2$ $(k = 0, 1, 2, \ldots)$ near here(see in Fig.1(b)). While when two moving atoms are initially in their maximal entangled state, the death of the entanglement with long time appears at $gt = (2k + 1)\pi$ $(k = 0, 1, 2, \ldots)$ hereabout (see in Fig.1(c)), and anabiosis of the entanglement with long time at $gt = 2k\pi$ $(k = 0, 1, 2, \ldots)$ hereabout. For two moving atoms are initially in their ground states (direct product state) in Fig.1(d), the death of the entanglement with long time exhibits at $gt = 2k\pi$ near here and anabiosis of the entanglement with long time appears at $gt = (2k + 1)\pi$ near here. From Fig.1, we can also see that, when two moving atoms are initially in direct product states, the amplitude of anabiosis of their entanglement is small, while when two moving atoms are initially in their maximal entangled state, the amplitude of anabiosis of their entanglement is the largest.

The influence of the parameter p of the mode field on the entanglement between two moving atoms is shown in Fig.2. In Figs.2(a) and 2(b), when two

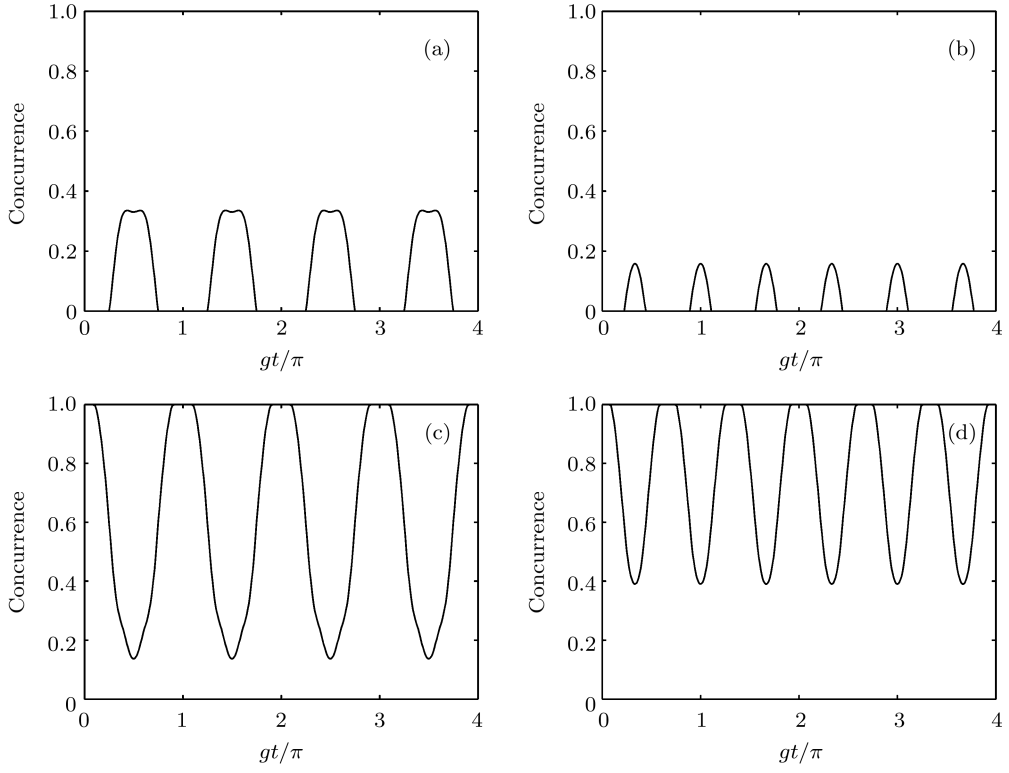


Fig.2. The time evolution of the concurrence C for the moving atom–atom entanglement with the initial atomic state $|\psi_{\rm atom}\rangle = \cos(\gamma/2)|++\rangle + \sin(\gamma/2)|--\rangle$ with $N = 5$, (a) $\gamma = 0$, $p = 2$; (b) $\gamma = 0$, $p = 3$; (c) $\gamma = \pi/2$, $p = 2$; (d) $\gamma = \pi/2$, $p = 3$.

moving atoms are initially in their excited states, we show two cases $p = 2$ and $p = 3$, respectively. It is explicit that the period shortens, and the amplitude decreases for the concurrence C as p increases, and there are periodic death and anabiosis of the entanglement between two moving atoms. While when two moving atoms initially are in the maximal entangled state, we also explore two cases $p = 2$ and $p = 3$, respectively, which are shown in Figs.2(c) and 2(d). It is obvious that with increase of parameter p, the death phenomena of entanglement between two moving atoms disappear when we compare the Figs.2(c) and 2(d) with Fig.1(c).

4. Conclusion

In this paper, we have studied the periodic death and anabiosis of entanglement between two moving atoms interacting with the mode field. We can conclude as follows:

(1) The atomic motion leads to the periodic evolution of the entanglement between two moving atoms. The period depends on the parameter p. The larger p is, the shorter the period is.

(2) The time of the death and the amplitude of the anabiosis of the entanglement between two moving atoms depend on the initial states of two moving atoms. When two moving atoms are initially in the direct product state, the entanglement exhibits longer death time and smaller anabiosis amplitude. Whereas when two moving atoms are initially in the maximal entangled state, it shows shorter death time and the largest anabiosis.

(3) The influence of the parameter p of the mode field on the entanglement between two moving atoms is: when two moving atoms are initially in the direct product state, the death of the entanglement still exists with the change of the parameter p; while when two moving atoms are initially in the maximal entangled state, the increase of parameter p results in disappearance of the death of the entanglement between two moving atoms.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[4] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
[5] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[6] Yu T and Eberly J H 2006 *Opt. Commun.* **264** 393
[7] Yönac M, Yu T and Eberly J H 2006 *J. Phys.* B **39** S621
[8] Anastopoulos C, Shresta S and Hu B L 2006 quant-ph/061007
[9] Aty M A 2006 quant-ph/0610186
[10] Cui H T 2007 *Phys. Lett.* A **365** 44
[11] Yu T and Eberly J H 2007 *Quantum Information and Computation* **7** 459
[12] Yönac M, Yu T and Eberly J H 2007 *J. Phys.* B **40** S45
[13] Lastra F, Romero G, López C E, Santos M F and Retamal J C 2007 *Phys. Rev.* A **75** 062324
[14] Zhang G F 2007 *Opt. Commun.* **275** 274
[15] Vaglica A and Vetric G 2007 *Phys. Rev.* A **75** 062120
[16] Aty M A and Cessa H M 2007 quant-ph/0703077
[17] Sainz I and Björk G 2007 *Phys. Rev.* A **76** 042313
[18] Cunha M O T 2007 quant-ph/0703121
[19] Almeida M P 2007 *Science* **316** 579
[20] Laurat J, Choi K S, Deng H, Chou C W and Kimble H J 2007 *Phys. Rev. Lett.* **99** 180504
[21] Schlicher R R 1989 *Opt. Commun.* **70** 97
[22] Hou L Z and Zeng G J 2001 *Chin. Phys.* **10** 821
[23] Fujii K, Higashida K, Kato R and Wada Y 2005 *The Bulletin of Yokohama City University* **56** 51
[24] Basharov A M 2002 *Jetp. Lett.* **75** 123