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Effect of electric field on the spectrum and the persistent current of a quantum ring with two electrons (II)—additional effect of a charged impurity*

Wu Hong(吴 洪)†

Department of Physics, School of Sciences, Jimei University, Xiamen 361021, *China*

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This paper studies the effect of a charged impurity together with or without an external homogeneous electric field on a quantum ring threaded by a magnetic field B and containing two electrons. The potential caused by the impurity has been plotted which is helpful to the understanding of the electronic structures inside the ring. The deep valley appearing in the potential curve is the source of localization, which affects seriously the Aharonov–Bohm oscillation (ABO) of the energy and persistent current. It also causes the fluctuation of the total orbital angular momentum L of the pair of electrons. It is found that the appearance of the impurity reduces the domain of the fractional ABO. During the increase of B, the domain of the integral ABO may appear earlier when B is even quite small. The transition from the localized states to extended states has also been studied. Furthermore, it has deduced a set of related formulae for a transformation, by which an impurity with a charge e_p placed at an arbitrary point $\boldsymbol{R}_p$ is equivalent to an impurity with a revised charge $\tilde{e}_p$ placed at the X-axis with a revised radial distance $\tilde{R}_p$. This transformation facilitates the calculation and make the analysis of the physical result clearer.

Keywords: charged impurity, quantum ring, ABO, potential
PACC: 7115P, 7320D

1. Introduction

The study of semiconductor quantum ring as a low-dimensional structure is one of the important topics in the domain of semiconductor physics and technology currently. As we know, when the quantum ring is threaded by an adjustable magnetic field, the energy, persistent current, and a number of other physical properties would oscillate with the variation of the magnetic field. This is the well known Aharonov–Bohm oscillation (ABO).[1−3] The period of ABO is h/e, the flux quantum. In recent years, quantum rings of 20–100 nm have been fabricated in laboratories.[4,5] The observations of ABO have been reported.[6,7] Owing to its unique optical and electrical properties, the quantum ring plays a very important role in nanoelectronic technology and devices.[8] In the aspect of application, the crucial problem is to control or adjust the physical properties of the system. This is usually realized by applying various external fields. Therefore, the study of the effect of these external fields is an important object of current research works.[9] In our previous paper,[10] we have studied the effects of an uniform external electric field together with the e–e correlation on the physical properties of quantum rings. The motivation of the present paper is to extend our previous study to include the effect of a charged impurity. To this goal we deduce an 'equivalent charge transformation formula' and emphasize the analysis of the potential well. Applying these two mathematical skills, we find that the calculation and analysis of the results become more easy and clear. We also discuss both the localized and extended states in detail, the transition between these two types of states plays the role of a switch in electron transfer.

2. Hamiltonian and its matrix element

We consider here a one-dimensional quantum ring placed on the $X-Y$ plane with radius R_0, as shown in Fig.1. Two electrons e_1 and e_2 are situated on the

*Project supported by the Natural Science Foundation of Fujian Province of China (Grant No T0650010).
†E-mail: Hwu@jmu.edu.cn

ring with their spherical coordinates $\left(R_0, \frac{\pi}{2}, \varphi_1\right)$ and $\left(R_0, \frac{\pi}{2}, \varphi_2\right)$, respectively. An impurity with charge $-N_p e$. (e is the charge of an electron having a negative value) is located at an arbitrarily given point p with the coordinates $\boldsymbol{R}_p(R_p, \theta_p, \varphi_p)$, where N_p is an integer. An external uniform magnetic field parallel to the Z-axis $\boldsymbol{B} = B\boldsymbol{e}_Z$ is applied and threads the ring. In addition, as in our previous study,[10] an uniform electric field is also applied along the X-axis, $\boldsymbol{E} = E\boldsymbol{e}_x$. Then, the Hamiltonian can be written as

$$\begin{aligned} H &= \sum_{i=1}^{2} \frac{1}{2m^*}\left(\boldsymbol{P}_i + \frac{e}{c}\boldsymbol{A}_i\right)^2 + \frac{e^2}{\varepsilon\,|\boldsymbol{r}_1 - \boldsymbol{r}_2|} \\ &\quad + H_{\text{Zeeman}} - eER_0\sum_{i=1}^{2}\cos\varphi_i - \sum_{i=1}^{2}\frac{N_p e^2}{\varepsilon\, l_i} \\ &\equiv T_1 + T_2 + V_{ee} + H_{\text{zeeman}} \\ &\quad + V_f + V_{p_1} + V_{p_2}, \end{aligned} \tag{1}$$

where m^* is the effective mass of the electron and l_i is the distance from the i-th electron to the impurity (refer to Fig.1). Comparing with the Hamiltonian in Ref.[10], we find that the last term in Eq.(1) is the additional term arising from the interaction between the electrons and the impurity. For the vector potential of the magnetic field, $\boldsymbol{A} = A\boldsymbol{e}_\varphi = \left(\frac{1}{2}R_0 B\right)\boldsymbol{e}_\varphi$, where $\boldsymbol{e}_\varphi$ is the unit vector along the tangential direction of the ring at this point. $\boldsymbol{e}_\varphi = -\sin\varphi\,\boldsymbol{e}_x + \cos\varphi\,\boldsymbol{e}_y$, and V_{ee} is the Coulomb correlation between the pair of electrons. $H_{\text{Zeeman}} = g\mu_{\text{B}}BS_z$ is the Zeeman energy, where S_z is the z-component of the total spin of the pair of electrons. V_f represents the interaction between the electrons and the external electric field.

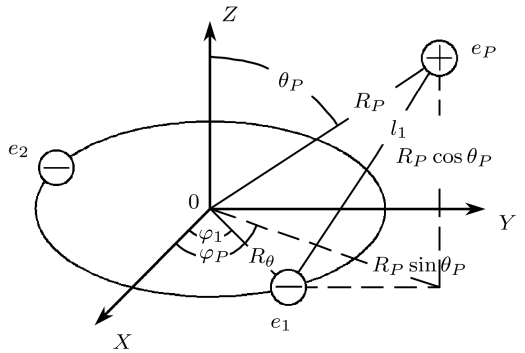


Fig.1. The quantum ring is located at the $X-Y$ plane. The impurity is placed at $\boldsymbol{R}_p(R_p, \theta_p, \varphi_p)$.

In order to calculate the matrix elements of the Hamiltonian, we choose a set of basis functions for the spatial part of the two-electron system as follows:

$$\varphi_{k_1,k_2} = \frac{1}{2\pi}\mathrm{e}^{\mathrm{i}(k_1\varphi_1 + k_2\varphi_2)}. \tag{2}$$

When the exchange symmetry is taken into consideration, the basis functions should be (anti) symmetrized as

$$\begin{aligned} \Phi_{k_1,k_2} &= \left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{k_1,k_2}} \frac{1}{2\pi}[\mathrm{e}^{\mathrm{i}(k_1\varphi_1 + k_2\varphi_2)} \\ &\quad + (-)^s \mathrm{e}^{\mathrm{i}(k_1\varphi_2 + k_2\varphi_1)}]. \end{aligned} \tag{3}$$

Where $S = 0$ or 1 is the total spin. With these basis functions, the eigen-states can be expanded as

$$\psi^{(m)} = \sum_j C_j^{(m)}\Phi_j, \tag{4}$$

Where the subscript j denotes the pair (k_1, k_2).

The matrix elements of each term of the Hamiltonian in Eq.(1) with respect to the Eq.(2) are derived as follows.

$$\begin{aligned} &\langle T_1 + T_2\rangle \\ &= \langle\varphi_{k_1',k_2'}\,|T_1 + T_2|\,\varphi_{k_1,k_2}\rangle \\ &= \alpha[(k_1+Q)^2 + (k_2+Q)^2]\delta_{k_1,k_1'}\delta_{k_2,k_2'}, \end{aligned} \tag{5}$$

where, the constant $\alpha = \dfrac{\eta^2}{2m^*R^2}$, $Q = \dfrac{\Phi}{\Phi_0}$, $\Phi = \pi R_0^2 B$ is the flux threading the ring, $\Phi_0 = \dfrac{hc}{e}$ is the flux quanta.

$$\langle V_{ee}\rangle = \frac{e^2}{\pi\varepsilon}I(|k_2 - k_2'|)\delta_{k_1+k_2,k_1'+k_2'},$$

where,

$$I(k) = \int_0^{\frac{\pi}{2}} \mathrm{d}\beta\,\frac{\cos(2k\beta)}{\sqrt{R_0^2\sin^2\beta + \delta^2}}. \tag{6}$$

In Eq.(6) δ is a parameter used to compensate the effects of the width of the ring.

$$\langle H_{\text{Zeeman}}\rangle = g\mu_{\text{B}}BS_z\delta_{k_1',\,k_1}\delta_{k_2',k_2}, \tag{7}$$

$$\langle V_f\rangle = \frac{-eER_0}{2}[(\delta_{k_1,k_1'-1} + \delta_{k_1,k_1'+1})\delta_{k_2,k_2'} + (\delta_{k_2,k_2'-1} + \delta_{k_2,k_2'+1})\delta_{k_1,k_1'}], \tag{8}$$

$$\langle V_{p,1}\rangle = \frac{-N_p e^2}{\pi\varepsilon}\{\cos[(k_1-k_1')\varphi_p] + \mathrm{i}\sin[(k_1-k_1')\varphi_p]\}J(|k_1-k_1'|)\delta_{k_2',\,k_2},$$
$$\langle V_{P,2}\rangle = \frac{-N_p e^2}{\pi\varepsilon}\{\cos[(k_2-k_2')\varphi_p] + \mathrm{i}\sin[(k_2-k_2')\varphi_p]\ \}\ J(|k_2-k_2'|)\delta_{k_1',\,k_1}, \tag{9}$$

where

$$J(k) = \int_0^\pi \frac{\cos(k\beta)}{\sqrt{R_0^2+R_p^2-2R_0R_p\sin\theta_p\cos\beta}}\mathrm{d}\beta,$$

Combining the Eqs.(5) to (9), we can obtain the matrix elements of the whole Hamiltonian with respect to the basis functions Eq.(2). The matrix elements with respect to the basis functions Eq.(3) can be thereby obtained. After the diagonalization, eigen-states $\psi_{s,j}$ and eigen-energies $\varepsilon_{s,j}$ can be obtained, where the subscript j denotes the order of states with the given S in increasing energy.

To lighten the strenuous computation works and to simplify the complexity in the analysis of the obtained results, we deduced an 'equivalent charge transformation formula' (see Appendix). Using this formula, we find that the effect of an impurity with charge e_p located at any 3-dimensional point $\boldsymbol{R}_p(R_p,\theta_p,\varphi_p)$ is equivalent to a revised charge $\tilde{e}_p$ located at a revised position on the $X-Y$ plane, $\tilde{\boldsymbol{R}}_p(\tilde{R}_p,\tilde{\theta}_p=90^\circ,\tilde{\varphi}_p=\varphi_p)$. Further considering the axial symmetry property in question, we can change again the revised position to be $\tilde{\boldsymbol{R}}_p(\tilde{R}_p,\tilde{\theta}_p=90^\circ,\tilde{\varphi}_p=0^\circ)$. i.e., $\tilde{e}_p$ is located at X-axis.

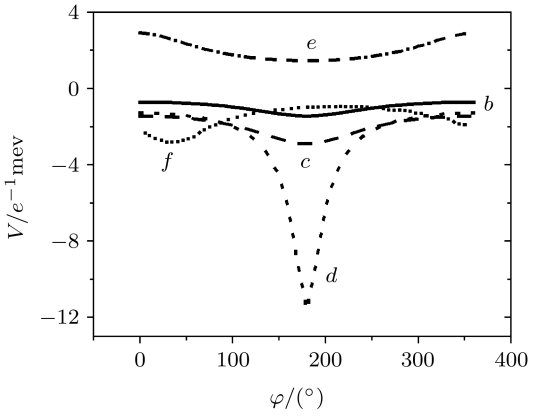


Fig.2. Potential wells on the ring for cases (b–f).

Since the effect of an external field is embodied by the associated potential well, so we emphasize the analysis of the potential wells in our discussions. Several typical potential wells which will be discussed below are sketched in Fig.2. In this figure, the curve d is amazing. It implies that, when the impurity is close to the ring, serious effect may be caused.

3. Calculated results and discussion

In our calculation, we used the same parameters as adopted in Ref.[10]: $m^* = 0.067\,m_\mathrm{e}$, $\varepsilon = 12.4$ (for GaAs rings), $R_0 = 40\,\mathrm{nm}$, $\delta = 0.05\,R_0$. The unit of electric field is e^{-1} meV/nm and the unit of magnetic field is Tesla. Our discussions focus at the following points:

(i) The potential well.

(ii) The low-lying eigen-energies ε and eigen-states.

(iii) The period of ABO.

(iv) The average angular momentum, $\bar{L} = \langle\psi|L|\psi\rangle$, and it's fluctuation, $\Delta L = \langle\psi|(L-\bar{L})^2|\psi\rangle^{1/2}$.

(v) The persistent current

$$J = \frac{\eta}{m^*R^2}\int_0^{2\pi}\mathrm{d}\varphi_2\left[\psi^*\left(-\mathrm{i}\frac{\partial}{\partial\varphi}+\frac{\Phi}{\Phi_0}\right)\psi + \mathrm{c.c.}\right]$$

(vi) The e–e correlated density $\rho(\varphi_1,\varphi_2) = |\psi(\varphi_1,\varphi_2)|^2(\mathrm{radian})^{-2}$. The charge-density $\rho_1(\varphi) = \rho_2(\varphi) = \int_0^{2\pi}\mathrm{d}\varphi_2\rho\ (\varphi,\varphi_2) = \int_0^{2\pi}\mathrm{d}\varphi_1\rho(\varphi_1,\varphi)$ $(\mathrm{radian})^{-1}$. The equal signs in these identities are based on the fact that e_1 and e_2 are identical particles.

Although we have computed for a wide range of scenarios, we present here only the results for six typical cases. These cases, denoted (a)–(f), are ordered roughly according to how serious the effect of the impurity is. The lowest states of $S=0$ and $S=1$ are studied first then the study will be extended to higher excited states.

3.1. Impurity located at the positive Z-axis $\boldsymbol{R}_p(R_p = 60\,\mathbf{nm},\ \theta_p = 0°, \varphi_p \sim$ any). The external electric field $E = 0$ and $N_p = 1$ (Case (a)).

This is a trivial case because the potential energy arising from the impurity can be written as:

$$v_p(\varphi) = \frac{e_p}{\varepsilon}\frac{1}{\sqrt{R_0^2 + R_p^2}} = \text{constant}, \tag{10}$$

and correspondingly,

$$\langle V_p \rangle = -N_p \times 3.22\,\text{meV} = \text{constant}. \tag{11}$$

Equation (10) shows that the potential well raised from the impurity is a constant. Equation (11) shows that the effect of e_p on the ring is simply adding a constant to the total Hamiltonian. In particular, the whole system remains to have rotational invariance. Now, let us compare the results of Fig.1(a) of Ref.[10] with our present results. In previous case, no electric field is involved. However, in the present case, the charged impurity is located at the symmetric axis. We find that all the values of ε, J, $\bar{L}$ and ΔL for both cases are exactly the same as expected, only the values of ε are shifted by a constant value of Eq.(11).

The correlated density of the ground state $\rho(\varphi_1, \varphi_2)$ is plotted in Fig.3(a). In the present special case, this contour map appears as a set of parallel straight lines. It implies that $\rho(\varphi_1,\varphi_2)$ depends only on the relative positions $\varphi_1 - \varphi_2$. Furthermore, when $\varphi_1 - \varphi_2 = \pm 180°$, $\rho(\varphi_1,\varphi_2)$ arrives at its maximum.

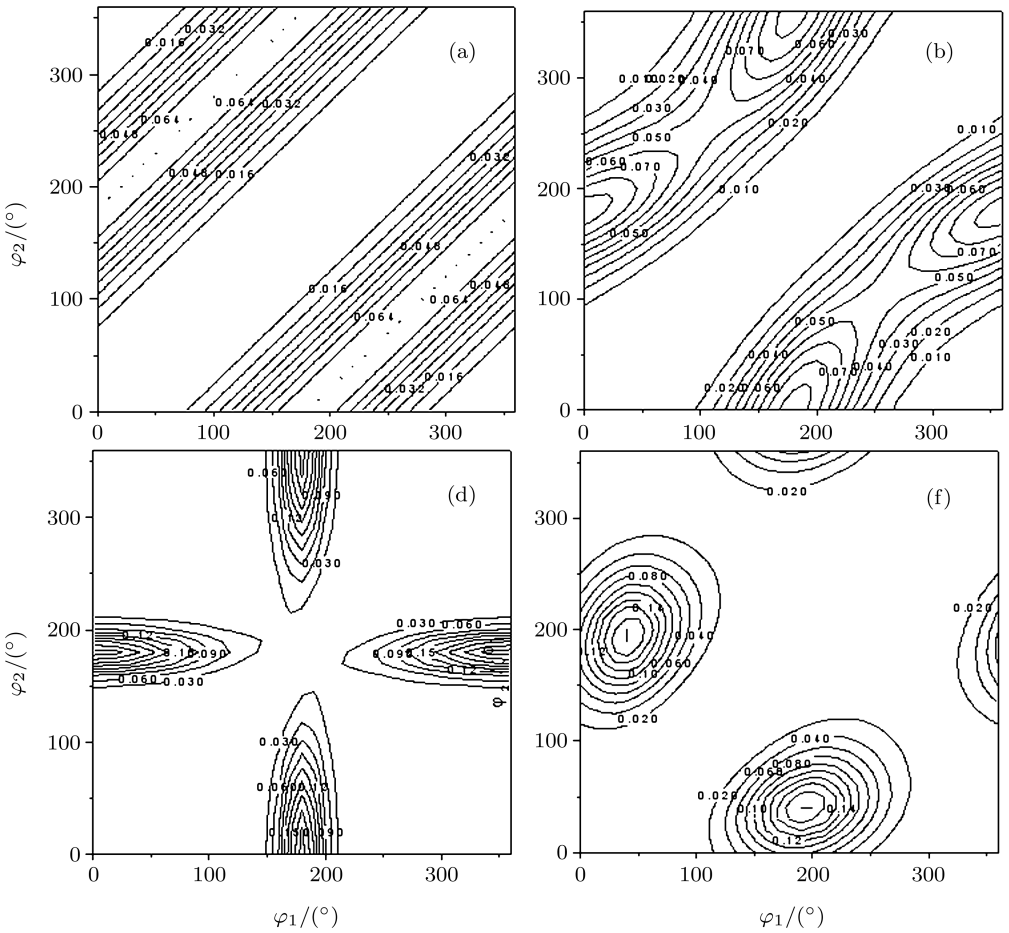


Fig.3. Contour maps of the correlated density $\rho(\varphi_1, \varphi_2) = |\psi(\varphi_1, \varphi_2)|^2$ for cases (b)–(f). The unit of $\rho(\varphi_1, \varphi_2)$ is (radian)$^{-2}$. $\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}\varphi_1 \mathrm{d}\varphi_2 \rho(\varphi_1, \varphi_2) = 1$. Case (a) impurity charge located at Z-axis. $E = 0$. Case (b) and (d) impurities located at X-axis, $E = 0$. Case (f) impurities located at $\boldsymbol{R}_p(R_p = 80\,\text{nm}$, $\theta_p = 80°, \varphi_p = 35°)$. $E = 0.02e^{-1}$ meV/nm is applied along the X-axis.

3.2. Charged impurity e_p located at an arbitrary 3-dimensional point $\boldsymbol{R}_p(R_p, \theta_p, \varphi_p)$, the external electric field $E = 0$.

Applying the 'equivalent charge transformation formula', (see Appendix), all our 3-dimensional problem can be reduced to the following simpler revised ones: a problem of a revised charge $\tilde{e}_p$ located at a revised position $\boldsymbol{R}_p(\tilde{R}_p, \tilde{\theta}_p = 90°, \tilde{\varphi}_p = \text{any})$. The value of $\tilde{\varphi}_p$ is irrelevant, usually we choose it to be 0° or 180° for our convenience. The computed results of ε, J, $\bar{L}$ and ΔL are exactly the same for both the original and the revised cases. But the calculated charge distribution of the electrons on the ring of the original problem is rotated by an angle $\tilde{\varphi}_p - \varphi_p$. Now, we need only to study the cases that the charge is located at the positive or negative X-axis, i.e. $\tilde{\varphi}_p$ is either 0° or 180°. Still, we need only to study the cases for $R_p > R_0$, i.e., the impurity on the $X-Y$ plane is outside the quantum ring. When the impurity located inside the ring, $R_p < R_0$, using Eq.(A7), we can treat the image of this charge instead. Then the position of image charge is outside the ring.

Now, let us assume first $E = 0$ and consider the following representative cases:

(b) $\boldsymbol{R}_p(R_p = 120\,\text{nm},\ \theta_p = 90°,\ \varphi_p = 180°)$, $N_p = 1$;

(c) $\boldsymbol{R}_p(R_p = 120\,\text{nm},\ \theta_p = 90°,\ \varphi_p = 180°)$, $N_p = 2$;

(d) $\boldsymbol{R}_p(R_p = 50\,\text{nm},\ \theta_p = 90°,\ \varphi_p = 180°)$, $N_p = 1$;

(e) $\boldsymbol{R}_p(R_p = 120\text{nm},\ \theta_p = 90°,\ \varphi_p = 0°)$, $N_p = -2$.

The potential wells of these cases are sketched in Fig.2. The values of ε, J, $\bar{L}$, ΔL of our calculated results are given in Fig.4. When the impurity is positively (negatively) charged, the electrons inside the ring are attracted (repelled) toward and localized around the dip of the valley of the well. This effect will limit the motion of the electrons, so that the ABO amplitudes of both the eigen-energy ε and the persistent current J are suppressed.

It was found that the appearance of the impurity, just as the appearance of the electric field discussed in Ref.[10], does not alter the period of the ABO. This can be understood from the discussion on the origin of the fractional ABO given in Refs.[10,11]. Now, it is well known that, preceding the normal ABO, the oscillation is a fractional ABO when the magnetic field B is weak. We found that the domain of the fractional ABO would be shortened remarkably due to the additional impurity. It is reminded that the $S = 1$ states are benefited from the Zeeman energy H_{Zeeman} while the $S = 0$ states are not, therefore the ground state will be eventually dominated by $S = 1$ states when B increases. When B exceeds a critical point so that the $S = 0$ states appear no more as a ground state, the domain of the fractional ABO ended. Comparing Fig.4 with Fig.1(a) of Ref.[10], it is clear that, due to the impurity, the difference of energy of the lowest $S = 0$ and the lowest $S = 1$ states becomes much smaller. Therefore the latter is easier to be lower than the former with the aid of the Zeeman energy. This explains why the domain of the fractional ABO becomes shorter. This is a noticeable effect of the impurity.

If the effect of the external field is ignored, the fluctuation of angular momenta $\Delta L = 0$. It implies the conservation of the angular momentum due to the axial symmetry. This is shown in Fig.1(a) of Ref.[10], where L is a constant inside a period of the ABO, and the whole curve of L looks like a number of steps. However, when the impurity appears not at the Z-axis, the axial symmetry would be broken. Accordingly, L is no more conserved. For the case (b), in each period, L deviates slightly from a constant as shown in Fig.4. However, the deviation would become very serious if the impurity is close to the ring as in the case (d), where the curve of L declines smoothly. We have also studied directly ΔL which is small in the case (b) but much large in (d). It is interesting to see that ΔL undergoes also the ABO with the same period as the ground state energy, this is shown in these figures.

Now, we examine the contour maps of $\rho(\varphi_1, \varphi_2)$, due to the permutation symmetry possessed by the wave functions and due to the fact that the impurity is located at the X-axis, the contour map of the correlated density $\rho(\varphi_1, \varphi_2) = |\psi(\varphi_1,\varphi_2)|^2$ should be symmetric with respect to the reflection against both of the diagonals. See Figs.3(b), (c). For case (b), the peaks appear at ($\varphi_1 = 0°$, $\varphi_2 = 180°$) and ($\varphi_1 = 180°$, $\varphi_2 = 0°$), $\rho_{\max} = 0.095(\text{radian})^{-2}$. Performing integration: $\rho_1(\varphi) = \rho_2(\varphi) = \int_0^{2\pi} \mathrm{d}\varphi_2 \rho(\varphi,\ \varphi_2)$, we can find that the density distribution, $\rho_1(\varphi)$ or $\rho_2(\varphi)$, along the whole ring. We could find that the electrons are localized toward the 180° side. For case (d), the position of the peaks unaltered, but the peak values become higher, $\rho_{\max} = 0.29\,(\text{radian})^{-2}$.

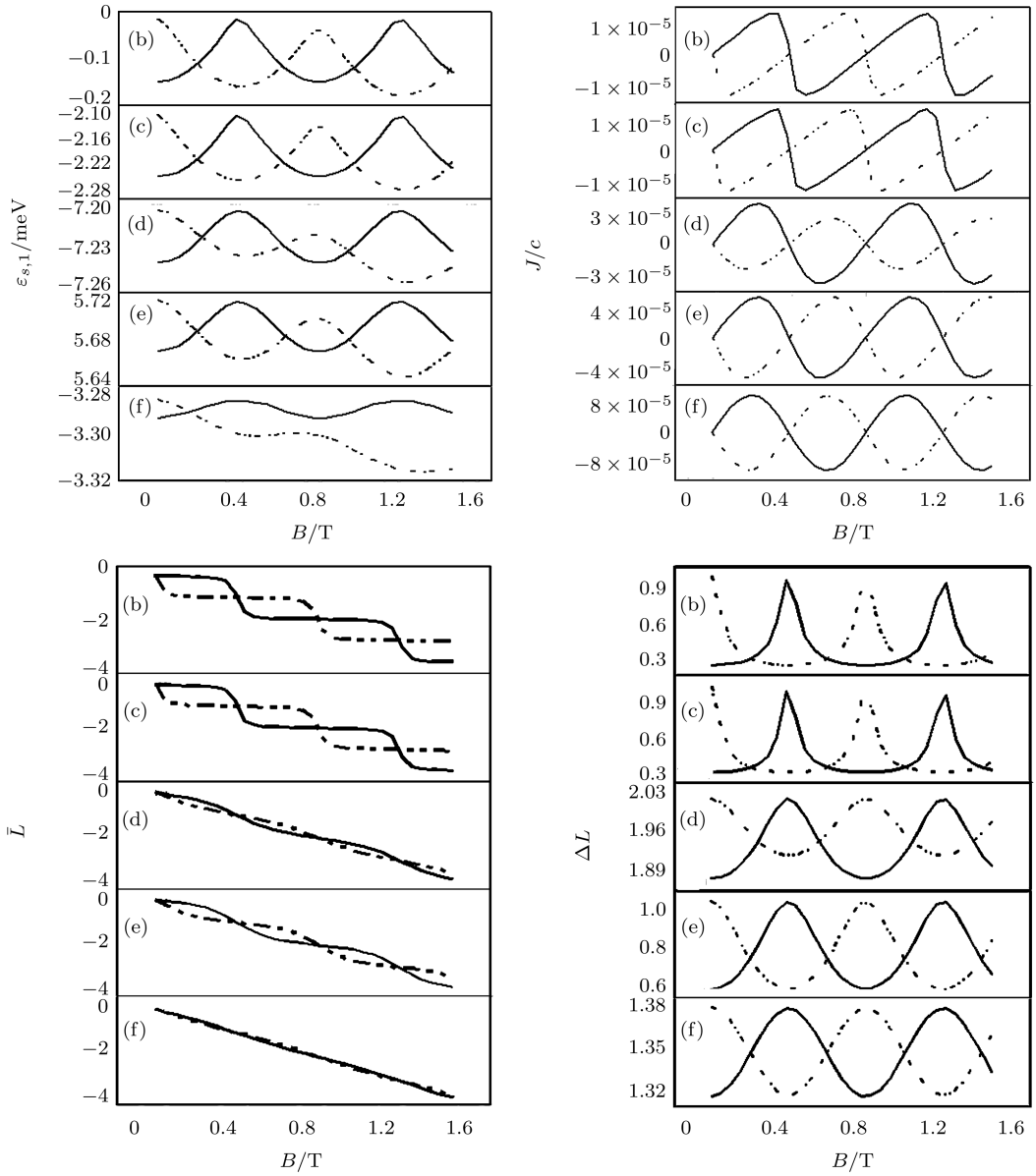


Fig.4. The evolution of $\varepsilon_{s,1}$, J, $\bar{L}$, ΔL, against B. Solid (dash) lines are for the $S = 0$ ($S = 1$) lowest state. J is in units of $c = C/R$ (where C is the speed of light in vacuum, $R = 40\,\mathrm{nm}$).

3.3. Both the electric field and the charged impurity exist

Case (f) $\boldsymbol{R}_p(R_p = 80\,\mathrm{nm},\ \theta_p = 80°,\ \varphi_p = 35°)$, $N_p = 1$, an external uniform electric field $E = 0.02\,e^{-1}\mathrm{meV/nm}$ parallel to the X-axis is assumed.

We can use the above method to treat this case. But it is important to be careful that, when applying the 'equivalent charge transformation formula', the revised $\tilde{\varphi}_p$ should always be φ_p. This is because the combined field comes from both the electric field and the charged impurity.

In this case the combined effect of the electric field and the impurity is serious as shown in the figure. The ABO of the ground state energy is nearly spoiled, the persistent current becomes very small (about one order smaller), $\bar{L}$ becomes nearly a declining straight line. The localization is also very strong as shown in Fig.3(f).

3.4. Delocalization

All the cases discussed above are for the lowest energy states of $S = 0$, 1. In those cases the electrons

may be localized around the dip of the potential valley. The ABO amplitudes of the energies and persistent current J may be suppressed. However, for the higher excited states, the electrons possess more energy and therefore they are more free from the localization. For an example, for the case with $\boldsymbol{R}_p(R_p = 50\,\text{nm}$, $\theta_p = 90°$, $\varphi_p = 180°)$, $N_p = 1$, and with an external electric field, $E = 0.05\ e^{-1}\,\text{meV/nm}$ along the X-axis together with a magnetic field $B = 0.95$ Tesla along the Z-axis. The density distribution of the electron, $\rho_1(\varphi) = \rho_2(\varphi)$, for different energy states are shows in Fig.5.

From these curves 1 of the lowest state, 80% of the density is localized inside a narrow region of only 79° of the ring, and the persistent current disappears nearly.

For higher states the distribution of the electron extends and the persistent current increases remarkably. Accordingly the states become extended states, this is shown by the data listed in Table 1. From the table we can see that the extended states have not only a very wide distribution, but also a very strong persistent current. The persistent current of the fourth state is the third order larger than the lowest state, this is a remarkable feature of the extended states.

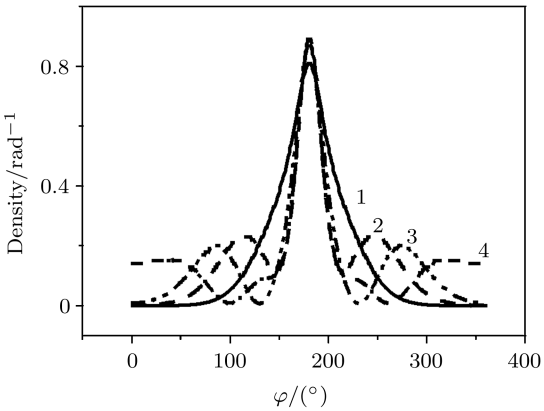


Fig.5. The density distributions of the electron of different states along the quantum ring. The impurity is given at $\varphi_p = 180°$. The states from the lowest to the fourth lowest are marked by 1 to 4 by the curves.

Table 1. The eigen-energy, persistent current and the width of region where 80% of the electron density is concentrated.

state	Eigen-energy $\varepsilon\,/\,e^{-1}\,\text{meV}$	Persistent current J/c	Width of the region where 80% of the electron density is concentrated /(°)
$S_{0,1}$	-10.0	1.8×10^{-6}	79
$S_{0,2}$	-8.9	6.2×10^{-5}	152
$S_{0,3}$	-8.0	6.1×10^{-4}	162
$S_{0,4}$	-7.4	2.0×10^{-3}	280

4. Summary

In this paper, we study the effect of a charged impurity on the physical properties of quantum rings. This work is a continuation of our previous research Ref.[1], in which only an external electric field was applied.

When the external electric field is not considered but only the impurity, using the transformation given in the appendix, disregarding where the impurity is actually located, we can treat it theoretically as if it is at the X-axis.

Results of the present study can be summarized as follows:

1) The effect of the charged impurity is embodied by its potential well. A deeper potential valley confines and localizes the motion of the electrons. The deeper the potential well, the stronger the localization effect. Once the electrons are localized, the amplitudes of the ABO of the ground state energy ε_{g} and persistent current will be suppressed.

2) Serious fluctuation of the orbital angular momentum can be caused by the impurity.

3) H_{Zeeman} is the major source that allows the fractional ABO to recover to the normal ABO. The recovering will be greatly speeded up due to the impurity.

4) For excited states, the electrons may have sufficient energy to be free from the confinement arising from the impurity. In this case, the state is no more localized but an extended state. For certain case, when the energy of the system increases and exceeds a critical value, the extended states appear suddenly, and will cause a great increase in persistent current. The strength of the current may increase suddenly by 10^2 times or more.

Finally, since the physical properties of the ring are remarkably affected by the impurity if they are

close to each other, the addition of an impurity is one of the practicable ways to control the micro-devices.

Appendix: Equivalent charge transformation formula

In Fig.1, an impurity with charge e_p is located at $\boldsymbol{R}_p$ in a 3-dimensional space. The associated potential acting on the electrons is

$$V_{p_i} = \frac{e_p}{\varepsilon}\frac{1}{l_i} = \frac{-N_p e}{\varepsilon}\frac{1}{l_i},$$
$$l_i = \sqrt{R_0^2 + R_P^2 - 2R_0 R_p \sin\theta_p \cos(\varphi_p - \varphi_i)}. \quad \text{(A1)}$$

Where φ_i is the longitudinal angle of the i-th electron at the ring. Let

$$l_i = \sqrt{\bar{R}_0^2 + \bar{R}_P^2 - 2\bar{R}_0\bar{R}_p \cos(\varphi_p - \varphi_i)} \quad \text{(A2)}$$

and

$$\bar{R}_0^2 + \bar{R}_p^2 = R_0^2 + R_p^2,$$
$$2\bar{R}_0\bar{R}_p = 2R_0 R_p \sin\theta_p. \quad \text{(A3)}$$

From Eq.(A3), we have

$$\bar{R}_0 = \frac{a_+ + a_-}{2}R_0,$$
$$\bar{R}_p = \frac{a_+ - a_-}{2}R_0, \quad \text{(A4a)}$$
$$\bar{R}_0 = \frac{a_+ - a_-}{2}R_0,$$
$$\bar{R}_p = \frac{a_+ + a_-}{2}R_0, \quad \text{(A4b)}$$

and

$$a_\pm = \left|\sqrt{1 + \left(\frac{R_p}{R_0}\right)^2 \pm 2\left(\frac{R_p}{R_0}\right)\sin\theta_p}\right|.$$

Substituting Eq.(A4) into Eqs.(A1) and (A3), we can rewrite Eq.(A1) as

$$V = \frac{e_p\left(\frac{R_0}{R_p}\right)}{\varepsilon}\frac{1}{\sqrt{R_0^2 + \left(\frac{R_0}{\bar{R}_0}\bar{R}_p\right)^2 - 2R_0\left(\frac{R_0}{\bar{R}_0}\bar{R}_p\right)\cos(\varphi_p - \varphi_i)}} = \frac{\tilde{e}_p}{\varepsilon}\frac{1}{\sqrt{R_0^2 + \tilde{R}_p^2 - 2R_0\tilde{R}_p\cos(\varphi_p - \varphi_i)}}, \quad \text{(A5)}$$

where $\tilde{e}_p$ and $\tilde{R}_p$ have two sets of different solutions:

$$\tilde{e}_1 = \frac{2}{a_+ - a_-}e_p,$$
$$\tilde{R}_1 = \frac{a_+ + a_-}{a_+ - a_-}R_0, \quad \text{(A6a)}$$
$$\tilde{e}_2 = \frac{2}{a_+ + a_-}e_p,$$
$$\tilde{R}_2 = \frac{a_+ - a_-}{a_+ + a_-}R_0. \quad \text{(A6b)}$$

Equations (A6a)–(A6b) show that the effect of charge e_p located at $\boldsymbol{R}_p(R_p, \theta_p, \varphi_p)$ on the quantum ring can be replaced by its equivalent charge, $\tilde{\varepsilon}_p$, located at $\boldsymbol{R}_p(\tilde{R}_p, \tilde{\theta}_p = 90°, \tilde{\varphi}_p = \varphi_p)$. The latter is located in a 2-dimensional $X - Y$ plane and is easier to handle. Equation (A6a) is the mirror image of Eq.(A6b) of the 'same charged sign' at the ring and vice versa.

In the case that $\boldsymbol{R}_p$ is in a 2-dimensional $X - Y$ plane already,

$$\tilde{e}_1 = e_p,$$
$$\tilde{R}_1 = R_p; \quad \text{(A7a)}$$
$$\tilde{e}_2 = \frac{R_0}{R_p}e_p,$$
$$\tilde{R}_2 = \frac{R_0}{R_p}R_0. \quad \text{(A7b)}$$

Equation (A7a) is the original charge, whereas Eq.(A7b) is it's mirror image.

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