Vol 17 No 8, August 2008
1674-1056/2008/17(08)/2989-05

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

The interaction of nonlinear waves in two-dimensional lattice*

Yang Xiao-Xia(杨晓霞), Duan Wen-Shan(段文山)†,
Li Sheng-Chang(栗生长), and Han Jiu-Ning(韩久宁)

College of Physics and Electronics Engineering, Northwest Normal University, Lanzhou 730070, China

(Received 28 October 2007; revised manuscript received 27 November 2007)

This paper investigates the collision between two nonlinear waves with arbitrary angle in two-dimensional nonlinear lattice. By using the extended Poincaré–Lighthill–Kuo perturbation method, it obtains two Korteweg–de Vries equations for nonlinear waves in both the ξ and η directions, respectively, and derives the analytical phase shifts after the collision of two nonlinear waves. Finally, the solution of $u(v)$ up to $O(\epsilon^3)$ order is given.

Keywords: nonlinear waves, two-dimensional nonlinear lattice, phase shift
PACC: 4630M, 5335M

1. Introduction

As is well known, the first recorded sighting of a soliton was made in 1834 by the Scottish scientist and engineer John Scott Russell. Since their discoveries, solitons have fascinated scientists in many widely different fields.[1−9] Probably the best studied nonlinear waves are those of the famous Korteweg–de Vries (KdV) equation. The KdV equation was found to describe not only the shallow water waves, but also hydromagnetics wave in cold plasma, ion acoustic waves, dust acoustic waves, and dust ion acoustic waves.[10] Extensive studies of the KdV equation have been made since then.[11−13] It is probably fair to say that the first important progress in the study of nonlinear (anharmonic) lattice began seriously in the 1950s with the numerical work by Fermi, Pasta, and Ulam (FPU).[14]

The FPU computer experiment was one of those cases where numerical computations revealed quite unexpected results, and led to the synergetic enhancement of understanding through combined computational and analytical works. Toda proposed a nonlinear lattice with a nearest neighbour interaction potential, and opened a door for a better understanding of nonlinear phenomena, not only the FPU recurrence, but also those in other fields related to the nonlinear mechanics.[15] As is well known, the waves propagating in one-dimensional (1D) lattice have been extensively studied during the last decades. Ilzuka and Wadati[16] studied the nonlinear waves in lattice with some discontinuities. Wadati and Toda have done many research works and obtained many important results on nonlinear waves in the lattice as well. These results not only enrich the soliton theory in the 1D system but also provide some theoretical basis for other theoretical and experimental works. The 1D chain of atoms is the simplest lattice model.[17−19] We assume that there are N same particles which are all in the x direction. Each particle interacts only with nearest neighbours. The potential between two particles is $\phi = \phi(r)$,

so the force between two neighbours is $f(r) = -\dfrac{\partial\phi(r)}{\partial r}$,

where r is the distance between two neighbours. For different potential, the properties of nonlinear waves in the lattice are different. If the force between two neighbours is linear, it is easier to find out the solution of the lattice. However, if the force is nonlinear, it is very difficult to find out the analytical solution of the lattice. We, then, use perturbation technique or numerical method to know the approximate answer. Much works have been done. For example, Tsai studied the approximate solutions of nonlinear lattice, in which the nonlinear potential are cubic

*Project supported by the National Natural Science Foundation of China under (Grant Nos 10575082 and 10247008), the Scientific Research Foundation (SRF) for the Returned Overseas Chinese Scholars (ROCF), State Education Ministry (SEM).
†Corresponding author. E-mail: duanws@nwnu.edu.cn

and quadratic.[20] However, most of these works are mainly concentrated on 1D systems,[21−23] few theoretical works have been done on two-dimensional (2D) nonlinear lattice. We also should note that studies of the interaction between nonlinear waves are frequently reported in recent years.[24,25]

In this paper, based on the 1D model, we try to study the nonlinear waves propagating in the 2D lattice. In 2D systems, the general case is $0 \leq \alpha \leq \pi$, the overtaking collision and head-on collision are only two special cases of $\alpha = 0$ and $\alpha = \pi$, respectively, where α is the angle between two propagation directions of two nonlinear waves. In order to investigate the general case, we use the extended Poincaré–Lighthill–Kuo (PLK) perturbation method and obtain two KdV equations to describe nonlinear waves propagating in two different directions respectively. Moreover, we can obtain their phase shifts after collision of two nonlinear waves.

2. Equation of motion

To investigate the nonlinear waves propagating in the (2D) lattice, we first study the nonlinear waves propagating in the simple cubic lattice, and only consider the forces among the neighbours, and then we will discuss the motion of the (n, m)th particle. It is assumed that we only consider forces from neighbouring particles. It is easy to note that there are four forces to the (n, m)th particle, the forces between two neighbours in the x and y directions are the functions of $f(r)$ and $g(r)$ respectively, where r is the distance between two neighbours. In general case, they are two different functions, but in a symmetric case they are the same function after continuum approximation. The basic equations of motion in the x and y directions respectively for this system are[26]

$$-m\frac{\partial^2 u}{\partial t^2} = f'(0)a^2\frac{\partial^2 u}{\partial x^2} + \frac{f'(0)}{12}a^4\frac{\partial^4 u}{\partial x^4} + \left[f''(0) - \frac{1}{a}f'(0)\right]a^3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + f'(0)a^2\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2} + g'(0)b^3\left[\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y^2}\frac{\partial v}{\partial y}\right], \quad (1)$$

$$-m\frac{\partial^2 v}{\partial t^2} = g'(0)b^2\frac{\partial^2 v}{\partial y^2} + \frac{g'(0)}{12}b^4\frac{\partial^4 v}{\partial y^4} + \left[g''(0) - \frac{1}{b}g'(0)\right]b^3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} + g'(0)b^2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + f'(0)a^3\left[\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x^2}\frac{\partial u}{\partial x}\right]. \quad (2)$$

In the above equations, u and v are the displacements of the particles from its equilibrium positions in the x and y directions respectively, a and b are the lattice constants with m the mass of the atom, $f'(0) = \frac{\partial f(r)}{\partial r}\,|_{r=0}$, $f''(0) = \frac{\partial^2 f(r)}{\partial^2 r}\,|_{r=0}$, $g'(0) = \frac{\partial g(r)}{\partial r}\,|_{r=0}$, $g''(0) = \frac{\partial^2 g(r)}{\partial^2 r}\,|_{r=0}$, and the other notations have their usual meaning.

We assume two nonlinear waves 1 and 2 in the lattice, which are, asymptotically, travelling with an arbitrary angle in the initial state. After some time they interact, collide, and then depart. we also assume that solitons have small amplitudes ϵ (where ϵ is a small formal perturbation parameter) and the interactions between nonlinear waves are weak. Hence, we expect that the collision will be quasi-elastic, so it will cause only shifts of the post-collision trajectories (phase shift). In order to analyse the effects of collision, we employ an extended PLK perturbation method.[27−29] This extended PLK method is a combination of the standard reductive perturbation method[30,31] with the technique of strained coordinates. The main idea of this perturbation method is as follows. In the limits of the long wavelength approximation, asymptotic expansions for both the flow field variables and the spatial or time coordinates are used. This makes an uniformly valid asymptotic expansion (i.e., eliminates secular terms) and at the same time obtains the changes of the trajectories (i.e., phase shifts) of the nonlinear waves after the collision.

According to this method, we introduce the stretched coordinates

$$\xi = \epsilon(k_1 x + l_1 y - c_1 t) + \epsilon^2 P^{(0)}(\eta, \tau) + \dots, \quad (3)$$

$$\eta = \epsilon(k_2 x + l_2 y - c_2 t) + \epsilon^2 Q^{(0)}(\xi, \tau) + \dots, \quad (4)$$

$$\tau = \epsilon^3 t, \quad (5)$$

where ξ and η denote the trajectories of the two solitons, respectively, c_1 and c_2 are the propagation speeds of two nonlinear waves in different directions. We assume that the velocities of c_1 and c_2 are constants, the functions $P^{(0)}(\eta, \tau)$ and $Q^{(0)}(\xi, \tau)$ are to be determined. From Eqs.(3)–(5), we get

$$\frac{\partial}{\partial t} = -\epsilon \left[c_1 \frac{\partial}{\partial \xi} + c_2 \frac{\partial}{\partial \eta} \right] - \epsilon^3 \left[c_2 \frac{\partial P^{(0)}}{\partial \eta} \frac{\partial}{\partial \xi} + c_1 \frac{\partial Q^{(0)}}{\partial \xi} \frac{\partial}{\partial \eta} - \frac{\partial}{\partial \tau} \right] + \dots, \quad (6)$$

$$\frac{\partial}{\partial x} = \epsilon \left[k_1 \frac{\partial}{\partial \xi} + k_2 \frac{\partial}{\partial \eta} \right] + \epsilon^3 \left[k_2 \frac{\partial P^{(0)}}{\partial \eta} \frac{\partial}{\partial \xi} + k_1 \frac{\partial Q^{(0)}}{\partial \xi} \frac{\partial}{\partial \eta} \right] + \dots, \quad (7)$$

$$\frac{\partial}{\partial y} = \epsilon \left[l_1 \frac{\partial}{\partial \xi} + l_2 \frac{\partial}{\partial \eta} \right] + \epsilon^3 \left[l_2 \frac{\partial P^{(0)}}{\partial \eta} \frac{\partial}{\partial \xi} + l_1 \frac{\partial Q^{(0)}}{\partial \xi} \frac{\partial}{\partial \eta} \right] + \dots. \quad (8)$$

Introducing the asymptotic expansions for the quantities u and v

$$u = \epsilon u_1 + \epsilon^3 u_2 + \dots, \quad (9)$$

$$v = \epsilon v_1 + \epsilon^3 v_2 + \dots, \quad (10)$$

then substituting Eqs.(6)–(10) into Eqs.(1) and (2), and equating the powers of ϵ, we obtain a hierarchy of coupled equations in different orders of ϵ. To the leading order, we have

$$c_1^2 \frac{\partial^2 u_1}{\partial \xi^2} + 2c_1 c_2 \frac{\partial^2 u_1}{\partial \xi \partial \eta} + c_2^2 \frac{\partial^2 u_1}{\partial \eta^2} = a_1 \left[k_1^2 \frac{\partial^2 u_1}{\partial \xi^2} + 2k_1 k_2 \frac{\partial^2 u_1}{\partial \xi \partial \eta} + k_2^2 \frac{\partial^2 u_1}{\partial \eta^2} \right], \quad (11)$$

$$c_1^2 \frac{\partial^2 v_1}{\partial \xi^2} + 2c_1 c_2 \frac{\partial^2 v_1}{\partial \xi \partial \eta} + c_2^2 \frac{\partial^2 v_1}{\partial \eta^2} = b_1 \left[l_1^2 \frac{\partial^2 v_1}{\partial \xi^2} + 2l_1 l_2 \frac{\partial^2 v_1}{\partial \xi \partial \eta} + l_2^2 \frac{\partial^2 v_1}{\partial \eta^2} \right], \quad (12)$$

where $a_1 = -\dfrac{f'(0)a^2}{m}$, $b_1 = -\dfrac{g'(0)b^2}{m}$, the solutions of Eqs.(11) and (12) are

$$u_1 = u_\xi(\xi, \tau) + u_\eta(\eta, \tau), \quad (13)$$

$$v_1 = v_\xi(\xi, \tau) + v_\eta(\eta, \tau). \quad (14)$$

Substituting Eqs.(13) and (14) into Eqs.(11) and (12), we obtain the following equations

$$2c_1^2 = a_1 k_1^2 + b_1 l_1^2, \quad (15)$$

$$2c_2^2 = a_1 k_2^2 + b_1 l_2^2, \quad (16)$$

$$u_\xi = v_\xi, \quad (17)$$

$$u_\eta = v_\eta. \quad (18)$$

The unknown functions u_ξ and u_η will be determined at higher order. Equations (17) and (18) imply that, in the leading order, we have two nonlinear waves, one of which, $u_\xi(\xi, \tau)$, is travelling in the ξ direction, and the other one, $u_\eta(\eta, \tau)$, is travelling in the η direction.

At the next order, after tedious calculations, we obtain

$$\gamma \frac{\partial^2 u_2}{\partial \xi \partial \eta} + \gamma P_\eta^{(0)} \frac{\partial^2 u_\xi}{\partial \xi^2} + \gamma Q_\xi^{(0)} \frac{\partial^2 u_\eta}{\partial \eta^2} + 4c_1 \frac{\partial^2 u_\xi}{\partial \xi \partial \tau} + 4c_2 \frac{\partial^2 u_\eta}{\partial \eta \partial \tau} + \beta_1 4c_1 \frac{\partial^4 u_\xi}{\partial \xi^4} + \beta_2 4c_2 \frac{\partial^4 u_\eta}{\partial \eta^4}$$
$$+\alpha_1 4c_1 \frac{\partial u_\xi}{\partial \xi} \frac{\partial^2 u_\xi}{\partial \xi^2} + \alpha_2 4c_2 \frac{\partial u_\eta}{\partial \eta} \frac{\partial^2 u_\eta}{\partial \eta^2} + \lambda_1 \frac{\partial u_\eta}{\partial \eta} \frac{\partial^2 u_\xi}{\partial \xi^2} + \lambda_2 \frac{\partial u_\xi}{\partial \xi} \frac{\partial^2 u_\eta}{\partial \eta^2} = 0, \quad (19)$$

where $\gamma = 2a_1k_1k_2 + 2b_1l_1l_2 - 4c_1c_2$, $\lambda_1 = a_4k_1^2k_2 + a_1k_1^2k_2 + 2a_3k_1^2k_2 + b_4l_1^2l_2 + b_1l_1^2l_2 + 2b_3l_1^2l_2$, $\lambda_2 = a_4k_1k_2^2 + a_1k_1k_2^2 + 2a_3k_1k_2^2 + b_4l_1l_2^2 + b_1l_1l_2^2 + 2b_3l_1l_2^2$, $\alpha_1 = [a_4k_1^3 + a_1k_1^3 + 2b_3l_1^3 + b_4l_1^3 + b_1l_1^3 + 2a_3k_1^3]/4c_1$, $\alpha_2 = [a_4k_2^3 + a_1k_2^3 + 2b_3l_2^3 + b_4l_2^3 + b_1l_2^3 + 2a_3k_2^3]/4c_2$, $\beta_1 = [a_2k_1^4 + b_2l_1^4]/4c_1$, $\beta_2 = [a_2k_2^4 + b_2l_2^4]/4c_2$, $a_2 = -f'(0)a^4/12m$, $b_2 = -g'(0)b^4/12m$, $a_3 = -f'(0)a^3/m$, $b_3 = -g'(0)b^3/m$, $a_4 = -[f''(0) - \frac{1}{a}f'(0)]/m$, $b_4 = -[g''(0) - \frac{1}{b}g'(0)]/m$. In order to avoid spurious resonance, we must eliminate all secular terms.[27,28] Letting $R = \frac{\partial u_\xi}{\partial \xi}$, $S = \frac{\partial u_\eta}{\partial \eta}$, we have

$$\frac{\partial R}{\partial \tau} + \alpha_1 R\frac{\partial R}{\partial \xi} + \beta_1\frac{\partial^3 R}{\partial \xi^3} = 0, \tag{20}$$

$$\frac{\partial S}{\partial \tau} + \alpha_2 S\frac{\partial S}{\partial \eta} + \beta_2\frac{\partial^3 S}{\partial \eta^3} = 0, \tag{21}$$

$$\gamma P_\eta^{(0)} + \lambda_1\frac{\partial u_\eta}{\partial \eta} = 0, \tag{22}$$

$$\gamma Q_\xi^{(0)} + \lambda_2\frac{\partial u_\xi}{\partial \xi} = 0, \tag{23}$$

Equations (20) and (21) are the two-side travelling wave KdV equation in the reference frames of ξ and η, respectively.

3. Phase shifts after interaction of shock waves

There are many solutions for both KdV Eqs.(20) and (21), for example, one soliton solution, two soliton solution, etc. For simplicity, we now study one soliton solution. It is well known that one soliton solution of Eqs.(20) and (21) are $R = r_{m\xi}\mathrm{sech}^2\frac{\xi - u_{0\xi}\tau}{w_\xi}$ and $S = s_{m\eta}\mathrm{sech}^2\frac{\eta - u_{0\eta}\tau}{w_\eta}$. $r_{m\xi} = \frac{3u_{0\xi}}{\alpha_1}$, $s_{m\eta} = \frac{3u_{0\eta}}{\alpha_2}$ and $w_\xi = \left(\frac{4\beta_1}{u_{0\xi}}\right)^{1/2}$, $w_\eta = \left(\frac{4\beta_2}{u_{0\eta}}\right)^{1/2}$ are the amplitudes and widths of two solitons, respectively. $u_{0\xi}$ and $u_{0\eta}$ are two arbitrary constants. As a result, we obtain

$$u_\xi = r_{m\xi}w_\xi\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi}, \tag{24}$$

$$u_\eta = s_{m\eta}w_\eta\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}. \tag{25}$$

Obviously, u_ξ and u_η are two shock waves. We denote u_ξ as shock wave 1 and u_η as shock wave 2. The angle α between two shock waves propagation directions can be given by $\cos\alpha = (k_1k_2 + l_1l_2)/(k_1^2 + l_1^2)^{1/2}(k_2^2 + l_2^2)^{1/2}$. Here we must know that $\alpha \neq 0$ or $\cos\alpha \neq 1$. Otherwise, our perturbation method in this paper is not valid. It is noted that $a_1k_1k_2 + b_1l_1l_2 = 2c_1c_2\cos\alpha$. By using Eqs.(22)–(25), one can obtain the phase functions of $P^{(0)}$ and $Q^{(0)}$ as follows:

$$P^{(0)}(\eta,\tau) = \frac{\lambda_1 s_{m\eta}w_\eta}{4(1-\cos\alpha)}\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}, \tag{26}$$

$$Q^{(0)}(\xi,\tau) = \frac{\lambda_2 r_{m\xi}w_\xi}{4(1-\cos\alpha)}\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi}. \tag{27}$$

At the same time, we can obtain the magnitude of $u(v)$ up to $O(\epsilon^3)$ order

$$\begin{aligned} &u(v)\\ &= \epsilon\left[r_{m\xi}w_\xi\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi}\right.\\ &\quad\left. + s_{m\eta}w_\eta\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}\right] + O(\epsilon^3) + \ldots. \end{aligned} \tag{28}$$

In order to obtain the phase shifts after the collision of two shock waves, we let $P_1 = P^{(0)}(\eta,\tau)\mid_{(\eta - u_{0\eta}\tau)\to-\infty}$, which is the phase value of shock wave 1 described by Eq.(24) before the collision, and $P_2 = P^{(0)}(\eta,\tau)\mid_{(\eta - u_{0\eta}\tau)\to\infty}$, which corresponds to that of shock wave 1 after the collision. It is noted from Eq.(26) that $P_1 = -\frac{\lambda_1 s_{m\eta}w_\eta}{4(1-\cos\alpha)}$, and $P_2 = \frac{\lambda_1 s_{m\eta}w_\eta}{4(1-\cos\alpha)}$. Similarly, we define Q_1, the phase value of shock wave 2 described by Eq.(25) before the interaction, and Q_2, which corresponds to that of shock wave 2 after the interaction. $Q_1 = Q^{(0)}(\xi,\tau)\mid_{(\xi - u_{0\xi}\tau)\to-\infty}$, and $Q_2 = Q^{(0)}(\xi,\tau)\mid_{(\xi - u_{0\xi}\tau)\to\infty}$. $Q_1 = -\frac{\lambda_2 r_{m\xi}w_\xi}{4(1-\cos\alpha)}$, and $Q_2 = \frac{\lambda_2 r_{m\xi}w_\xi}{4(1-\cos\alpha)}$. Therefore, we can obtain the corresponding phase shifts ΔP and ΔQ

$$\Delta P = P_2 - P_1 = \frac{\lambda_1 s_{m\eta}w_\eta}{2(1-\cos\alpha)}, \tag{29}$$

$$\Delta Q = Q_2 - Q_1 = \frac{\lambda_2 r_{m\xi}w_\xi}{2(1-\cos\alpha)}. \tag{30}$$

4. Discussion

From Eqs.(29) and (30), we see that, due to the collision, each shock wave has a positive phase shift in its travelling direction. The magnitudes of the phase shifts are related to the physical parameters (i.e., the arbitrary constants $u_{0\xi}$, $u_{0\eta}$, the shock wave velocities c_1, c_2, the angle α, and the forces $f(r)$, $g(r)$). For convenience, we now discuss the special cases. First,

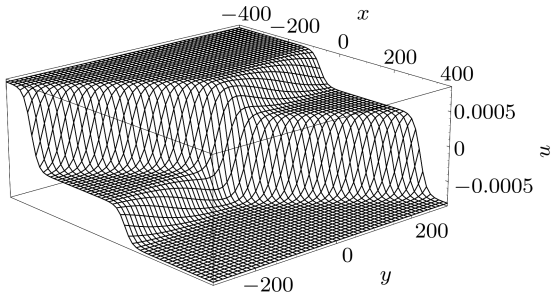


Fig.1. The interaction between two shock waves in 2D lattice, where $u_{0\xi} = 1, u_{0\eta} = 1, k_1 = 5, l_1 = -5, k_2 = 8, l_2 = 2, f'(0) = g'(0) = -0.1, f''(0) = g''(0) = 15, m = 1, a = b = 1$.

for this symmetric case between x and y directions, the forces between two nearest particles, are given by the equation of $f(r) = g(r) = Ar + Br^2 + Cr^3$ where A, B and C are constants. Thus we can obtain $f'(0) = g'(0) = A$ and $f''(0) = g''(0) = 2B$. We consider the collision between two shock waves with same amplitude, for this case $c_1 = c_2$, $u_{0\xi} = u_{0\eta}$, then we find $\Delta P = \Delta Q$. Figure 1 shows the profile of interaction between two shock waves with an arbitrary propagation angle. For head-on collision of shock waves, $\alpha = \pi$, we obtain $\Delta P = \frac{\lambda_1 s_{m\eta} w_\eta}{4}$, $\Delta Q = \frac{\lambda_2 r_{m\xi} w_\xi}{4}$. It is the special case of our 2D situation, we find that the value of ΔP or ΔQ is independent of the parameters B and C. Figure 2 shows the variation of phase shift with A for $\alpha = \pi$. From Fig.2, we can find that the value of ΔP or ΔQ increases as A increases.

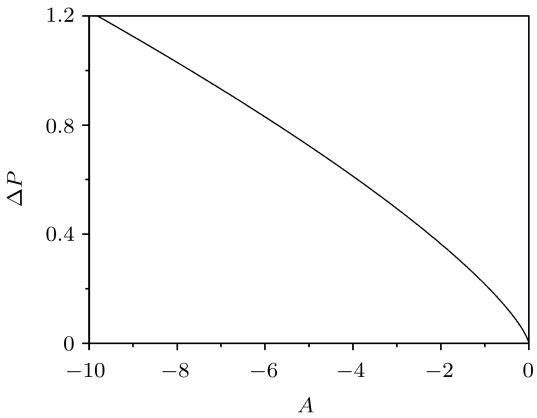


Fig.2. The phase shift ΔP as a function of A, where $u_{0\xi} = 1, u_{0\eta} = 1, k_1 = l_1 = 0.5, k_2 = l_2 = -0.5, m = 1, a = b = 1$.

In summary, by using the extended PLK perturbation method, the leading-order analytical phase shifts of collision between two shock waves in 2D nonlinear lattice are derived. We expect that the present investigation will provide insight and understanding of the phenomenon related to waves collision.

References

[1] Jiang D S and She W L 2007 *Acta Phys. Sin.* **56** 245 (in chinese)
[2] He B G, Xu C Z and Zhang J F 2005 *Acta Phys. Sin.* **54** 5525 (in chinese)
[3] Xia T C, Zhang H Q and Yan Z Y 2001 *Chin. Phys.* **10** 694
[4] Lü K P and Duan W S 2000 *Chin. Phys.* **9** 81
[5] Zhang C W, Liu J, Raizen M G and Niu Q 2004 *Phys. Rev. Lett.* **93** 074101
[6] Chen L L and Lou S Y 1999 *Chin. Phys.* **8** 285
[7] Lou S Y 2000 *Acta Phys. Sin.* **49** 1657 (in chinese)
[8] Xia Q L, Yi J H, Peng Y D, Ye T M, Li L Y and Wang H Z 2007 *Chin. Phys.* **16** 223
[9] Lin J and Wang K L 2001 *Acta Phys. Sin.* **50** 13 (in chinese)
[10] Tskhakaya D D and Shukla P K 2001 *Phys. Lett.* A **286** 277
[11] Tang J S and Yang X L 2007 *Chin. Phys.* **16** 310
[12] Dong H H 2007 *Chin. Phys.* **16** 1177
[13] Li B A and Wang M L 2005 *Chin. Phys.* **14** 1698
[14] Feimi E, Pasta J R and Ulam S M 1955 *Los Alamos Report* LA-1940
[15] Liu J, Wu B and Niu Q 2003 *Phys. Rev. Lett.* **90** 170404
[16] Ilzuka T and Wadati M 1993 *J. Phys. Soc. Jpn.* **62** 1932
[17] Yuan X P and Zheng Z G 2007 *Chin. Phys. Lett.***24** 2513
[18] Yuan X P, Chen H B and Zheng Z G 2006 *Chin. Phys.* **15** 1464
[19] Hu B B, Li B W and Zhao H 1998 *Phys. Rev.* E **57** 2992
[20] Tasi J 1980 *J. Appl. Phys.* **51** 5816
[21] Feng P C, Tang Y and Wang D L 2001 *Acta Phys. Sin.* **50** 1110 (in chinese)
[22] Huang G X 2001 *Chin. Phys.* **10** 523
[23] Li B, Zhao H and Hu B B 2001 *Phys. Rev. Lett.* **86** 63
[24] Huang G X, Velarde M G and Makarov V A 2001 *Phys. Rev.* E **64** 013617
[25] Xue J K 2004 *Phys. Rev.* E **69** 016403
[26] Duan W S, Shi Y R, Zhang L, Lin M M and Lü K P 2005 *Chaos, Solitons Fractals* **25** 957
[27] Su C H and Mirie R M 1980 *J. Fluid. Mech* **98** 509
[28] Jeffery A and Kawahawa T 1982 *Asymptotic Methods in Nonlinear Wave Theory* (London: Pitman)
[29] Huang G X and Velarde M G 1996 *Phys. Rev.* E **53** 2988
[30] Taniuti T And Wei C C 1968 *J. Phys. Soc. Jpn.* **24** 941
[31] Kakutani T, Ono H, Taniuti T and Wei C C 1968 *J. Phys. Soc. Jpn.* **24** 1159