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Stability analysis of viscous Z-pinch plasma with a sheared axial flow*

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Within the magnetohydrodynamics (MHD) frame, we analyse the effect of viscosity on magneto-Rayleigh–Taylor (MRT) instability in a Z-pinch configuration by using an exact method and an approximate method separately. It is demonstrated that the plasma viscosity indeed has a stabilization effect on the MRT mode in the whole wavenumber region, and its influence increases with the perturbation wavenumber increasing. After the characteristics and feasibility of the approximate method have been investigated, we apply it to the stability analysis of viscous plasma where a sheared axial flow (SAF) is involved, and we attain an analytical dispersion relation. It is suggested that the viscosity and the SAF are complemental with each other, and a wide wavenumber range of perturbation is possible to be restrained if the SAF and the viscosity are large enough. Finally, we calculate the possible value of viscosity parameter according to the current experimental conditions, and the results show that since the value of viscosity is much less than the threshold value, its mitigation effect is small enough to be neglected. The role of the viscosity in the stabilization becomes considerable only if special techniques are so developed that the Z-pinch plasma viscosity can be increased greatly.

Keywords: Z-pinch, magneto-Rayleigh-Taylor (MRT) instability, viscosity, sheared axial flow
PACC: 5235P, 5255E, 5230

1. Introduction

As a result of a series of experimental breakthroughs in the study of Z-pinch plasmas in the late 1990s, the efficiency and cost effectiveness of such x-ray radiation resource devices have been well recognized.[1,2] However, it is well known that the magnetically imploding cylindrical load is unstable, and any perturbation that is initially present or develops during the implosion itself will grow rapidly. One of the most serious types of perturbations is magneto-Rayleigh–Taylor (MRT) instability, which shares many features with hydrodynamic Rayleigh-Taylor (RT) instability in a nonmagnetic fluid.[3−5] The latter arises at a perturbed interface between two incompressible fluids, when the light fluid exerts an acceleration on the heavy one. In the magnetic analog, the heavy fluid is represented by plasma and the light fluid by the magnetic field that accelerates the plasma in the inward radial direction. As the general Z-pinch imploding time is about a few nanoseconds and the plasma acceleration is about $10^{12}\,\mathrm{m/s^2}$ in magnitude, the MRT instability develops at a great speed and forms a typical spike-bubble construction. The bubbles represent the magnetic flux penetrating into the plasma, while the spikes represent the plasma that has been squeezed back and left behind by the net inward motion of the plasma. Different from the Rayleigh-Taylor instability in fluid, the plasma sheath in the Z-pinch problem is a medium of finite thickness, and thus these spike-bubble structures can not only broaden the radial thickness of the sheath, thereby reducing the temperature of the pinching plasma, but also eventually burst through into the inner vacuum region, thus terminating the plasma discharge within about ten Alfven transit times.[6] Consequently, the study on how to mitigate or eliminate the MRT instability, which plays a destructive role in imploding, attracts more attention and becomes increasingly important for enhancing the implosion quality and improving the x-ray radiation output.

To improve the stability of the Z-pinch imploding, one has concentrated on different mechanisms including the improvement of the initial symmetry,[1,2] hourglass configuration,[7,8] the finite Larmor radius (FLR) effect,[9,10] and the sheared axial flow (SAF) effect.[10−13] However, all these mechanisms each have their own shortages or limitations. For example, it is suggested theoretically that a sufficiently large SAF

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is effective in restraining the middle wavenumber perturbation ($2.4 < K < 10$, where K is the dimensionless disturbance wavenumber, see Refs.[12, 13]), but its stabilization effect in the $K > 10$ region is limited, and a hybrid MRT/Kelvin–Helmholtz (MRT/KH) instability is aroused with $K < 2.4$. Therefore, it is impracticable to use a single mechanism to attain a universally satisfying stabilization effect in the whole region covering all perturbation wavenumbers, and the study of synergistic effect of different mechanisms becomes much more instructive. We have analysed the coupling effect of plasma compressibility and SAF, which suggests that the former has a mitigation effect on the SAF-induced MRT/KH instability and will optimize the stabilization effect of SAF.[14,15] In this paper, we will discuss another dissipative process—the plasma viscosity, and analyse the case in which the SAF and the viscosity both are involved.

The effects of plasma viscosity and their influence on the RT instability have been studied in great detail, and they were first studied in the Chandrasekhar's monograph,[16] in which the stability of a boundary between two semi-infinite viscous fluids was considered. A careful analysis of viscous effects in the hydrodynamic case of complex density profile was performed by Mikaelian.[17,18] In these studies, analytical and numerical results show that as one of important dissipative processes, plasma viscosity plays an important role in assuaging the MRT instability, especially when the characteristic spatial scale of the disturbance is relatively small. However, all those studies were concerned with the cases of two distinct fluid-slabs or continuous variation of density, and none of them belonged to any one of Z-pinch configurations, until Qiu *et al*[19] studied the case of a plasma slab surrounded by vacuum magnetic field, namely a Z-pinch configuration. In Qiu *et al*'s paper, by assuming the wavelength to be small compared with the characteristic scale of the density gradient, a clear analytical dispersion relation was attained and showed that the mitigation effect of the viscosity increased as the plasma temperature was enhanced. In this paper, we try to attain an exact analytic dispersion relation without using any approximation in Section 2 and we further extend it to a case with a sheared axial flow in Section 3. The latter problem is so complicated that it is impossible to obtain an analytic solution without using any assumption. Therefore, we use an approximate approach, which Mikaelian suggested earlier for the inviscid and tensionless case,[18] and later we use it for a viscosity analysis.[20] Finally, we make some discussion based on the practical experimental conditions, and draw conclusions in Section 4.

2. Effect of viscosity

Z-pinch implosion begins with the explosion of metal wires and the formation of plasma cloud. And according to Haines model,[6] in the inward acceleration phase, the thickness of the plasma cloud, sometimes considered approximately as an annular shell, is sufficiently smaller than the pinch radius, thereby allowing us to use a slab geometry instead of the annular one.

Within the viscid MHD frame, the momentum equation can be written as

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u}=-\nabla p+\boldsymbol{J}\times\boldsymbol{B}+\rho\boldsymbol{g}\nabla\cdot\boldsymbol{P}, \quad (1)$$

where all the notations are conventional, but for $\boldsymbol{g}$ and $\boldsymbol{P}$, which are the magnetic field caused effective acceleration and the plasma viscosity tensor respectively.

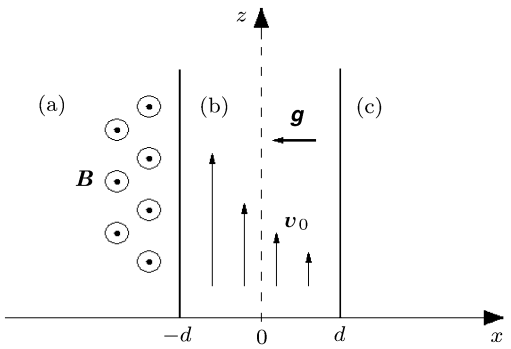


Fig.1. The slab geometry used in the stability analysis, where Symbols a, b and c denote different zones separately.

Using the Cartesian coordinate setup as in Ref.[14] (see Fig.1), except for the steady axial speed v_0=0 in this section, and assuming that all variables vary in the x direction and can be expressed as $\xi_0+\xi_1$, where subscript 1 denotes perturbations of steady state quantities that are marked with subscript 0, the linearized equations become

$$\rho_0\frac{\partial u_{1x}}{\partial t}=-\frac{\partial}{\partial x}p_1-(J_0B_{1y}+J_{1z}B_0)-\rho_1 g+\frac{\partial}{\partial x}P_{x_i}, \quad (2)$$

$$\rho_0\frac{\partial u_{1y}}{\partial t}=-\frac{\partial}{\partial y}p_1+J_0B_{1x}+\frac{\partial}{\partial y}P_{y_i}, \quad (3)$$

$$\rho_0\frac{\partial u_{1z}}{\partial t}=-\frac{\partial}{\partial z}p_1+J_{1x}B_0+\frac{\partial}{\partial z}P_{z_i}, \quad (4)$$

where P_{k_i} is given by $P_{k_i} = \mu(\partial u_k/\partial x_i + \partial u_i/\partial x_k)$ and μ is the dynamic plasma viscosity parameter. Since modes with k parallel to the magnetic field cannot grow,[12,19] perturbations are assumed to vary in the form of $\exp(-\mathrm{i}\omega t + \mathrm{i}kz)$. Inserting P_{k_i} into Eqs.(2)–(4) and making use of the continuity, incompressibility and Maxwell equations, we ultimately obtain the equation of perturbation

$$\begin{aligned} & k^2\left[\rho_0 u_{1x} + u_{1x}\frac{g}{\omega^2}\mathrm{D}\rho_0 + 2\frac{\mathrm{D}\mu}{\mathrm{i}\omega}Du_{1x}\right. \\ & \left. + \frac{\mu}{\mathrm{i}\omega}\left(\mathrm{D}^2 - k^2\right)u_{1x}\right] \\ =& \mathrm{D}\left[\rho_0 \mathrm{D}u_{1x} + \frac{\mathrm{D}\mu}{\mathrm{i}\omega}\left(\mathrm{D}^2 + k^2\right)u_{1x}\right. \\ & \left. + \frac{\mu}{\mathrm{i}\omega}\left(\mathrm{D}^2 - k^2\right)\mathrm{D}u_{1x}\right], \end{aligned} \tag{5}$$

where D denotes $\mathrm{d}/\mathrm{d}x$. Clearly, Eq.(5) is the same as the one derived by Chandrasekhar[16] in the case of a hydrodynamic analysis, where the magnetic field and Lorentz force were absent. This is because during the derivation we have used the incompressible consumption, which makes the effect of the magnetic field undetectable.[14] Therefore the magnetic field related terms disappear here.

2.1. Results from an exact method

We emphasize that Eq.(5) is exact, general, and able to be applied to fluids of arbitrary density profile $\rho(x)$ and viscosity $\mu(x)$. If we make an assumption that $k^{-2}\mathrm{d}^2/\mathrm{d}x^2 \ll 1$, when we concentrate on the analysis of the small scale perturbation, Eq.(5) is reduced to the form of Eq.(13) in Ref.[19]. Here we try to attain a general solution suitable to perturbations in the whole region covering all wavenumbers, and therefore Eq.(5) is used as a basic condition without making any simplification.

Different from the cases studied by Mikaelian, the Z-pinch plasma is figured as a three-region-two-interface MHD problem, which makes the number of the boundary conditions doubled. Therefore, this problem becomes more complicated. To obtain the conditions at the vacuum-plasma interfaces, it is necessary to specify which quantities should be continuous and which jump conditions should be satisfied. According to Chandrasekhar's suggestion,[16] all three components of the velocity, as well as the tangential viscous stresses, must be continuous, and the continuity of Du follows from the continuity equation. The value of D^2u at the interface is specified by the following equation:

$$\begin{aligned} & k^2 u_{1x}\frac{g}{\omega^2}\Delta\left[\rho_0\right]_{\pm d} - 2\mathrm{i}\frac{k^2}{\omega}\mathrm{D}u_{1x}\Delta\left[\mu\right]_{\pm d} \\ =& \Delta\left[\left(\rho_0 - \mathrm{i}\frac{\mu}{\omega}(\mathrm{D}^2 - k^2)\right)\mathrm{D}u_{1x}\right. \\ & \left. - \mathrm{i}(\mathrm{D}^2 + k^2)\frac{u_{1x}}{\omega}\mathrm{D}\mu\right]_{\pm d}, \end{aligned} \tag{6}$$

which is the result of integrating Eq.(5) over an infinitesimal element of x including $x = \pm d$, the plasma-vacuum interfaces, and $\Delta[f]_x$ is defined as $\Delta[f]_x = f(x+0) - f(x-0)$.

To obtain an analytical solution, we consider an ideal case, in which the mass density and the viscosity are uniform. Therefore the perturbation equation becomes

$$(\mathrm{D}^2 - k^2)\left[1 - \mathrm{i}\frac{\gamma}{\omega}(\mathrm{D}^2 - k^2)\right]u_{1x} = 0, \tag{7}$$

where $\gamma = \mu \,/\, \rho_0$. For any inviscid plasma, this equation reduces to $(\mathrm{D}^2 - k^2)u_{1x} = 0$ with a familiar solution formed as $\mathrm{e}^{\pm kx}$, which is widely used in the MRT instability analysis. Such an inviscid case has been studied widely, and here we will not discuss it any more.

The general solution of Eq.(7) is a linear combination of the solutions $\mathrm{e}^{\pm kx}$ and $\mathrm{e}^{\pm qx}$, where $q = (k^2 - \mathrm{i}\omega/\gamma)^{1/2}$. Since the perturbation must vanish when $x \to \pm\infty$, according to Fig.1, the solutions of Eq.(7) in zones a, b and c are respectively expressed as

$$u_{1xa} = A_a\mathrm{e}^{kx} + C_a\mathrm{e}^{q_a x}, \qquad x < -d \quad \text{in zone } a, \tag{8}$$

$$u_{1xb} = A_b\mathrm{e}^{kx} + B_b\mathrm{e}^{-kx} + C_b\mathrm{e}^{q_b x} + D_b\mathrm{e}^{-q_b x}, \qquad -d < x < d \text{ in zone } b, \tag{9}$$

$$u_{1xc} = B_c\mathrm{e}^{-kx} + D_c\mathrm{e}^{-q_c x}, \qquad x > d \text{ in zone } c, \tag{10}$$

where the coefficients are determined by the jump conditions at both interfaces. Substituting eigenfunctions (8)–(10) into the interface jump conditions, the eight equations can be written in the form $\boldsymbol{MV}=0$, where $\boldsymbol{M}$ is an 8×8 matrix and $\boldsymbol{V}$ is a vector with eight constants A_a through D_c as its elements. A nontrivial solution is obtained only if

$$\mathrm{Det}\,(\boldsymbol{M})=0. \tag{11}$$

And from such a condition, we can obtain the following relationship between K and ω, namely the dispersion relation:

$$\begin{aligned}&3q_bk^3+\frac{q_b^5}{k}-\left(q_b^2+k^2\right)^2\left(\mathrm{cth}\,(2\mathrm{d}k)\,\mathrm{cth}\,(2\mathrm{d}q_b)\right.\\&\left.-4\,\mathrm{csc}\,h\,(2\mathrm{d}k)\,\mathrm{csc}\,h\,(2\mathrm{d}q_b)\right)\\=&\frac{k^2g^2-\omega^4}{8\gamma_b^4q_bk^3}.\end{aligned} \tag{12}$$

If the plasma in zone b is inviscid, dispersion relation Eq.(12) reduces to a familiar form $\omega^2=-kg$. But for the viscid case, Eq.(12) should be solved exactly. After a simple analysis of the asymptotic behaviour of ω for $k\to\infty$ and 0, we find that when $k\to 0$, then $-\mathrm{i}\omega\to(kg)^{1/2}$. On the other hand, when $k\to\infty$, then Eq.(12) becomes $-\mathrm{i}\omega\to g/(2k\gamma^2)$. It is clear that the growth rate goes to zero when $k\to 0$ and $k\to\infty$. Therefore, a mode of maximum instability appears, which is similar to the case analysed by Chandrasekhar.[16] To have a straightforward impression of the mitigation effect of the viscosity, we present the relations between the growth rate and perturbation wavenumber, which are normalized respectively by setting $\Gamma=\mathrm{Re}(-\mathrm{i}\omega)/(kg)^{1/2}$ and $K=2kd$, with different values of viscosity parameter in Fig.2.

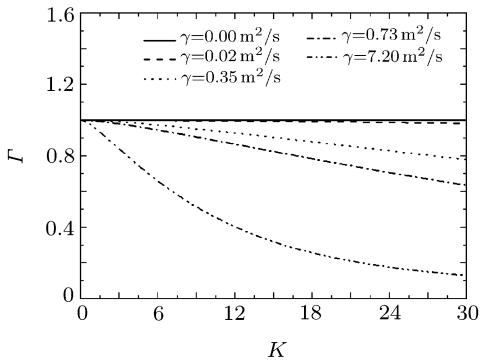


Fig.2. The exact relations between the normalized growth rate and normalized wavenumber with different values of viscosity parameter.

In this figure, the four curves correspond to γ=0.02, 0.35, 0.73, and 7.2 m^2/s separately, which are identified with different plasma statuses (see Ref.[19]). Although these results are quantitively different from the results given by Qiu *et al*, their trends of the growth rate varying with either the wavenumber or viscosity are similar: the mitigation effect of viscosity is strengthened with the perturbation wavenumber K and viscosity parameter γ increasing. With a small viscosity, e.g. γ=0.02 m^2/s, the mitigation effect is not obvious (less than 2% even for a large wavenumber perturbation K=30). Therefore, in such cases the plasma viscosity in estimating instability growth rate can be ignored. However, as the viscosity increases, the growth rate is reduced rapidly. When the viscosity is enhanced to 0.35 and 0.73 m^2/s, the growth rates decrease 22% and 36% respectively. We emphasize that one of the obvious differences in result between Qiu *et al* and us is that according to our calculations the MRT instability cannot be eliminated completely based on such a mechanism within a finite wavenumber, but it can in Qiu *et al*'s paper. For example, in Fig.1 of Ref.[19] for a large viscosity, 7.2 m^2/s, the MRT instability is eliminated completely by such a viscosity at a very small value of K, no more than 1, while for our calculations in Fig.2, the mitigated growth rate is about 13% of the original one at K=30. Such a value can also be achieved according to our analysis of foregoing asymptotic behaviour for Eq.(12), which shows that the growth rate goes to zero only when $k\to 0$ and $k\to\infty$. This disagreement maybe results from the small wavelength assumption in Qiu *et al*'s paper, that is,

$$\frac{1}{k^2}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\ll 1, \tag{13}$$

where $|(\mathrm{d}/\mathrm{d}x)^{-1}|$ represents the space-scale of the plasma variation and k^{-1} denotes the wavelength of perturbation. Considering the similarity between Eq.(13) and the asymptotic condition $k\to\infty$, it is reasonable to expect a quenching point for the MRT instability to be in a limited wavenumber region.

2.2. Results from an approximate method

The foregoing analysis is based on a general procedure of solving eigenvalue problems. Fortunately, we gain an analytical function of dispersion relation Eq.(12) due to the ideal uniform model we used. However, in order to estimate the actual situation one should deal with models which are much closer to the

real condition and usually include various dissipative mechanisms and complicated parameter profiles. In most cases it is hard to obtain an analytical result or to make analytical analysis. Therefore, it is necessary to develop an effective method which can deal with a wide range of similar questions and give approximate results in an analytical form. In fact, based on such a consideration, Mikaelian[18] has already developed a simple analytical approximate method and applied it to the RT instability analysis. All we need to do here is just apply it to the Z-pinch case.

Generally, for any moment equation, such as Eq.(5), there are infinitely many growth rates and associated eigenfunctions, both of which vary synchronously. To settle such a problem one must deal with the moment equation with a number of boundary and interface conditions. Considering that in most cases the growth rate has both real and imaginary parts, the actual number of equations to be resolved is doubled, which is the location of all the difficulties. To avoid these difficulties, this approximate technique must be based on the assumption: all the RT (MRT) dominated instabilities have a classical sharp-boundary RT mode eigenfunction, i.e.

$$f(x) = \mathrm{e}^{-k|x-x_0|}. \tag{14}$$

Since the eigenfunction is designated, the growth rate can be resolved directly from the moment equation. We must admit that this approximate method does greatly simplify the difficulty in solving such a kind of RT problem. However it also has an obvious limitation: its reliability greatly depends on the similarity between the real eigenfunction and Eq.(14). In many cases in which, for example, there exist arbitrary mass profiles, Mikaelian found it to work well and the approximate result to be very similar to the exact one. However, our calculations show that this technique does not serve the turn when dealing with cases in which the eigenfunction greatly differs from the classical one. Only after its advantages and shortages have been well understood, can we exert its predominance in dealing with the practical questions qualitatively and analytically.

In order to make a detailed comparison between the approximate results and the exact results, now we use the approximate method to solve the foregoing uniform-profile problem again, and in the next section we will apply it to more complicated cases. To reduce the fourth-order moment equation we multiply Eq.(5) by u_x^m and integrate the resulting equation over $(-\infty, +\infty)$. Many terms can be integrated by parts, and the resulting "surface terms" may be set to be zero. Mikaelian's study indicates that the m=0 equation gives the best results, and therefore we will follow this point. By setting m=0 we obtain

$$-\omega^2 \int \rho u_{1x} \mathrm{d}x - \mathrm{i}\omega \int \mu \left(\mathrm{D}^2 + k^2\right) u_{1x} \mathrm{d}x - g \int \mathrm{D}\rho u_{1x} \mathrm{d}x = 0. \tag{15}$$

Obviously, until this step, Eq.(15) is an exact function which can be applied to arbitrary profiles, and will produce the exact growth rate provided that one uses the exact u_{1x}. As is known, the most severe MRT instability in a Z-pinch plasma is located at the outer interface between the plasma and the vacuum, namely $x = -d$. Therefore we set the approximate u_{1x} to be

$$u_{1x} = A\mathrm{e}^{-k(x+d)}. \tag{16}$$

By substituting it into Eq.(15) and integrating, with the uniform condition, i.e. ρ, μ=constant employed, we obtain

$$-\omega^2 - \mathrm{i}2k^2\upsilon\omega - kgA = 0, \tag{17}$$

where

$$\upsilon = \frac{(\mu_a + \mu_b) + (\mu_c - \mu_b)\,\mathrm{e}^{-2kd}}{(\rho_a + \rho_b) + (\rho_c - \rho_b)\,\mathrm{e}^{-2kd}}$$

and

$$A = \frac{(\rho_b - \rho_a) + (\rho_c - \rho_b)\,\mathrm{e}^{-2kd}}{(\rho_a + \rho_b) + (\rho_c - \rho_b)\,\mathrm{e}^{-2kd}}, \tag{18}$$

and the analytical dispersion relation becomes

$$\omega = -\mathrm{i}k^2\upsilon \pm \sqrt{k^4\upsilon^2 + kgA}. \tag{19}$$

First, we discuss two kinds of extreme situations. When we make the small-wavelength assumption, that is $k \gg d^{-1}$, we obtain

$$\upsilon = \frac{(\mu_a + \mu_b)}{(\rho_a + \rho_b)} \quad \text{and} \quad A = \frac{(\rho_b - \rho_a)}{(\rho_a + \rho_b)}, \tag{20}$$

and the dispersion relation reduces to the case of the single sharp-edge profile. This result is consistent with the fact that as its space scale is much less than the plasma scale, the perturbation is more localized, and its growth rate relates only to parameters around the outer interface (μ_a, μ_b, ρ_a, ρ_b).

In the long-wavelength limit $k \ll d^{-1}$, we have

$$\upsilon = \frac{(\mu_a + \mu_c)}{(\rho_a + \rho_c)} \quad \text{and} \quad A = \frac{(\rho_c - \rho_a)}{(\rho_a + \rho_c)}, \tag{21}$$

In this case, the plasma thickness is much less than the perturbation scale and the information of quantities in zone b is neglectable.

For more general discussion, we set

$$\rho_a, \mu_a = 0 \quad \text{and} \quad \rho_c, \mu_c = 0. \tag{22}$$

Therefore, we have

$$\upsilon = \gamma_b = \frac{\mu_b}{\rho_b}, \quad A = 1, \tag{23}$$

and

$$\omega = -\mathrm{i}k^2\gamma_b \pm \mathrm{i}\sqrt{k^4\gamma_b^2 + kg}. \tag{24}$$

It is easy to find that if the viscosity is finite then we have a dimensionless growth rate $\Gamma = \mathrm{Re}(-\mathrm{i}\omega)/(kg)^{1/2} < 1$, which means that the viscosity plays a role in stabilizing the MRT instability in the whole region covering all wavenumbers. And from expression (24) we also find that as the wavenumber k increases the mitigation effect becomes important. From these analyses we are impressed by the simplicity, straightforwardness and significance of the approximate technique. Curves shown in Fig.3 also demonstrate the acceptable accuracy of the results. In this figure, the solid curve represents the exact result the same as that in Fig.2, while the dash curve denotes the approximate result calculated from expression (24). From their comparison, we find that although they are not exactly the same, their differences are within a tolerable level. From the calculations we find that when the viscosity is less than $0.73\,\mathrm{m}^2/\mathrm{s}$, the difference between the results, which indicates the deviation of the approximate eigenfunction from the exact one, increases with K increasing, and when the γ=$7.2\,\mathrm{m}^2/\mathrm{s}$ the difference appears in a small wavenumber region. The acceptable accuracy of these results manifests that the eigenfunction Eq.(16) is an effective approach to practical problems. Owing to the effectiveness of the approximate technique, we will apply it to the following discussion.

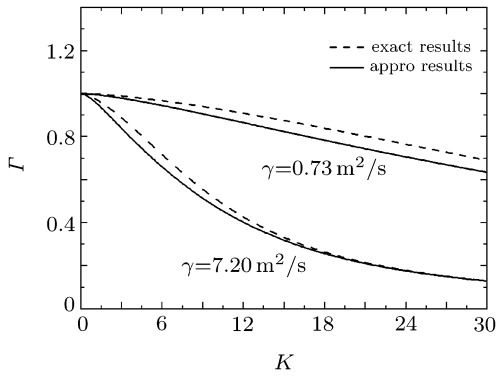


Fig.3. Comparison between results from two different methods, where solid line denotes the exact result and the dash represents the approximate one.

3. Viscous plasma with an SAF

From many researches (for example, Shumlak and Roderick[12] and Qiu *et al*,[10,13]) and our studies,[14,21] we know that the SAF has an efficient mitigation effect on perturbations within a moderate wavenumber region ($2.4 < K < 10$), but this effect is less efficient in the remaining regions. For perturbations with small values of K, the instability has an MRT/KH hybrid mode, whose growth rate exceeds the pure MRT mode's. And, because the stabilization effect of the SAF decreases with the enhancement of K, in a large wavenumber region the mitigation effect is insufficient. Enlightened by the analysis above, we believe that it is instructive to discuss the case with both viscosity and SAF, expecting the mitigation effect for the high K region of the viscosity to be able to give a positive complementarity to the SAF.

According to the discussion in Section 2, we will redefine the steady speed by adding an x distributed axial flow. According to our studies before, a linear profile speed is used[14,21] and expressed as

$$v_0 = V_0\left(1 - x/d\right)/2. \tag{25}$$

And after the same treatment, we obtain the differential moment equation as follow:

$$\begin{aligned} & k^2\left[\rho_0\left(n + \mathrm{i}kv_0\right)u_{1x} - \frac{g}{n + \mathrm{i}kv_0}\mathrm{D}\rho_0 u_{1x} - 2\mathrm{D}\mu\mathrm{D}u_{1x} - \mu\left(\mathrm{D}^2 - k^2\right)u_{1x}\right] \\ =& \mathrm{D}\Big[\rho_0\left(n + \mathrm{i}kv_0\right)\mathrm{D}u_{1x} - \mathrm{i}k\rho_0 u_{1x}\mathrm{D}v_0 - \mathrm{D}\mu\left(k^2 + D^2\right)u_{1x} - \mu\left(\mathrm{D}^2 - k^2\right)\mathrm{D}u_{1x} \\ & + \mathrm{i}k\mathrm{D}\left(\mu\mathrm{D}v_0\right)\Big], \end{aligned} \tag{26}$$

in which there exist not only the separate terms of viscosity μ and SAF v_0, but also their coupling term (the

last term on the right side-hand). Integrating Eq.(26) with respect to x, we obtain

$$\omega^2+\left(\mathrm{i}2\gamma k^2+\frac{V_0}{2d}+\frac{2kV_0}{\mathrm{e}^{-2kd}-1}\right)\omega-\frac{\left(k^2-k/2d\right)V_0^2-2\mathrm{i}\gamma V_0k^3}{\mathrm{e}^{-2kd}-1}+kg=0. \tag{27}$$

From Eq.(27), it is easy to derive an approximate dispersion relation

$$\omega=\frac{kV_0}{1-\mathrm{e}^{-2kd}}-\frac{V_0}{4d}-\mathrm{i}k^2\gamma\pm\sqrt{\left(\frac{kV_0}{1-\mathrm{e}^{-2kd}}-\frac{V_0}{4d}-\mathrm{i}k^2\gamma\right)^2+\frac{2\mathrm{i}\gamma V_0k^3-\left(k^2-k/2d\right)V_0^2}{1-\mathrm{e}^{-2kd}}-kg}. \tag{28}$$

Comparing the above approximate dispersion relation with the exact one shows that, the approximate technique provides us with an analytical dispersion relation which is much more meaningful and straightforward to understand the effect of different mechanisms. Let the axial speed V_0=0, expression (28) reduces to a familiar expression (19) for the non-flow case. Or if we set γ=0, expression (28) is transformed into

$$\omega=-\left(\frac{1}{4d}+\frac{k}{\mathrm{e}^{-2kd}-1}\right)V_0\pm\sqrt{\left[\frac{1}{16d^2}+\frac{\mathrm{e}^{-2kd}k^2}{\left(\mathrm{e}^{-2kd}-1\right)^2}\right]V_0^2-kg}, \tag{29}$$

which is an approximation to the inviscid SAF case. As the first term in the square root is constantly positive, the growth rate is definitely less than those for the non-axial-flow cases, and the SAF plays a stabilization effect in the whole region covering all wavenumbers.

However, this is inconsistent with the well-accepted prediction that when the SAF is sufficiently large a drastic enhancement of the growth rate will appear in a small wavenumber region. Figure 4 shows a comparison between the results from these two methods, where the sold line denotes the exact result gained from Ref.[14] and the dash line represents the result from expression (29). These two curves fit well in the large wavenumber region, but deviate from each other as K decreases. This is because when an SAF is involved, the instability acts in an MRT/KH hybrid way. In the large wavenumber region, usually $K>10$, the MRT mode is predominant, and the perturbation behaves as an MRT instability. On the contrary, in the small wavenumber region where the KH mode becomes important, the eigenfunction deviates from expression (16), and it is beyond the range in which Mikaelian's approximate technique works. That is the reason why expression (29) cannot simulate the cases with a small value of K. Fortunately, because the viscosity effect increases with the wavenumber increasing, in the region with a small value of K the effect due to the plasma viscosity may be neglectable. Moreover, experimental data suggest that perturbations with wavelengths of about 1mm, corresponding to the wavenumber $K\approx 20$, develop most rapidly, so the region with a large value of K is the most attractive to us.

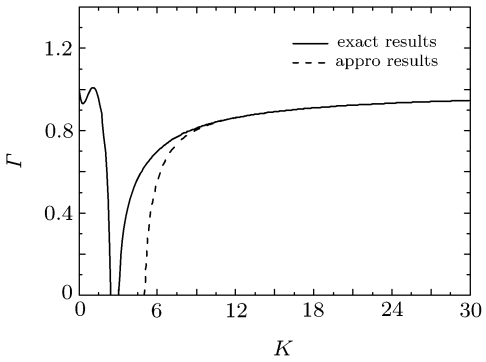


Fig.4. Comparison between results from two different methods for V_0=3×10^5 m/s, where solid line denotes the exact result, and the dash line represents the approximate one.

Figure 5 shows the result for the case of a viscosity-SAF coexisting, where the pure SAF and the pure viscosity both are also presented for comparison. As we have analysed before, since such an approximate technique approximates the viscous case and the SAF within an acceptable accuracy as $K>10$, the information involved with the large wavenumber region is instructive. Results in Fig.5 demonstrate that the viscosity can stabilize small wavelength perturbations while the mitigation effect of the SAF is not prominent, and these curves also reveal a similar tendency in the region with a large value of K to that shown in Fig.3. In fact, it is clear that the hy-

brid case performs as a combination of both effects from the SAF and the viscosity. In the region with a large value of K, the SAF effect is on the wane, thus the hybrid case acts more like the pure viscosity case, while in the region with $K < 20$, the SAF effect is enhanced, and it becomes important as the wavenumber decreases. Figure 6 shows the dispersion relations for $V_0 = 3 \times 10^5$ m/s with various values of viscosity. Compared with the inviscid case which is represented by solid curve, the stable status is improved by taking the viscosity into consideration. And the influence of the viscosity tends to increase with K increasing, which is the exact complement of the SAF's. If the plasma is sufficiently viscous, the joint action of the viscosity and the SAF causes the growth rate to decrease in a large wavenumber region compared with a classical case ($\Gamma = 1.0$).

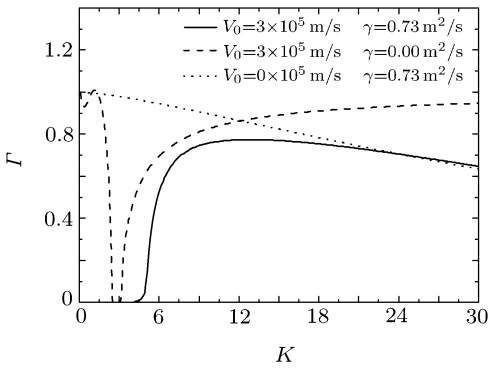


Fig.5. The normalized growth rates for three different cases: viscous plasma with an SAF (solid line); inviscid plasma with an SAF (dash line); viscous plasma without an SAF (dotted line).

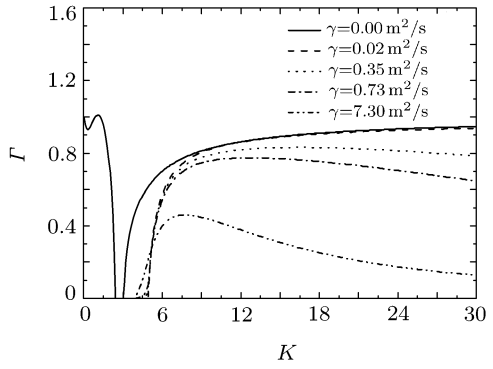


Fig.6. The normalized growth rate for SAF-involved cases with different values of viscosity and $V_0 = 3 \times 10^5$ m/s.

Fortunately, the two mechanisms cooperate well, and there exists no evidence of negative interaction between them. However, considering the practical experiment conditions, we have to admit that the prospect of using viscosity is not so encouraging. The viscosity parameter is defined as the following expression:

$$\gamma = \frac{T\tau_i}{m_i} = 67.5 \times 10^{18} \frac{(T/e)^{5/2}}{A^{1/2} Z^4 n_i}, \qquad (30)$$

where all the notations are conventional. The units of T and (T/e) are J and eV respectively, and the other quantities are in MKS units. Making use of a series of typical parameters available for an inward accelerating tungsten plasma of a 10-MA facility, such as Saturn, the value of viscosity gained from Eq.(30) is about 10^{-6} m^2/s (T_i is about $10^2 - 10^3$ eV, and n_i is about $10^{24} - 10^{25}$ m^{-3}). As the pinch develops, n_i and Z keep increasing while the ion temperature T_i rises at a higher rate. As a result, the calculated γ sustains a constant on the order of 10^{-6} m^2/s. Therefore, from the foregoing analysis we could see that although a sufficiently high viscosity is able to effectively reduce the instability growth rate, especially, to the high wavenumber perturbations, its actual value for current experiments is much less than the threshold value. It can be concluded that only if special techniques are used to increase the Z-pinch plasma viscosity or to enhance the ion temperature greatly, can the viscosity play a considerable role in the stabilization.

4. Conclusions

For a long time, one has paid a lot of attention to different mechanisms of mitigating the MRT instability. Compared with the methods of using SAF effect or hour-glass effect, the ways of using plasma viscosity, compressibility and finite Larmor radius (FLR) effect to stabilize pinch stability are completely different techniques and have obvious advantages. They are methods of utilizing inborn mechanisms of the plasma native qualities, and they can be achieved without difficulties such as exerting external forces or changing the initial geometry of load. Therefore, studying these plasma characteristics is usually instructive and inspiring.

In this paper, we analyse the effect of plasma viscosity on the MRT instability in the whole wavenumber region. All the results from two different methods demonstrate that the plasma viscosity indeed has a stabilization effect and that the effectiveness of such a mechanism is enhanced with the perturbation wavenumber and the value of the viscosity parameter increasing. According to our calculations, we find that

for a small viscosity less than $0.02\,\mathrm{m}^2/\mathrm{s}$, the mitigation effect is so tiny that it can be ignored. With a sufficiently large viscosity, the MRT instability can be assuaged to a very small degree in a large wavenumber region, but it cannot be eliminated completely. By comparing results from both methods, we discuss the advantages and the restrictions of the approximate method. By assuming that the eigenfunction of the perturbation equation has a classical RT-form, the approximate method greatly simplifies the difficulty, and it can present a relatively clear picture for much more complex parameter profiles and cases. However, the reliability greatly relies on the similarity between the exact eigenfunction and the supposed one. According to these considerations we discuss a configuration in which an SAF is involved. Since the small K section corresponds to a KH instability dominated region, the approximate method works only for large K perturbations. Results from the discussion on this point indicate that the viscosity plays a stabilization role in the SAF-involved cases, and that there appears no evidence for negative cooperation. Because the viscosity effect mainly is located in the small wavelength region, where the mitigation effect of the SAF is not prominent, the former acts as a good complement of the latter. This analysis suggests a possible method to improve the stability of the Z-pinch implosion by enhancing the viscosity and inducing a large SAF at the same time. However, when discussing the value of the viscosity parameter, we have to admit that according to the present experiments, the available plasma viscosity is so small that its mitigation effect is inconspicuous. To make the viscosity effect practicable, one must take a particular measure to increase the plasma viscosity greatly.

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