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Teleportation of arbitrary unknown two-atom state with cluster state via thermal cavity*

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This paper proposes a scheme for implementing the teleportation of an arbitrary unknown two-atom state by using a cluster state of four identical 2-level atoms as quantum channel in a thermal cavity. The two distinct advantages of the present scheme are: (i) The discrimination of 16 orthonormal cluster states in the standard teleportation protocol is transformed into the discrimination of single-atom states. Consequently, the discrimination difficulty of states is degraded. (ii) The scheme is insensitive to the cavity field state and the cavity decay for the thermal cavity is only virtually excited when atoms interact with it. Thus, the scheme is more feasible.

Keywords: teleportation, thermal cavity, atomic entangled state, atomic cluster state
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The nonlocal property of quantum mechanics enables a striking phenomenon called quantum teleportation. In terms of a classical communication channel and a quantum resource, the teleportation protocols give ways to teleport arbitrary unknown quantum information from a sender to a remote receiver. Hence, quantum teleportation plays an important role in quantum information processing. After Bennett *et al*'s pioneering work[1] presented in 1993, this topic has already attracted much attention so far.[2−36] As far as all the relevant issues discussed in various literatures are concerned, they can be simply classified into three areas: (i) what kind of states are to be teleported; (ii) what kind of quantum channels inhabited by quantum entanglement are to be used; (iii) how to implement the physical realization. As a matter of fact, in the theoretical aspects, people have allowed for teleporting various quantum states, such as the qubit states,[2,3] the qudit states,[4−6] the continuous-variable states,[7−9] the discrete-variable states,[10−13] etc.[14] When designing the teleportation protocols, people have already tried different quantum channels. For examples, the maximally entangled Bell states are employed in the original standard teleportation protocol,[1] the non-maximally entangled states in Ref.[15], the mixed states in Refs.[16–18], the W-class states in Refs.[19,20], the genuine states in Ref.[21] and the cluster states in Ref.[22]. As far as the physical realization, many protocols based on different physical systems have been proposed, e.g., the iron-trap system,[23,24] the nuclear magnetic resonance system,[26] the atom–cavity system,[27,28] the cavity quantum electrodynamics (QED) systems,[29−36] and so on.

In 2001, Briegel and Raussendorf[37] introduced a new sort of entangled state named as the cluster state. In quantum computation the cluster state has several advantages (e.g., it is more immune to decoherence than GHZ states.[38]) Hence recently they have attracted much attention[22,39−48] and various schemes for generating them in different physical systems have been proposed, including those in cavity QED systems.[44−47] It is worth stressing that, as a kind of entangled state, cluster states can also be used as the quantum channel candidate for perfect teleportation in some special conditions. This feature has

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already been revealed very recently.[22]

In this paper, we will propose a scheme for implementing the teleportation of an arbitrary unknown two-atom state by using a cluster state of four identical 2-level atoms as quantum channel. The present scheme owns the following two advantages: (1) the discrimination of 16 orthonormal cluster states in the standard teleportation protocol is transformed into the discrimination of single-atom states through controlling atom–cavity–field interaction. Consequently the discrimination difficulty is degraded. (2) The scheme is insensitive to the cavity field state and the cavity decay for the thermal cavity is only virtually excited when atoms interact with it. Thus, the scheme is more feasible. One will see these advantages later.

Before presenting our scheme in detail, let us concisely review the atom–cavity–field model and work out in prior some useful formulae within the model frame. Suppose there are N identical 2-level atoms. Those atoms are sent to pass simultaneously through a single-mode cavity and meanwhile be driven by a classical field. For such an atom–cavity–field interacting system, in the rotating-wave approximation the general Hamiltonian reads[46−49] ($\hbar = 1$, same hereafter)

$$H = \omega_{\rm a} S_z + \omega_{\rm c} a^+ a + \sum_{j=1}^{N} \left[\frac{\kappa}{2} (a^+ S_j^- + a S_j^+) + \frac{\Omega}{2} (S_j^+ {\rm e}^{-{\rm i}\omega_{\rm f} t} + S_j^- {\rm e}^{{\rm i}\omega_{\rm f} t}) \right], \quad (1)$$

where $S_z = \frac{1}{2} \sum_{j=1}^{N} (|e_j\rangle \langle e_j| - |g_j\rangle \langle g_j|)$, $S_j^+ = |e_j\rangle \langle g_j|$, $S_j^- = |g_j\rangle \langle e_j|$, $|g_j\rangle$ and $|e_j\rangle$ denote the ground and excited states of the jth atom respectively, a^+ and a are respectively the creation and annihilation operators for the cavity mode, κ is the atom–cavity coupling strength, Ω is the Rabi frequency, $\omega_{\rm a}$ is the atomic transition frequency, $\omega_{\rm c}$ is the cavity frequency, $\omega_{\rm f}$ is the frequency of the classical field, and N is the atom number. Choosing $\omega_{\rm a} = \omega_{\rm f}$, then in the interaction picture, we get the interaction Hamiltonian

$$H_{\rm I} = \sum_{j=1}^{N} \left\{ \Omega \sigma_{z,j} + \frac{\kappa}{2} \left[{\rm e}^{-{\rm i}\delta t} a^+ \left(\sigma_{z,j} + \frac{1}{2} \sigma_j^+ - \frac{1}{2} \sigma_j^- \right) + {\rm e}^{{\rm i}\delta t} a \left(\sigma_{z,j} + \frac{1}{2} \sigma_j^- - \frac{1}{2} \sigma_j^+ \right) \right] \right\}, \quad (2)$$

where δ is the detuning between the atomic transition frequency $\omega_{\rm a}$ and the cavity frequency $\omega_{\rm c}$, $\sigma_{z,j} = \frac{1}{2}(|+_j\rangle \langle +_j| - |-_j\rangle \langle -_j|)$, $\sigma_j^+ = |+_j\rangle \langle -_j|$, $\sigma_j^- = |-_j\rangle \langle +_j|$, $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$, and $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$. Performing the unitary transformation $|\Psi(t)\rangle = {\rm e}^{-{\rm i}H_0 t} |\psi(t)\rangle$ with $H_0 = \sum_{j=1}^{N} \Omega \sigma_{z,j}$, one can obtain the effective Schrödinger equation[49]

$${\rm i}[{\rm d}|\psi(t)\rangle/{\rm d}t] = \mathcal{H}_{\rm I} |\psi(t)\rangle \quad (3)$$

with

$$\mathcal{H}_{\rm I} = \sum_{j=1}^{N} \frac{\kappa}{2} \left[{\rm e}^{-{\rm i}\delta t} a^+ \left(\sigma_{z,j} + \frac{1}{2} \sigma_j^+ {\rm e}^{{\rm i}\Omega t} - \frac{1}{2} \sigma_j^- {\rm e}^{-{\rm i}\Omega t} \right) + {\rm e}^{{\rm i}\delta t} a \left(\sigma_{z,j} + \frac{1}{2} \sigma_j^- {\rm e}^{-{\rm i}\Omega t} - \frac{1}{2} \sigma_j^+ {\rm e}^{{\rm i}\Omega t} \right) \right]. \quad (4)$$

Further, if $\Omega \gg \delta$ and $\Omega \gg \kappa$ are assumed, then the terms oscillating fast can be eliminated and the effective interaction Hamiltonian is reduced to[49]

$$\mathcal{H}_{\rm I} = \frac{\kappa}{2} ({\rm e}^{-{\rm i}\delta t} a^+ + {\rm e}^{{\rm i}\delta t} a) S_x, \quad (5)$$

where $S_x = \frac{1}{2} \sum_{j=1}^{N} (S_j^+ + S_j^-)$. In this case, the evolution operator can be written as[46,47]

$$U'(t) = {\rm e}^{-{\rm i}A(t) S_x^2} {\rm e}^{-{\rm i}B(t) S_x a} {\rm e}^{-{\rm i}C(t) S_x a^+}, \quad (6)$$

where $A(t) = \frac{\kappa^2}{4\delta} \left[t + \frac{1}{{\rm i}\delta} ({\rm e}^{-{\rm i}\delta t} - 1) \right]$, $B(t) = \frac{\kappa}{2{\rm i}\delta} ({\rm e}^{{\rm i}\delta t} - 1)$ and $C(t) = -\frac{\kappa}{2{\rm i}\delta} ({\rm e}^{-{\rm i}\delta t} - 1)$. Choosing an appropriate interaction time τ such that $\delta\tau = 2\pi$, one can obtain $B(\tau) = C(\tau) = 0$. In this case, the evolution of the system is[49]

$$|\Psi\rangle \to {\rm e}^{-{\rm i}H_0 \tau} U'(\tau) |\Psi\rangle = {\rm e}^{-{\rm i}\Omega\tau S_x - {\rm i}\lambda\tau S_x^2} |\Psi\rangle, \quad (7)$$

where $\lambda = \frac{\kappa^2}{4\delta}$. Note that the operator ${\rm e}^{-{\rm i}\Omega\tau S_x - {\rm i}\lambda\tau S_x^2}$ is independent of the cavity field state and accordingly it is allowed to be in a thermal state. Using Eq.(7) one can work out the wavefunction of the N atoms after the interaction time τ. We have considered two cases. To be more specific, in the case that $N = 2$ and the coupling strength κ and Rabi frequency Ω are appropriately chosen such that $\Omega\tau = 2n\pi$ and $\lambda\tau = \pi/2$ (i.e., $\Omega\tau = 2n\pi$ and $\kappa\tau = 2\pi$), we have worked out the states of the two atoms before and after the interaction:

$$|gg\rangle \to \frac{1}{\sqrt{2}} {\rm e}^{-{\rm i}\frac{\pi}{4}} (|gg\rangle - {\rm i}\,|ee\rangle),$$

$$|ge\rangle \to \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}\left(|ge\rangle - \mathrm{i}\,|eg\rangle\right),$$
$$|eg\rangle \to \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}\left(|eg\rangle - \mathrm{i}\,|ge\rangle\right),$$
$$|ee\rangle \to \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}\left(|ee\rangle - \mathrm{i}\,|gg\rangle\right). \quad (8)$$

We also considered the case of $N = 4$. The state transformation of the four atoms is as follows

$$|xymn\rangle \to \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|xymn\rangle + \mathrm{i}|(x+1) \times(y+1)(m+1)(n+1))), \quad (9)$$

where $|x\rangle$, $|y\rangle$, $|m\rangle$ and $|n\rangle$ denote the atomic state $|g\rangle$ or $|e\rangle$, and $|g+1\rangle = |e\rangle$ and $|e+1\rangle = |g\rangle$ are defined. Note that the Eqs.(8) and (9) will be useful in our teleportation scheme presented later.

Now let us turn to the detailed presentation of our teleportation scheme. Suppose Alice and Bob are in two different spaces. They both have six 2-level atoms in total. Alice has four (named atoms 1, 3, 5 and 6 hereafter) while Bob has two (named atoms 2 and 4 hereafter). The atoms 1, 2, 3 and 4 are in a cluster state, which takes the form of

$$|\Psi\rangle_{1234} = \frac{1}{2}(|gggg\rangle_{1234} + |ggee\rangle_{1234} + |eegg\rangle_{1234} - |eeee\rangle_{1234}). \quad (10)$$

This state is the quantum channel linking two places for teleportation. Incidentally, this state can be prepared via thermal cavity (e.g., see Refs.[44–47]). The atoms 5 and 6 are in an arbitrary unknown state, which can be written as

$$|\Phi\rangle_{56} = \alpha\,|gg\rangle_{56} + \beta\,|ge\rangle_{56} + \gamma\,|eg\rangle_{56} + \zeta\,|ee\rangle_{56}, \quad (11)$$

where α, β, γ and ζ are complex and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\zeta|^2 = 1$. In this case, the state of the 6-atom system is

$$\begin{aligned}|\Upsilon\rangle_{123456} &= \frac{1}{2}\left(|gggg\rangle_{1234} + |ggee\rangle_{1234} + |eegg\rangle_{1234} - |eeee\rangle_{1234}\right) \otimes \left(\alpha\,|gg\rangle_{56} + \beta\,|ge\rangle_{56} + \gamma\,|eg\rangle_{56} + \zeta\,|ee\rangle_{56}\right)\\ &= \frac{1}{2}\{[\alpha\left(|gggg\rangle_{1245} + |eegg\rangle_{1245}\right) + \gamma\left(|ggge\rangle_{1245} + |eege\rangle_{1245}\right)]\,|gg\rangle_{36}\\ &\quad + [\beta\left(|gggg\rangle_{1245} + |eegg\rangle_{1245}\right) + \zeta\left(|ggge\rangle_{1245} + |eege\rangle_{1245}\right)]\,|ge\rangle_{36}\\ &\quad + [\alpha\left(|ggeg\rangle_{1245} - |eeeg\rangle_{1245}\right) + \gamma\left(|ggee\rangle_{1245} - |eeee\rangle_{1245}\right)]\,|eg\rangle_{36}\\ &\quad + [\beta\left(|ggeg\rangle_{1245} - |eeeg\rangle_{1245}\right) + \zeta\left(|ggee\rangle_{1245} - |eeee\rangle_{1245}\right)]\,|ee\rangle_{36}\}. \quad (12)\end{aligned}$$

Both Alice and Bob want to teleport this arbitrary state initially in the atoms 5 and 6 to the atoms 2 and 4. To achieve this goal, Alice first lets the atoms 3 and 6 pass through a single-mode cavity and meanwhile be driven by a classical field. As explained before, setting $\delta t = 2\pi$ (also, $\Omega t = 2n\pi$ and $\kappa t = 2\pi$), then after the atoms 3 and 6 interact with the cavity and the classical field, in terms of Eq.(8) one knows that the state of the six atoms is

$$\begin{aligned}|\Upsilon'\rangle_{123456} &= \frac{1}{2\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}\{[\alpha\left(|gggg\rangle_{1245} + |eegg\rangle_{1245}\right) + \gamma\left(|ggge\rangle_{1245} + |eege\rangle_{1245}\right)]\left(|gg\rangle_{36} - \mathrm{i}\,|ee\rangle_{36}\right)\\ &\quad + [\beta\left(|gggg\rangle_{1245} + |eegg\rangle_{1245}\right) + \zeta\left(|ggge\rangle_{1245} + |eege\rangle_{1245}\right)]\left(|ge\rangle_{36} - \mathrm{i}\,|eg\rangle_{36}\right)\\ &\quad + [\alpha\left(|ggeg\rangle_{1245} - |eeeg\rangle_{1245}\right) + \gamma\left(|ggee\rangle_{1245} - |eeee\rangle_{1245}\right)]\left(|eg\rangle_{36} - \mathrm{i}\,|ge\rangle_{36}\right)\\ &\quad + [\beta\left(|ggeg\rangle_{1245} - |eeeg\rangle_{1245}\right) + \zeta\left(|ggee\rangle_{1245} - |eeee\rangle_{1245}\right)]\left(|ee\rangle_{36} - \mathrm{i}\,|gg\rangle_{36}\right)\}. \quad (13)\end{aligned}$$

It can be rewritten as

$$\begin{aligned}|\Upsilon'\rangle_{123456} &= \frac{1}{2\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}[|gggg\rangle_{1356}\left(\alpha\,|gg\rangle_{24} - \mathrm{i}\beta\,|ge\rangle_{24}\right)\\ &\quad + |ggge\rangle_{1356}\left(-\mathrm{i}\alpha\,|ge\rangle_{24} + \beta\,|gg\rangle_{24}\right) + |ggeg\rangle_{1356}\left(\gamma\,|gg\rangle_{24} - \mathrm{i}\zeta\,|ge\rangle_{24}\right)\\ &\quad + |ggee\rangle_{1356}\left(-\mathrm{i}\gamma\,|ge\rangle_{24} + \zeta\,|gg\rangle_{24}\right) + |gegg\rangle_{1356}\left(\alpha\,|ge\rangle_{24} - \mathrm{i}\beta\,|gg\rangle_{24}\right)\\ &\quad + |gege\rangle_{1356}\left(-\mathrm{i}\alpha\,|gg\rangle_{24} + \beta\,|ge\rangle_{24}\right) + |geeg\rangle_{1356}\left(\gamma\,|ge\rangle_{24} - \mathrm{i}\zeta\,|gg\rangle_{24}\right)\\ &\quad + |geee\rangle_{1356}\left(-\mathrm{i}\gamma\,|gg\rangle_{24} + \zeta\,|ge\rangle_{24}\right) + |eggg\rangle_{1356}\left(\alpha\,|eg\rangle_{24} + \mathrm{i}\beta\,|ee\rangle_{24}\right)\\ &\quad + |egge\rangle_{1356}\left(\mathrm{i}\alpha\,|ee\rangle_{24} + \beta\,|eg\rangle_{24}\right) + |egeg\rangle_{1356}\left(\gamma\,|eg\rangle_{24} + \mathrm{i}\zeta\,|ee\rangle_{24}\right)\end{aligned}$$

$$
\begin{aligned}
&+|egee\rangle_{1356}(\mathrm{i}\gamma|ee\rangle_{24}+\zeta|eg\rangle_{24})+|eegg\rangle_{1356}(-\alpha|ee\rangle_{24}-\mathrm{i}\beta|eg\rangle_{24})\\
&+|eege\rangle_{1356}(-\mathrm{i}\alpha|eg\rangle_{24}-\beta|ee\rangle_{24})+|eeeg\rangle_{1356}(-\gamma|ee\rangle_{24}-\mathrm{i}\zeta|eg\rangle_{24})\\
&+|eeee\rangle_{1356}(-\mathrm{i}\gamma|eg\rangle_{24}-\zeta|ee\rangle_{24})].
\end{aligned}\tag{14}
$$

After this, Alice then lets the atoms 1, 3, 5 and 6 pass simultaneously through a single-mode cavity and meanwhile be driven by a classical field. Also setting $\delta t=2\pi$, $\Omega t=2n\pi$ and $\lambda t=\pi/2$, then after the second atom–cavity–field interaction, in terms of Eq.(9) one knows that the state of the six atoms is

$$
\begin{aligned}
&|\Upsilon''\rangle_{123456}\\
&=\frac{1}{4}\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}[(|gggg\rangle_{1356}+\mathrm{i}|eeee\rangle_{1356})(\alpha|gg\rangle_{24}-\mathrm{i}\beta|ge\rangle_{24})+(|ggge\rangle_{1356}+\mathrm{i}|eeeg\rangle_{1356})(-\mathrm{i}\alpha|ge\rangle_{24}\\
&+\beta|gg\rangle_{24})+(|ggeg\rangle_{1356}+\mathrm{i}|eege\rangle_{1356})(\gamma|gg\rangle_{24}-\mathrm{i}\zeta|ge\rangle_{24})+(|ggee\rangle_{1356}+\mathrm{i}|eegg\rangle_{1356})(-\mathrm{i}\gamma|ge\rangle_{24}\\
&+\zeta|gg\rangle_{24})+(|gegg\rangle_{1356}+\mathrm{i}|egee\rangle_{1356})(\alpha|ge\rangle_{24}-\mathrm{i}\beta|gg\rangle_{24})+(|gege\rangle_{1356}+\mathrm{i}|egeg\rangle_{1356})(-\mathrm{i}\alpha|gg\rangle_{24}\\
&+\beta|ge\rangle_{24})+(|geeg\rangle_{1356}+\mathrm{i}|egge\rangle_{1356})(\gamma|ge\rangle_{24}-\mathrm{i}\zeta|gg\rangle_{24})+(|geee\rangle_{1356}+\mathrm{i}|eggg\rangle_{1356})(-\mathrm{i}\gamma|gg\rangle_{24}\\
&+\zeta|ge\rangle_{24})+(|eggg\rangle_{1356}+\mathrm{i}|geee\rangle_{1356})(\alpha|eg\rangle_{24}+\mathrm{i}\beta|ee\rangle_{24})+(|egge\rangle_{1356}+\mathrm{i}|geeg\rangle_{1356})(\mathrm{i}\alpha|ee\rangle_{24}\\
&+\beta|eg\rangle_{24})+(|egeg\rangle_{1356}+\mathrm{i}|gege\rangle_{1356})(\gamma|eg\rangle_{24}+\mathrm{i}\zeta|ee\rangle_{24})+(|egee\rangle_{1356}+\mathrm{i}|gegg\rangle_{1356})(\mathrm{i}\gamma|ee\rangle_{24}\\
&+\zeta|eg\rangle_{24})+(|eegg\rangle_{1356}+\mathrm{i}|ggee\rangle_{1356})(-\alpha|ee\rangle_{24}-\mathrm{i}\beta|eg\rangle_{24})+(|eege\rangle_{1356}+\mathrm{i}|ggeg\rangle_{1356})(-\mathrm{i}\alpha|eg\rangle_{24}\\
&-\beta|ee\rangle_{24})+(|eeeg\rangle_{1356}+\mathrm{i}|ggge\rangle_{1356})(-\gamma|ee\rangle_{24}-\mathrm{i}\zeta|eg\rangle_{24})+(|eeee\rangle_{1356}+\mathrm{i}|gggg\rangle_{1356})(-\mathrm{i}\gamma|eg\rangle_{24}\\
&-\zeta|ee\rangle_{24})].
\end{aligned}\tag{15}
$$

It can be rewritten as

$$
\begin{aligned}
|\Upsilon''\rangle_{123456}=\;&\frac{1}{4}\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}[|gggg\rangle_{1356}(\alpha|gg\rangle_{24}-\mathrm{i}\beta|ge\rangle_{24}+\gamma|eg\rangle_{24}-\mathrm{i}\zeta|ee\rangle_{24})\\
&+|ggge\rangle_{1356}(-\mathrm{i}\alpha|ge\rangle_{24}+\beta|gg\rangle_{24}-\mathrm{i}\gamma|ee\rangle_{24}+\zeta|eg\rangle_{24})\\
&+|ggeg\rangle_{1356}(\alpha|eg\rangle_{24}-\mathrm{i}\beta|ee\rangle_{24}+\gamma|gg\rangle_{24}-\mathrm{i}\zeta|ge\rangle_{24})\\
&+|ggee\rangle_{1356}(-\mathrm{i}\alpha|ee\rangle_{24}+\beta|eg\rangle_{24}-\mathrm{i}\gamma|ge\rangle_{24}+\zeta|gg\rangle_{24})\\
&+|gegg\rangle_{1356}(\alpha|ge\rangle_{24}-\mathrm{i}\beta|gg\rangle_{24}-\gamma|ee\rangle_{24}+\mathrm{i}\zeta|eg\rangle_{24})\\
&+|gege\rangle_{1356}(-\mathrm{i}\alpha|gg\rangle_{24}+\beta|ge\rangle_{24}+\mathrm{i}\gamma|eg\rangle_{24}-\zeta|ee\rangle_{24})\\
&+|geeg\rangle_{1356}(-\alpha|ee\rangle_{24}+\mathrm{i}\beta|eg\rangle_{24}+\gamma|ge\rangle_{24}-\mathrm{i}\zeta|gg\rangle_{24})\\
&+|geee\rangle_{1356}(\mathrm{i}\alpha|eg\rangle_{24}-\beta|ee\rangle_{24}-\mathrm{i}\gamma|gg\rangle_{24}+\zeta|ge\rangle_{24})\\
&+|eggg\rangle_{1356}(\alpha|eg\rangle_{24}+\mathrm{i}\beta|ee\rangle_{24}+\gamma|gg\rangle_{24}+\mathrm{i}\zeta|ge\rangle_{24})\\
&+|egge\rangle_{1356}(\mathrm{i}\alpha|ee\rangle_{24}+\beta|eg\rangle_{24}+\mathrm{i}\gamma|ge\rangle_{24}+\zeta|gg\rangle_{24})\\
&+|egeg\rangle_{1356}(\alpha|gg\rangle_{24}+\mathrm{i}\beta|ge\rangle_{24}+\gamma|eg\rangle_{24}+\mathrm{i}\zeta|ee\rangle_{24})\\
&+|egee\rangle_{1356}(\mathrm{i}\alpha|ge\rangle_{24}+\beta|gg\rangle_{24}+\mathrm{i}\gamma|ee\rangle_{24}+\zeta|eg\rangle_{24})\\
&+|eegg\rangle_{1356}(-\alpha|ee\rangle_{24}-\mathrm{i}\beta|eg\rangle_{24}+\gamma|ge\rangle_{24}+\mathrm{i}\zeta|gg\rangle_{24})\\
&+|eege\rangle_{1356}(-\mathrm{i}\alpha|eg\rangle_{24}-\beta|ee\rangle_{24}+\mathrm{i}\gamma|gg\rangle_{24}+\zeta|ge\rangle_{24})\\
&+|eeeg\rangle_{1356}(\alpha|ge\rangle_{24}+\mathrm{i}\beta|gg\rangle_{24}-\gamma|ee\rangle_{24}-\mathrm{i}\zeta|eg\rangle_{24})\\
&+|eeee\rangle_{1356}(\mathrm{i}\alpha|gg\rangle_{24}+\beta|ge\rangle_{24}-\mathrm{i}\gamma|eg\rangle_{24}-\zeta|ee\rangle_{24})].
\end{aligned}\tag{16}
$$

The third, Alice performs single-atom state measurements on all the atoms in her site respectively and informs Bob of her measurement outcomes. From Eq.(16), one can see that with equal probability (i.e., 1/16) Alice can get any one of the sixteen possible results. Without loss of generality, for an example, suppose Alice's measurement result is $|gggg\rangle_{1356}$. In this case, after Alice's measurements, the state of the atoms 2 and 4 collapses into the state $\alpha|gg\rangle_{24}-\mathrm{i}\beta|ge\rangle_{24}+\gamma|eg\rangle_{24}-\mathrm{i}\zeta|ee\rangle_{24}$. Since Alice has informed

Bob of her measurement results via classical channel, based on Alice's message (4 bits), Bob finally performs the unitary operations Z and V on the atom 4, where $Z = |e\rangle\langle e| - |g\rangle\langle g|$ and $V = |e\rangle\langle e| + \mathrm{i}|g\rangle\langle g|$. After his performance, the arbitrary unknown state initially in the atoms 5 and 6 is successfully recovered in the atoms 2 and 4 now. That is,

$$\begin{aligned} & Z_2V_4(\alpha\,|gg\rangle_{24} - \mathrm{i}\beta\,|ge\rangle_{24} + \gamma\,|eg\rangle_{24} - \mathrm{i}\zeta\,|ee\rangle_{24}) \\ &= -\mathrm{i}(\alpha|gg\rangle_{24} + \beta\,|ge\rangle_{24} + \gamma\,|eg\rangle_{24} + \zeta\,|ee\rangle_{24}). \quad (17) \end{aligned}$$

Here the factor $-\mathrm{i}$ in the right hand of the above equality is trivial and unimportant. For another example, suppose Alice's measurement result is $|egge\rangle_{1356}$. In this case, after Alice's measurements, the state of the atoms 2 and 4 collapses into the state $\mathrm{i}\alpha\,|ee\rangle_{24} + \beta\,|eg\rangle_{24} + \mathrm{i}\gamma\,|ge\rangle_{24} + \zeta\,|gg\rangle_{24}$. Based on Alice's message, Bob finally performs the unitary operation X on the atom 2 and XV on the atom 4 to recover the unknown state, i.e.,

$$\begin{aligned} & X_2X_4V_4(\mathrm{i}\alpha\,|ee\rangle_{24} + \beta\,|eg\rangle_{24} + \mathrm{i}\gamma\,|ge\rangle_{24} + \zeta\,|gg\rangle_{24}) \\ &= \mathrm{i}(\alpha|gg\rangle_{24} + \beta\,|ge\rangle_{24} + \gamma\,|eg\rangle_{24} + \zeta\,|ee\rangle_{24}), \quad (18) \end{aligned}$$

where $X = |e\rangle\langle g| + |g\rangle\langle e|$. Similarly, if Alice gets any other measurement result and informs Bob of it, based on Alice's message Bob then can successfully reconstruct the unknown state in the atoms 2 and 4 by performing appropriate unitary operations. Here we do not depict them one by one in detail. The correspondences between Alice's measurement outcomes and Bob's unitary operations are summarized in Table 1. Incidentally, the unitary operations on a single atom can be easily implemented by using a classical field.

Table 1. Alice's measurement outcomes (AMO) and Bob's unitary operations (BUO). See text for more details.

AMO	BUO	AMO	BUO	AMO	BUO	AMO	BUO
$\|gggg\rangle_{1356}$	Z_4V_4	$\|ggge\rangle_{1356}$	V_4X_4	$\|ggeg\rangle_{1356}$	$X_2V_4Z_4$	$\|ggee\rangle_{1356}$	$X_2V_4X_4$
$\|gegg\rangle_{1356}$	$Z_2X_4V_4$	$\|gege\rangle_{1356}$	Z_2V_4	$\|geeg\rangle_{1356}$	$X_2Z_2X_4V_4$	$\|geee\rangle_{1356}$	$X_2Z_2V_4$
$\|eggg\rangle_{1356}$	X_2V_4	$\|egge\rangle_{1356}$	$X_2X_4V_4$	$\|egeg\rangle_{1234}$	V_4	$\|egee\rangle_{1356}$	X_4V_4
$\|eegg\rangle_{1356}$	$X_2Z_2V_4X_4$	$\|eege\rangle_{1356}$	$Z_2X_2Z_4V_4$	$\|eeeg\rangle_{1356}$	$Z_2V_4X_4$	$\|eeee\rangle_{1356}$	$Z_2Z_4V_4$

In summary, in this paper we have proposed a scheme for implementing the teleportation of an arbitrary unknown two-atom state by using a cluster state of four identical 2-level atoms as quantum channel in thermal cavity. In the present scheme the usual joint measurement is unnecessary and the scheme is insensitive to the cavity field state and the cavity decay.

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