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Phase transition on speed limit traffic with slope*

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Through introducing a generalized optimal speed function to consider spatial position, slope grade and variable safe headway, the effect of slope in a single-lane highway on the traffic flow is investigated with the extended optimal speed model. The theoretical analysis and simulation results show that the flux of the whole road with the upgrade (or downgrade) increases linearly with density, saturates at a critical density, then maintains this saturated value in a certain density range and finally decreases with density. The value of saturated flux is equal to the maximum flux of the upgrade (or downgrade) without considering the slight influence of the driver's sensitivity. And the fundamental diagrams also depend on sensitivity, slope grade and slope length. The spatiotemporal pattern gives the segregation of different traffic phases caused by the rarefaction wave and the shock wave under a certain initial vehicle number. A comparison between the upgrade and the downgrade indicates that the value of saturated flux of the downgrade is larger than that of the upgrade under the same condition. This result is in accordance with the real traffic.

Keywords: generalized optimal speed, car-following model, slope, phase transition
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1. Introduction

In the last decades, the traffic flow problems have aroused the interest of physicists, engineers and mathematicians as a typical example of non-equilibrium statistical physics. To describe the properties of real traffic flow, many mathematical models including microscopic ones, mesoscopic ones and macroscopic ones have been proposed.[1−8]

Up to now, with regard to the car-following model, numerous extensions and improvements have focused on two aspects: one is fundamental theory including the improvement of the model and relative stability analysis[9,10] and the other is practical applications, such as the effects of traffic light,[11] slow car,[12] night driving,[13] slowdown sections.[14,15] It has been also extended to describing two-lane traffic[16−18] and a multi-species system of road traffic.[19]

Roads with abruptly changing surface conditions are widely existent in real traffic flows. And the traffic jams coming from these roads have aroused a great deal of concern. The hydrodynamic models played an important role in this research initially,[20,21] and later, macroscopic models with spatially varying flux functions were discussed and analysed deeply by Zhang *et al.*[22−24] Recently, Meng and Zhang[25] applied the cellular automaton (CA) model to the study of traffic characteristic with deceleration strips. In addition, some researchers investigated a similar problem by using the CA model[26] and car-following model.[14] However, it should be noted that in real traffic, besides by usual bottleneck, working zone, accident, tunnel, etc, the change of road conditions can also be caused by the geographical situation, for example, the upgrade and the downgrade in highland and mountainous areas. Different from the common speed limit regions, at the joint between the normal section and the slope, the abrupt changing of road surface conditions causes the vehicle speed to vary discontinuously with respect to spatial location. Moreover, on the slope, the maximal speed and safety distance determined by main factors including slope grade, drivers' psychology and vehicles' capability cannot be constant. On these roads with slope, traffic jams, even traffic accidents, occur frequently compared with those of the traffic flows on normal highways.[27,28] One is interested in the structure and the formation of traffic jams induced by this slope. However, there is a lack of knowledge about this problem. How does the traffic flow change by introducing a slope? What

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is the difference in traffic property between the upgrade and the downgrade? And how do such factors as slope grade, slope length and sensitivity affect the occurrence of traffic jam? In a word, the traffic jam induced by the slope has been scarcely investigated by using the dynamic models.

In this paper, we extend the classical optimal speed model to investigate the traffic phase transition with the slope. The rest of the present paper is organized as follows. In Section 2, the slope model is set up by introducing a generalized optimal speed function. Section 3 gives the simulation results and corresponding discussion. The final section is devoted to a summary and prospect of further study.

2. Model

2.1. Generalized optimal speed

In 1995, Bando[7] proposed a traffic model called the optimal speed model, which is described by the following equation of motion of vehicle i:

$$\frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} = a\left\{V\left(\Delta x_i\right) - \frac{\mathrm{d}x_i}{\mathrm{d}t}\right\}, \tag{1}$$

where $V(\Delta x_i(t))$ is the optimal speed, $x_i(t)$ is the position of vehicle i at time t, $\Delta x_i(t)(= x_{i+1}(t) - x_i(t))$ is the headway of the ith vehicle at time t, and a is the sensitivity of a driver and given by the inverse of the delay time τ. $V(\Delta x_i(t))$ is often given by

$$V(\Delta x_i) = \frac{V_{\max}}{2}[\tanh(\Delta x_i - x_{\mathrm{c}}) + \tanh(x_{\mathrm{c}})], \tag{2}$$

where $V_{\max}$ is the maximal speed of vehicles and x_{c} is the safety distance of vehicles.

Here, we introduce a new generalized optimal speed $V_{\mathrm{G}}(\Delta x_i, \xi, \eta)$ related to multi-factors as follows:

$$V_{\mathrm{G}}(\Delta x_i, \xi, \eta) = \frac{V_{\max}(\xi,\eta)}{2}[\tanh(\Delta x_i - x_{\mathrm{c}}(\xi,\eta)) + \tanh(x_{\mathrm{c}}(\xi,\eta))], \tag{3}$$

where ξ and η are two non-dimensional parameters. ξ denotes the influence of external conditions, such as abruptly changing road surface conditions and geometrical design, speed limit for security, lane numbers, etc.; while η represents the influence of internal condition, such as drivers' driving habit and vehicle type. From the viewpoint that ξ and η should be independent, these two parameters in nature disclose the degree of drivers' response to the changing of road conditions and individual driving characteristics. As the road geometrical shape is given, for a single lane, single vehicle type and similar drivers' driving behaviour, expression (3) reduces to expression (2). We call the combination of Eq.(1) and expression (3) the generalized optimal speed model.

Note that the definition of expression (3) is based on the following two assumptions: (1) the type of the generalized optimal speed function is the same as the original one; (2) the parameters ξ and η mainly affect the maximal speed of vehicles and safe headway.

2.2. The slope optimal speed model

Expression (3) gives a general expression for the optimal speed. Here, for the purpose of real application, we take the road with a slope as a typical example. We consider a unidirectional vehicle flow on a single-lane highway. We assume that (1) all vehicles and drivers' habits are the same, i.e. without considering the influence of parameter η; (2) the whole road is a normal (flat) section plus a section of upgrade or downgrade; (3)the external influence is expressed by the parameter α denoting the slope grade. Figure 1 shows the schematic for the road with a slope.

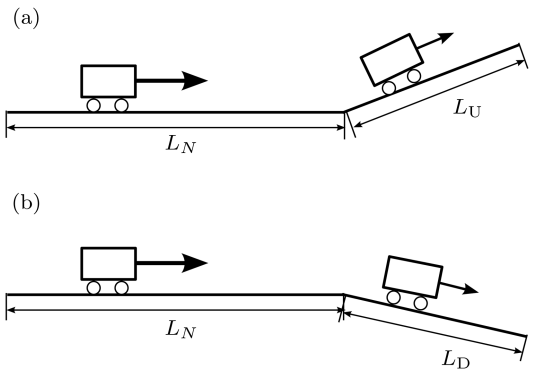


Fig.1. The single-lane highways each with a section of slope, where (a) is for upgrade and (b) for downgrade.

For convenience in the later discussion, we first give the definition of the slope grade α as follows:

$$\alpha = \frac{h}{l} \times 100\%, \tag{4}$$

where h is the vertical height of slope; l is the corresponding horizontal distance. For the downgrade, α is negative.

We apply the optimal speed model to the road configuration as shown in Fig.1. In a normal region, the optimal speed is given by expression (2). In a section of slope, vehicles move with a variable speed

which mainly depends on α. Now the optimal speed is given by

$$\begin{aligned} & V_{\mathrm{U(D)}}(\Delta x_i, \alpha) \\ = & \frac{V_{\mathrm{U(D),max}}(\alpha)}{2}[\tanh(\Delta x_i - x_{\mathrm{U(D),c}}(\alpha)) \\ & + \tanh(x_{\mathrm{U(D),c}}(\alpha))], \end{aligned} \tag{5}$$

where $V_{\mathrm{U(D)}}(\Delta x_i, \alpha)$, $V_{\mathrm{U(D),max}}(\alpha)$ and $x_{\mathrm{U(D),c}}(\alpha)$ denote the optimal speed function, the maximal speed, the safety distance for upgrade (downgrade), respectively. According to the empirical observation,[29] for $|\alpha| < 3\%$, the maximal speed of vehicles hardly changes compared with that on the normal road and the maximum speed decreases a little on the upgrade, and correspondingly increases a little on the downgrade. For $\alpha > 3\%$, the maximal speed decreases gradually with the increase of the grade, but as α increases to 8%, the maximal speed hardly varies. For the downgrade with $\alpha < -4.5\%$, the maximal speed decreases with the increase of the absolute value of grade. For a safety distance, either on the upgrade or on the downgrade, it always increases with the increase of the absolute value of grade. This is because when the vehicles move on the upgrade, the leading car may change gear lever, which makes the leading car have the tendency to glide downwards. However, as for the downgrade, when the absolute value of grade is not so large, the maximal speed will increase, so the safety distance should also increase. But in the circumstances of steep slope, the safe headway will also increase though the speed of vehicles drops due to the drivers' the psychological factors. From the above analysis, we give the approximate relationships between maximum speed and grade α, and between safety distance on upgrade (downgrade) and grade α as follows:

$$\begin{aligned} & V_{\mathrm{U,max}}(\alpha) \\ = & \begin{cases} 2, & 0 \le \alpha < 2\%, \\ -308\alpha^2 + 5.92\alpha + 2, & 2\% \le \alpha \le 8\%, \\ 0.5, & 8\% < \alpha \le 10\%, \end{cases} \end{aligned} \tag{6}$$

$$V_{\mathrm{D,max}}(\alpha) = -222\alpha^2 - 10\alpha + 2, \ -10\% \le \alpha \le 0, \tag{7}$$

$$x_{\mathrm{U,c}}(\alpha) = 20\alpha + 3, \ \ 0 \le \alpha \le 10\%, \tag{8}$$

$$x_{\mathrm{D,c}}(\alpha) = 310\alpha^2 - 12.5\alpha + 3, \ \ -10\% \le \alpha \le 0, \tag{9}$$

It is necessary to point out that in Eqs.(6)–(9) the coefficients before α are determined according to the assumption $V_{\max} = 2$ and $x_{\mathrm{c}} = 3$ in normal roads as well as the empirical observation in Ref.[29]. In addition, in a real road design, the grade is not too large, so we let $|\alpha|_{\max} = 10\%$.[29]

3. Simulation examples

We perform numerical simulation for the traffic model with the upgrade shown in Fig.1 under the periodic boundary condition. The simulation is performed till the traffic flow reaches a steady state. We solve numerically Eq.(1) with optimal speed functions (2) and (5) by using the fourth-order Runge-Kutta method where the time interval is $\Delta t = 1/20$.

We carry out simulation by varying initial headway, sensitivity, slope grade and slope length. The safety distance and the maximal speed are chosen according to Eqs.(6) and (8). The length of road is chosen as follows: $L = 1500$ and $L_{\mathrm{U}} = 300$ unless otherwise stated.

Figures 2(a)–2(c) show the plots of flux against density for the values of sensitivity a: 2.0 (a), a: 1.0 (b) and a: 0.7 (c) with the values of α: 0, 4%, 6% and 9%. The traffic flux is obtained by averaging the flux from $t = 2000$ to 5000. For comparison, we also show the theoretical flux curve in the slope section, which is given by $J = V_{\mathrm{U}}(\Delta x, \alpha)/\Delta x$, where Δx is the average value of headway. The case of $\alpha = 0$ corresponds to the situation that the entire road is flat. When the sensitivity is higher than the critical value 2.0, no traffic jam occurs and the current value is agreement with the theoretical current curve (see Fig.2(a)). If the sensitivity is lower than 2.0, traffic jams occur and the current value deviates from the theoretical flux curve in a region of the density at which traffic jams appear (see Figs.2(b)–2(c)). These results are similar to those for the case of a slowdown occurring in the road.[14]

In the case of $\alpha \neq 0$ for $a = 2$ (see Fig.2(a)), the flux increases linearly with density at low densities, but it is lower than the flux of no slope. Then, the current saturates at the first critical density and keeps a constant value till the occurrence of the second critical density. And this saturated value of flux is equal to the maximal flux of the upgrade. When the density is higher than the second critical density, the currents decrease with density increasing. Considering the effect of slope grade, it can be seen that with α increasing, the current decreases except for the consistence in a higher density ($\rho = 0.6$). The first critical density decreases but the second critical density

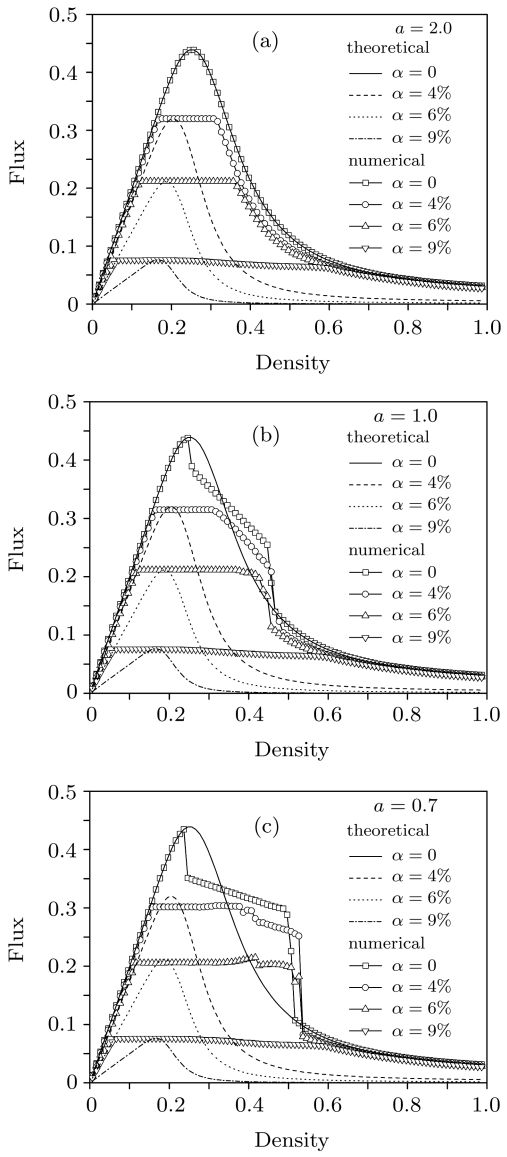


Fig.2. Fundamental diagrams with different parameters.

increases with the increase of α. This means for a larger α, the density range corresponding to the saturated current also increases. This is because with the increase of the slope grade, the speed-limiting ability strengthens, which leads the critical density from the free flow to the saturated flow to decrease. At the same time, the decreasing flux in the upgrade requires more vehicles for the occurring of congested phase, so the second critical density increases. In addition, the sensitivity has a certain influence on the fundamental diagrams. With the gradual decrease of a, the fundamental diagrams become more complicated. Firstly, the value of saturated flux decreases slightly, especially on a moderate slope. Secondly, the value of the first critical density does not depend strongly on the sensitivity and drops a little, but the value of the second critical density increases with sensitivity decreasing. Finally, at a lower sensitivity ($a = 0.7$), the instability is more obvious. In a saturated current region, there are some fluctuations. Note that for different sensitivities, the fundamental diagrams of $\alpha = 9\%$ are nearly the same, which indicates that for a steep slope, the speeds of all vehicles are very low and the sensitivity has no influence on the whole road.

Now we analyse the first critical density for the saturated flow, and let it be ρ_{c1}. Without considering the slight influence of sensitivity, as the flux of whole road reaches the saturated value, the outflow and the inflow of the upgrade attain an equilibrium, so we have

$$\rho_N V_{N,\max} = \rho_U V_U, \tag{10}$$

where ρ_N and $V_{N,\max}$ are the average density and the maximal speed for the normal section, respectively; ρ_U and V_U denote the average density and the speed for the upgrade, respectively. In view of the conservation of vehicles, it can be derived that

$$\rho_N(L - L_U) + \rho_U L_U = \rho L. \tag{11}$$

Then, the average density for the whole road is obtained from Eqs.(10) and (11) as

$$\rho = \frac{L_U}{L}\left(1 - \frac{V_U}{V_{N,\max}}\right)\rho_U + \frac{V_U}{V_{N,\max}}\rho_U. \tag{12}$$

As $\rho = \rho_{c1}$, the value of the saturated flow is equal to the maximal flow for the upgrade, one can obtain

$$\left.\frac{\mathrm{d}(\rho_U V_U)}{\mathrm{d}\rho_U}\right|_{\rho_U = \rho_{U,c}} = 0, \tag{13}$$

where $\rho_{U,c}$ is the density as the flow for the upgrade attains its maximum. Let $V_{U,c}$ be defined as the speed corresponding to $\rho_{U,c}$. Finally, we obtain

$$\rho_{c1} = \frac{L_U}{L}\left(1 - \frac{V_{U,c}}{V_{N,\max}}\right)\rho_{U,c} + \frac{V_{U,c}}{V_{N,\max}}\rho_{U,c}. \tag{14}$$

From expression (14), it can be seen that at given $V_{N,\max}$ and L, ρ_{c1} is related to L_U as well as α. Here we first give the approximate analytical results about the first critical density for the saturated flux.

As α is fixed, ρ_{c1} increases with slope length increasing. We can obtain $\lim\limits_{L_U \to L} \rho_{c1} = \rho_{U,c}$, i.e. the density range corresponding to the saturated flow shrinks to a point and the value of density for the maximal flow of the whole road tends to that for the maximal flow of the upgrade. Besides, when L_U is fixed, the increase of α could make $V_{U,c}$ and $\rho_{U,c}$ decrease, consequently, ρ_{c1} would decrease. And it can be obtained that $\lim\limits_{\alpha \to 0} \rho_{c1} = \rho_{N,c}$, namely, the saturated flow plateau gradually disappears, but the density for

the maximal flow of the whole road tends to that for the maximal flow of the normal section. This conclusion accords with the above numerical simulation (see Fig.2).

In the region of saturated current, a traffic jam appears just before the section of slope. We study the formation and the dissolution of shock wave and rarefaction wave through the headway profiles. Figure 3 gives the position-headway pattern obtained at $t = 20, 300, 700, 1000$ and Fig.4 gives a spatiotemporal pattern at $t = 1000$, related to Fig.3, where $\alpha = 6\%$, $N = 375$, and $a = 1.0$.

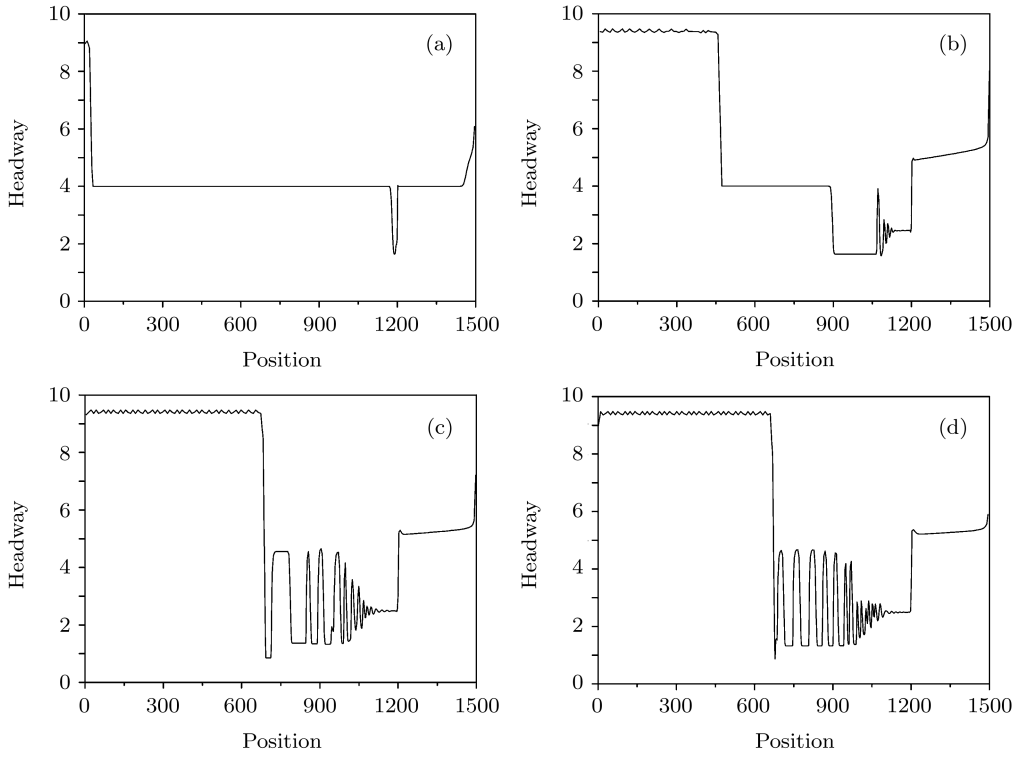


Fig.3. The headways against position of vehicles obtained at $t = 20, 300, 700, 1000$, where $\alpha = 6\%$, $N = 375$, and $a = 1.0$.

In Fig.3, we see a fully-developed shock wave and rarefaction wave. Initially, the vehicles are distributed uniformly on the whole road. With the time evolution, the neighbouring vehicles which are distributed at the joint between the normal section and upgrade section would move according to a respective optimal speed function. This difference leads the headway of the neighbouring vehicles to be less than the initial average headway. With time increasing, the headway of vehicles before the slope will gradually decrease. From the macroscopic viewpoint, a backward-propagating shock wave occurs. Based on the same analysis, at the end of the slope, a backward-propagating rarefaction wave is formed. Under the periodic boundary condition, this rarefaction wave propagates along the moving direction of vehicles till it encounters the shock wave. From the headway-position pattern, it can be found that a queue length is gradually increasing, which is shown as a congested region. Under the influence of the rarefaction wave coming from the end of the slope, a free flow appears in an upstream region of the congested section. In this free flow section, vehicles could move at a desired speed of the normal section, while in the upgrade, vehicles move at a speed near a desired maximum speed of the grade $\alpha = 6\%$. In addition, the traffic characteristic is also different in the whole congested section. About 100 before the slope, the headway is almost uniform and vehicles are moving at a very low speed, displaying a light moving jam; but between the moving jam and the free flow are the regions of high spatial frequency oscillation, which corresponds to the stop-and-go traffic. As time goes on, the system reaches its stable state, in which the shape of headway-position pattern no longer changes. From Fig.4, we can see clearly that there exist such traffic phenomena as two obvious free flow regions, moving jam, stop-and-go traffic, interaction between the shock and the rarefaction wave, etc.

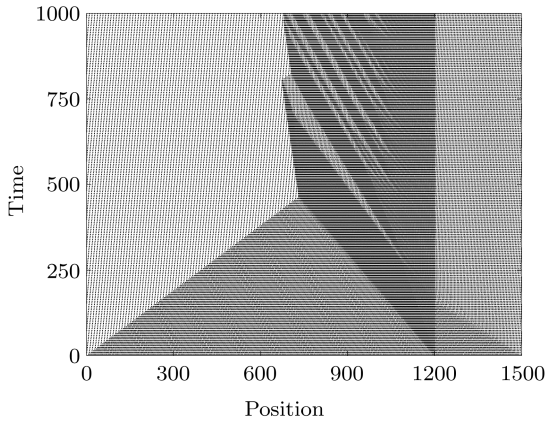


Fig.4. Spatiotemporal pattern up to time $t = 1000$, where $\alpha = 6\%$, $N = 375$, and $a = 1.0$

Under the same slope grade, the traffic characteristic varies with slope length. Figure 5 shows the fundamental diagrams for the values of $L_{\mathrm{U}} = 200, 300, 400$ and 500, where $a = 1$ and $\alpha = 4\%, 7\%$. We can find that for a small slope, L_{U} has no obvious influence on the fundamental diagram. The first critical density under different slope lengths is almost the same and the second critical density decreases a little with the increase of L_{U}, whereas, for $\alpha = 7\%$, the flux plateau decreases with the increase of slope length. And the first critical density increases slightly and the second critical density decreases both with the increase of slope length. In the two cases, the value of saturated flux cannot vary with slope length. All these results mean that for a small slope, the speed-limiting ability is very weak, i.e. the maximal speed and the safety distance in the normal section and the upgrade section have no obvious difference, so the increasing slope length dose not greatly affect the critical density from the free flow to the saturated flow, but it could influence the second critical density. When the slope is very steep, the speed-limiting ability is enhanced, the increase of slope length will make the plateau length diminish and traffic flux rapidly drop. This simulation result further validates the previous theoretical analysis.

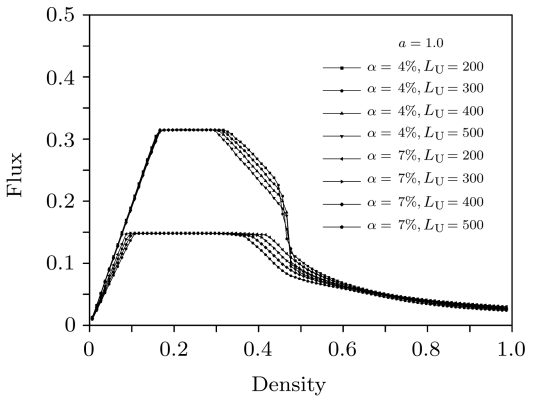


Fig.5. The fundamental diagrams with different values of slope length L_{U} : 200, 300, 400 and 500, where $a = 1$, $\alpha = 4\%$ and 7%.

Finally, we briefly investigate traffic characteristic induced by the downgrade. Figure 6 shows the fundamental diagrams of the whole road composed of the normal section plus upgrade, downgrade, respectively, where $L = 1500, L_{\mathrm{U}} = 300, L_{\mathrm{D}} = 300, a = 1.0$ and $\alpha = 2\%, 4\%, 6\%$ and 9% (for the downgrade, α is negative). It can be seen that under the same sensitivity, the configurations of fundamental diagrams hardly change. When the absolute value of slope grade is the same, the traffic current of the downgrade is larger than that of the upgrade. And at a high density ($\rho > 0.6$), all the curves tend to consistency. For $\alpha = \pm 2\%$, there is a small-range saturated flow in the fundamental diagram of the upgrade, but on the downgrade, this saturated flow does not exist. This is because for the whole road, the speed of the downgrade will increase and the safe headway does not change obviously, which could not generate an effective speed limit region. When the slope is very steep, the current of the downgrade is still larger than that of the upgrade, especially, the second critical density is still large. In such a case, drivers should drive quite carefully so as to avoid the accidents. This also reflects the fact that the accidents would occur more easily in the downgrade than in the upgrade under the same slope grade.[28]

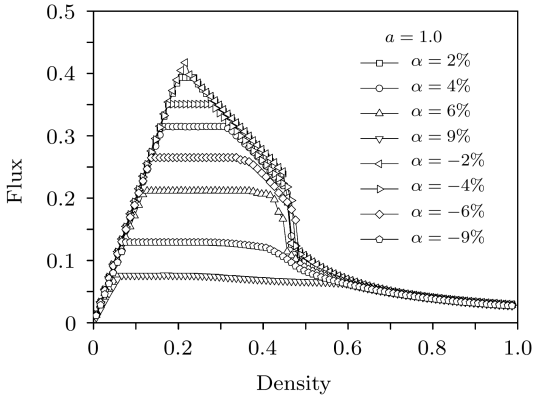


Fig.6. A comparison between the fundamental diagrams of the upgrade and the downgrade under different grades.

4. Summary and perspectives

We have extended the classical optimal speed model to investigate the traffic characteristic in a

single-lane highway with a slope. A generalized optimal speed function has been introduced to synthetically describe the effect of all the outer and inner factors on the optimal speed. Then we apply it to the analysing of the real road with an upgrade (or a downgrade). From numerical simulations, we obtain fundamental diagrams, headways against position plot and spatiotemporal patterns under different parameters. It is found that

- The saturated flux appears in the fundamental diagram. Its value depends mainly on the maximal speed and safe headway of the upgrade (downgrade) and has little relation with sensitivity.
- From the analytical viewpoint, the approximate expression of the first critical density corresponding to saturated flow is given. The theoretical analysis accords with the numerical simulation.
- When the sensitivity decreases, the system becomes more unstable and uniform jam gradually transforms into an oscillatory jam.
- The spatiotemporal pattern shows the segregation of different traffic phases caused by the rarefaction and the shock waves induced by slope.
- The value of saturated flux of the downgrade is larger than that of upgrade under the same conditions, which is consistent with the real traffic.

Different from the usual speed limit region caused by the traffic accident, toll station, working region, slowdown, etc. the upgrade (downgrade) road covers a wide kind of speed limit region coming from the road condition and spatial geometrical configuration, i.e. the speed dose not depend on the spatial situation continuously. The presented model, the method and the general conclusion can be conveniently extended to more common traffic flow with several different kinds of speed limit regions, such as urban underpass, harbour tunnel, the effect of fog, rain and snow. We hope that our primitive results here can be useful for controlling the traffic flow at the joint between the flat and the speed limit regions and also for making decisions in designing roads so as to avoid traffic jams.

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