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The q-analogues of two-mode squeezed states constructed by virtue of the IWOP technique*

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The q-analogues of two-mode squeezed states are introduced by virtue of deformation quantization methods and the technique of integration within an ordered product (IWOP) of operators. Some new completeness relations about these squeezed states composed of the bra and ket which are not mutually Hermitian conjugates are obtained. Furthermore, the antibunching effects of the two-mode squeezed vacuum state $S_2'(r)\,|00\rangle$ are investigated. It is found that, in different ranges of the squeezed parameter r, both modes of the state exhibit the antibunching effects and the two modes of the state are always nonclassical correlation.

Keywords: q-analogue of two-mode squeezed state, IWOP technique, completeness relation, antibunching effect

PACC: 4250, 0365

1. Introduction

In the past few years much attention has been paid to the q-deformed boson oscillator owing to its potential applications in studying the q-analogues of the quantum theory of the radiation light field.[1,2] In Refs.[3–10] some important physical concepts, such as the coherent states, even and odd coherent states and squeezed states for the ordinary oscillator, were extended to the q-deformed case and their nonclassical properties were discussed. Owing to the recently discovered connections between q-deformation and nonlinearity,[11,12] the exhaustive studies associated with the q-deformed quantum states have highly academic interest. In this paper, in the same way as in Ref.[10], we shall generalize the concept of q-deformed coherent states to the q-deformed two-mode squeezed states by using the technique of integration within an ordered product (IWOP) of operators and the properties of the inverses of q-deformed annihilation operator a_q and creation operator a_q^+. Moreover, the antibunching effects of the two-mode squeezed vacuum state $S_2'(r)\,|00\rangle$ are investigated in detail.

2. The q-analogues of two-mode squeezed states

The annihilation operator a_q and creation operator a_q^+ of the q-deformed oscillators are distortions of the annihilation operator a and creation operator a^+ of the usual harmonic oscillators. Then the q-deformed two-mode boson creation operator $a_{q,i}^+$ (here and hereafter $i = 1, 2$), annihilation operator $a_{q,i}$ and the number operator $N_{q,i}$ $(= a_{q,i}^+ a_{q,i})$ satisfy the following closed algebraic relations:

$$\begin{aligned} &a_{q,i}a_{q,i}^+ - q a_{q,i}^+ a_{q,i} = 1,\\ &[N_{q,i},\ a_{q,i}^+] = a_{q,i}^+,\\ &[N_{q,i},\ a_{q,i}] = -a_{q,i}, \end{aligned} \tag{1}$$

where q is a deformed parameter.

From Ref.[13] we know that the so-called q-Fock state $|n\rangle_q$ is the usual Fock state $|n\rangle$ in fact. Then operating the q-deformed operators $a_{q,i}$,$a_{q,i}^+$ and $N_{q,i}$ on the Fock states $|n\rangle_i$, we have

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$$
\begin{aligned}
&a_{q,i}\left|n\right\rangle_i = \sqrt{[n]}\left|n\right\rangle_i,\\
&a_{q,i}^{+}\left|n\right\rangle_i = \sqrt{[n+1]}\left|n+1\right\rangle_i,\\
&N_{q,i}\left|n\right\rangle_i = n\left|n\right\rangle_i,
\end{aligned}
\tag{2}
$$

where $[n]=(1-q^n)/(1-q)$. Therefore, we can easily obtain the inverses of the operators $a_{q,i}$ and $a_{q,i}^{+}$ as follows,

$$
\begin{aligned}
a_{q,i}^{-1} &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{[n+1]}}\left|n+1\right\rangle_{ii}\left\langle n\right|,\\
(a_{q,i}^{+})^{-1} &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{[n+1]}}\left|n\right\rangle_{ii}\left\langle n+1\right| = (a_{q,i}^{-1})^{+}.
\end{aligned}
\tag{3}
$$

It then follows that the inverses of the operators $a_{q,i}$ and $a_{q,i}^{+}$ satisfy the following non-commutative relations

$$
\begin{aligned}
&a_{q,i}a_{q,i}^{-1} = (a_{q,i}^{+})^{-1}a_{q,i}^{+} = 1,\\
&a_{q,i}^{-1}a_{q,i} = a_{q,i}^{+}(a_{q,i}^{+})^{-1} = 1-\left|0\right\rangle_{ii}\left\langle 0\right|,
\end{aligned}
\tag{4}
$$

which means that $a_{q,i}^{-1}$ is the right inverse of $a_{q,i}$, and $(a_{q,i}^{+})^{-1}$ is the left inverse of $a_{q,i}^{+}$. The results are completely analogous to the case of the inverses of ordinary two-mode boson operators.

By virtue of the inverse operators $a_{q,i}^{-1}$, $(a_{q,i}^{+})^{-1}$ and the number operator $N_{q,i}$, in the following we construct two new operators,

$$
\begin{aligned}
&A_{q,i}^{+} = N_{q,i}a_{q,i}^{-1},\\
&A_{q,i} = (a_{q,i}^{+})^{-1}N_{q,i}.
\end{aligned}
\tag{5}
$$

Using Eqs.(1), (4) and (5), we can prove that

$$
\begin{aligned}
[a_{q,i},A_{q,i}^{+}] =&(N_{q,i}+1)a_{q,i}a_{q,i}^{-1}\\
&-N_{q,i}(1-\left|0\right\rangle_{ii}\left\langle 0\right|) = 1,
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
[A_{q,i},a_{q,i}^{+}] =&(a_{q,i}^{+})^{-1}a_{q,i}^{+}(N_{q,i}+1)\\
&-(1-\left|0\right\rangle_{ii}\left\langle 0\right|)N_{q,i} = 1,
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&[A_{q,i}^{+},N_{q,i}] = -A_{q,i}^{+},\\
&[A_{q,i},N_{q,i}] = A_{q,i}.
\end{aligned}
\tag{8}
$$

Hence $A_{q,i}^{+}$ (or $a_{q,i}^{+}$) and $a_{q,i}$(or $A_{q,i}$) make up the fundamental canonical commutative relations.

In Ref.[14], Fan and Jing have proved that the normal ordering form of the vacuum projector is

$$
\left|0\right\rangle_{ii}\left\langle 0\right| =: \exp(-a_i^{+}a_i) :, \tag{9}
$$

where the normal ordering symbol :: is for a_i^{+} and a_i. In the same way, we can also prove that the normal ordering forms of the vacuum projector are

$$
\left|0\right\rangle_{ii}\left\langle 0\right| = {}^{\circ}_{\circ}\exp(-A_{q,i}^{+}a_{q,i}){}^{\circ}_{\circ} = \vdots\exp(-a_{q,i}^{+}A_{q,i})\vdots, \tag{10}
$$

where the normal ordering symbol ${}^{\circ}_{\circ}\ {}^{\circ}_{\circ}$ is for $A_{q,i}^{+}$ and $a_{q,i}$ and the normal ordering symbol $\vdots\ \vdots$ is for $a_{q,i}^{+}$ and $A_{q,i}$. Then the results in Eq.(10) are completely distinct from the normal ordering form of the vacuum projector $\left|0\right\rangle_{ii}\left\langle 0\right|$ in Eq.(9).

Subsequently, we construct the following eigenstates:

$$
\left\|\, x_1,x_2\right\rangle = \pi^{-1/2}\exp\left[-\frac{1}{2}x_i^2+\sqrt{2}x_iA_{q,i}^{+}-\frac{1}{2}A_{q,i}^{+2}\right]\left|0\right\rangle, \tag{11}
$$

$$
\left\langle x_1,x_2\right| = \pi^{-1/2}\left\langle 0\right|\exp\left[-\frac{1}{2}x_i^2+\sqrt{2}x_ia_{q,i}-\frac{1}{2}a_{q,i}^{2}\right], \tag{12}
$$

$$
\left\|\, x_1',x_2'\right\rangle = \pi^{-1/2}\exp\left[-\frac{1}{2}x_i'^2+\sqrt{2}x_i'a_{q,i}^{+}-\frac{1}{2}a_{q,i}^{+2}\right]\left|0\right\rangle, \tag{13}
$$

$$
\left\langle x_1',x_2'\right| = \pi^{-1/2}\left\langle 0\right|\exp\left[-\frac{1}{2}x_i'^2+\sqrt{2}x_i'A_{q,i}-\frac{1}{2}A_{q,i}^{2}\right], \tag{14}
$$

where we have let the operators $\hat{x}_i$ and $\hat{x}_i'$ be

$$
\begin{aligned}
&\hat{x}_i = \frac{1}{\sqrt{2}}(\hat{a}_{q,i}+\hat{A}_{q,i}^{+}),\\
&\hat{x}_i' = \frac{1}{\sqrt{2}}(\hat{A}_{q,i}+\hat{a}_{q,i}^{+}).
\end{aligned}
\tag{15}
$$

From Eqs.(10)–(15) by performing the following integration we can obtain two completeness relations,

$$
\int \mathrm{d}x_1\mathrm{d}x_2\left\|\, x_1,x_2\right\rangle\left\langle x_1,x_2\right| = \pi^{-1}\int \mathrm{d}x_1\mathrm{d}x_2{}^{\circ}_{\circ}\exp\left[-x_i^2+\sqrt{2}x_i(A_{q,i}^{+}+a_{q,i})-\frac{1}{2}(A_{q,i}^{+2}+a_{q,i}^{2})-A_{q,i}^{+}a_{q,i}\right]{}^{\circ}_{\circ} = 1, \tag{16}
$$

$$
\int \mathrm{d}x_1'\mathrm{d}x_2'\left\|\, x_1',x_2'\right\rangle\left\langle x_1',x_2'\right| = \pi^{-1}\int \mathrm{d}x_1'\mathrm{d}x_2'\vdots\exp\left[-x_i'^2+\sqrt{2}x_i'(a_{q,i}^{+}+A_{q,i})-\frac{1}{2}(a_{q,i}^{+2}+A_{q,i}^{2})-a_{q,i}^{+}A_{q,i}\right]\vdots = 1. \tag{17}
$$

From the above discussion, we find that the completeness relations given by Eqs.(16) and (17) are made up of the bra and ket which are not mutually Hermitian conjugate. It is rather different from the usual completeness relations constructed by bra and ket of mutually Hermitian conjugate.

By means of simple calculation, we can prove that the eigenvalue equation of the coordinate operators $\hat{x}_i$ and $\hat{x}'_i$ may be written as

$$\begin{aligned}\hat{x}_i \,\|\, x_1, x_2\rangle &= x_i \,\|\, x_1, x_2\rangle\,,\\ \langle x_1, x_2 |\, \hat{x}_i &= \langle x_1, x_2 |\, x_i,\end{aligned} \tag{18}$$

$$\begin{aligned}\hat{x}'_i \,\|\, x'_1, x'_2\rangle &= x'_i \,\|\, x'_1, x'_2\rangle\,,\\ \langle x'_1, x'_2 |\, \hat{x}'_i &= \langle x'_1, x'_2 |\, x'_i.\end{aligned} \tag{19}$$

Then using Eqs.(10)–(14) we can have the following squeezing operators

$$\begin{aligned}S_2(r) =& \iint \mathrm{d}x_1 \mathrm{d}x_2 \,\|\, x_1 \cosh r + x_2 \sinh r, x_1 \sinh r + x_2 \cosh r\rangle\, \langle x_1, x_2|\\ =& \pi^{-1} \iint \mathrm{d}x_1 \mathrm{d}x_2 {}^{\circ}_{\circ}\exp[-(x_1^2 + x_2^2)\cosh^2 r - x_1 x_2 \sinh 2r + \sqrt{2}(x_1 \cosh r + x_2 \sinh r)]A_{q,1}^+\\ &+ \sqrt{2}(x_1 \sinh r + x_2 \cosh r)A_{q,2}^+ - \frac{1}{2}(a_{q,1} + A_{q,1}^+)^2 - \frac{1}{2}(a_{q,2} + A_{q,2}^+)^2 + \sqrt{2}(x_1 a_{q,1} + x_2 a_{q,2})^{\circ}_{\circ}\\ =& \exp(A_{q,1}^+ A_{q,2}^+ \tanh r)\exp[(A_{q,1}^+ a_{q,1} + A_{q,2}^+ a_{q,2} + 1)\ln(\sec hr)]\exp(-a_{q,1}a_{q,2}\tanh r),\end{aligned} \tag{20}$$

$$\begin{aligned}S'_2(r) =& \iint \mathrm{d}x'_1 \mathrm{d}x'_2 \,\|\, x'_1 \cosh r + x'_2 \sinh r, x'_1 \sinh r + x'_2 \cosh r\rangle\, \langle x'_1, x'_2|\\ =& \exp(a_{q,1}^+ a_{q,2}^+ \tanh r)\exp[(a_{q,1}^+ A_{q,1} + a_{q,2}^+ A_{q,2} + 1)\ln(\sec hr)]\exp(-A_{q,1}A_{q,2}\tanh r),\end{aligned} \tag{21}$$

where r is a squeezed parameter, i.e.,

$$r = \mathrm{e}^{\mu}, \qquad \tanh r = (\mu^2 - 1)/(\mu^2 + 1). \tag{22}$$

From Eqs.(20) and (21) we have the following transformations

$$S_2(r) a_{q,1} S_2^{-1}(r) = a_{q,1}\cosh r - A_{q,2}^+ \sinh r,\ \ S_2(r) a_{q,2} S_2^{-1}(r) = a_{q,2}\cosh r - A_{q,1}^+ \sinh r, \tag{23}$$

$$S'_2(r) A_{q,1}^+ {S'}_2^{-1}(r) = A_{q,1}^+ \cosh r - a_{q,2}\sinh r,\ \ S'_2(r) A_{q,2}^+ {S'}_2^{-1}(r) = A_{q,2}^+ \cosh r - a_{q,1}\sinh r. \tag{24}$$

Operating the squeezed operators $S_2(r)$ and $S'_2(r)$ on the vacuum state $|00\rangle$ respectively, we can obtain two q-analogues of two-mode squeezed vacuum states

$$\begin{aligned}|r, q\rangle_1 =& S_2(r)\,|00\rangle\\ =& C_1 \exp(A_{q,1}^+ A_{q,2}^+ \tanh r)\,|00\rangle\\ =& C_1 \sum_{n=0}^{\infty} \frac{n! \tanh^n r}{[n]!}\,|n, n\rangle\,,\end{aligned} \tag{25}$$

$$\begin{aligned}|r, q\rangle_2 =& S'_2(r)\,|00\rangle\\ =& C_2 \exp(a_{q,1}^+ a_{q,2}^+ \tanh r)\,|00\rangle\\ =& C_2 \sum_{n=0}^{\infty} \frac{[n]! \tanh^n r}{n!}\,|n, n\rangle\,,\end{aligned} \tag{26}$$

where the normalization constants C_1 and C_2 are given by

$$\begin{aligned}|C_1|^2 &= \left(\sum_{n=0}^{\infty} \frac{(n!)^2 \tanh^{2n} r}{([n]!)^2}\right)^{-1},\\ |C_2|^2 &= \left(\sum_{n=0}^{\infty} \frac{([n]!)^2 \tanh^{2n} r}{(n!)^2}\right)^{-1},\end{aligned} \tag{27}$$

$$[n]! = [0][1][2]\ldots[n],\ \ [0]! = 1. \tag{28}$$

Here we define two displacement operators

$$\begin{aligned}D(z_1, z_2) &= \exp(z_i A_{q,i}^+ - z_i^* a_{q,i}),\\ D(z'_1, z'_2) &= \exp(z'_i a_{q,i}^+ - {z'}_i^* A_{q,i}).\end{aligned} \tag{29}$$

Then operate $D(z_1, z_2)$ and $D(z'_1, z'_2)$ on the vacuum states $|00\rangle$ respectively, we obtain two q-analogues of two-mode coherent states

$$\begin{aligned}\|\, z_1, z_2\rangle =& D(z_1, z_2)\,|00\rangle\\ =& \exp\left(-\frac{1}{2}|z_i|^2 + z_i A_{q,i}^+\right)|00\rangle\,,\end{aligned} \tag{30}$$

$$\begin{aligned}\|\, z'_1, z'_2\rangle =& D(z'_1, z'_2)\,|00\rangle\\ =& \exp\left(-\frac{1}{2}|z'_i|^2 + z'_i a_{q,i}^+\right)|00\rangle\,.\end{aligned} \tag{31}$$

By virtue of the IWOP technique, from Eqs.(30) and (31) we can easily obtain two over-completeness relations in the following,

$$\int \frac{\mathrm{d}z_1\mathrm{d}z_2}{\pi^2} \| z_1, z_2\rangle \langle z_1, z_2| = \int \frac{\mathrm{d}z_1\mathrm{d}z_2}{\pi^2} {}^{\circ}_{\circ}\exp(-|z_i|^2 + z_i A^+_{q,i} + z^*_i a_{q,i} - A^+_{q,i}a_{q,i}){}^{\circ}_{\circ} = 1, \tag{32}$$

$$\int \frac{\mathrm{d}z'_1\mathrm{d}z'_2}{\pi^2} \| z'_1, z'_2\rangle \langle z'_1, z'_2| = \int \frac{\mathrm{d}z'_1\mathrm{d}z'_2}{\pi^2} {:}\exp(-|z'_i|^2 + z'_i a^+_{q,i} + z'^*_i A_{q,i} - a^+_{q,i}A_{q,i}){:} = 1. \tag{33}$$

where

$$\langle z_1, z_2| = \langle 00| \exp\bigg(-\frac{1}{2}|z_i|^2 + z^*_i a_{q,i}\bigg), \ \ \langle z'_1, z'_2| = \langle 00| \exp\bigg(-\frac{1}{2}|z'_i|^2 + z'^*_i A_{q,i}\bigg). \tag{34}$$

On the other hand, if operating $D(z_1, z_2)$ and $D(z'_1, z'_2)$ on the squeezed vacuum states $|r, q\rangle_1$ and $|r, q\rangle_2$ respectively, we obtain two q-analogues of two-mode coherent squeezed states as follows:

$$\begin{aligned} \| z_1, z_2, r\rangle =& D(z_1, z_2)S_2(r)\,|00\rangle \\ =& \sec hr \exp[-\frac{1}{2}(|z_1|^2 + |z_2|^2) + (A^+_{q,1} - z^*_1)(A^+_{q,2} - z^*_2)\tanh r] \| z_1, z_2\rangle \ , \end{aligned} \tag{35}$$

$$\begin{aligned} \| z'_1, z'_2, r\rangle =& D(z'_1, z'_2)S'_2(r)\,|00\rangle \\ =& \sec hr \exp[-\frac{1}{2}(|z'_1|^2 + |z'_2|^2) + (a^+_{q,1} - z'^*_1)(a^+_{q,2} - z'^*_2)\tanh r] \| z'_1, z'_2\rangle \ . \end{aligned} \tag{36}$$

It then follows that we can prove that the states (35) and the following states:

$$\langle z_1, z_2, r| = \sec hr \langle z_1, z_2| \exp\bigg[-\frac{1}{2}(|z_1|^2 + |z_2|^2) + (a_{q,1} - z_1)(a_{q,2} - z_2)\tanh r\bigg] \tag{37}$$

make up the following completeness relations:

$$\begin{aligned} &\int \frac{\mathrm{d}z_1\mathrm{d}z_2}{\pi^2} \| z_1, z_2, r\rangle \langle z_1, z_2, r| \\ =& \sec h^2 r \int \frac{\mathrm{d}z_1\mathrm{d}z_2}{\pi^2} {}^{\circ}_{\circ}\exp[-(|z_1|^2 + |z_2|^2) + z_1 A^+_{q,1} + z_2 A^+_{q,2} + z^*_1 a_{q,1} + z^*_2 a_{q,2} \\ &+ (A^+_{q,1} - z^*_1)(A^+_{q,2} - z^*_2)\tanh r + (a_{q,1} - z_1)(a_{q,2} - z_2)\tanh r - A^+_{q,1}a_{q,1} - A^+_{q,2}a_{q,2}]{}^{\circ}_{\circ} \\ =& 1. \end{aligned} \tag{38}$$

where we have used Eq.(10).

In the same way, using the IWOP technique we can prove that the states (36) and the following states:

$$\langle z'_1, z'_2, r| = \sec hr \langle z'_1, z'_2| \exp\bigg[-\frac{1}{2}(|z'_1|^2 + |z'_2|^2) + (A_{q,1} - z'_1)(A_{q,2} - z'_2)\tanh r\bigg] \tag{39}$$

also compose the following completeness relations:

$$\int \frac{\mathrm{d}z'_1\mathrm{d}z'_2}{\pi^2} \| z'_1, z'_2, r\rangle \langle z'_1, z'_2, r| = 1. \tag{40}$$

Similarly, the over-completeness relations (32), (33), (38) and (40) are also made up of the bra and ket which are not mutually Hermitian conjugate.

3. Antibunching effects of the state $S'_2(r)\,|00\rangle$

In this section we shall study the antibunching effects of the state $S'_2(r)\,|00\rangle$. For the usual field, if the second-order correlation function $g^{(2)}(0) < 1$, then the field exhibits antibunching effect. For the q-deformed light fields we define the second-order correlation function for the ith mode as follows,

$$g^{(2)}_{q,i}(0) = \frac{\langle a^{+2}_{q,i}a^2_{q,i}\rangle}{\langle a^+_{q,i}a_{q,i}\rangle^2}. \tag{41}$$

If $g^{(2)}_{q,i}(0) < 1$, then it means the ith mode exhibits antibunching effect.

For the state $S'_2(r)\,|00\rangle$, the second-order correlation functions of two modes are given by

$$g^{(2)}_{q,1}(0) = g^{(2)}_{q,2}(0) = \frac{\sum\limits_{n=0}^{\infty} \tanh^{2n} r \dfrac{([n]!)^2[n][n-1]}{(n!)^2}}{|C_2|^2 \left(\sum\limits_{n=0}^{\infty} \tanh^{2n} r \dfrac{([n]!)^2[n]}{(n!)^2} \right)^2}. \tag{42}$$

By virtue of the results of numerical computations from Eq.(42), in Fig.1 we plot $g^{(2)}_{q,i}(0)$ $(i = 1, 2)$ against the squeezed parameter r while taking different values of q (q= 0.2, 0.4, 0.6 and 0.8) for the four curves. From Fig.1 we find that, in different ranges of r, both modes exhibit the antibunching effects. If keeping r fixed, for the smaller values of q the antibunching effects in both modes are remarkable and as q increases the antibunching effects are weakened. For the smaller values of q the antibunching effects are weak with respect to the parameter r increasing, however for the greater values of q the antibunching effects are strengthened with r increasing.

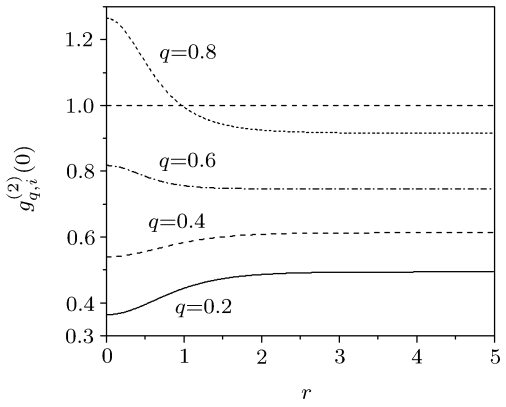


Fig.1. The variations of the second-order correlation function $g^{(2)}_{q,i}(0)$ $(i = 1, 2)$ for the ith mode with the squeezed parameter r.

On the other hand, the second-order correlation function between two modes is defined as

$$g^{(2)}_{q,12}(0) = \frac{\left\langle a^{+}_{q,1} a_{q,1} a^{+}_{q,2} a_{q,2} \right\rangle}{\left\langle a^{+}_{q,1} a_{q,1} \right\rangle \left\langle a^{+}_{q,2} a_{q,2} \right\rangle}. \tag{43}$$

If $g^{(2)}_{q,12}(0) > 1$, it means the two modes are correlated.

For the state $S'_2(r)\,|00\rangle$, the second-order correlation function between two modes is given by

$$g^{(2)}_{q,12}(0) = \frac{\sum\limits_{n=0}^{\infty} \tanh^{2n} r \dfrac{([n]![n])^2}{(n!)^2}}{|C_2|^2 \left(\sum\limits_{n=0}^{\infty} \tanh^{2n} r \dfrac{([n]!)^2[n]}{(n!)^2} \right)^2}. \tag{44}$$

Similarly, by virtue of the results of numerical computations from Eq.(44), in Fig.2 we plot $g^{(2)}_{q,12}(0)$ against the squeezed parameter r while taking different values of q (q=0.2, 0.4, 0.6 and 0.8) for the four curves. From Fig.2 it is clearly seen that, in the whole range of r, the second-order correlation function $g^{(2)}_{q,12}(0)$ is always greater than one, namely the two modes in the state $S'_2(r)\,|00\rangle$ are always correlated. If keeping r fixed, for the smaller value of q the correlation between two modes is remarkable and as q increases the correlation is weakened. If keeping q fixed, the correlation firstly weakens and then keeps constant with r increasing. The influence of the parameters q and r on the correlation in the state $S'_2(r)\,|00\rangle$ is sensitive and remarkable.

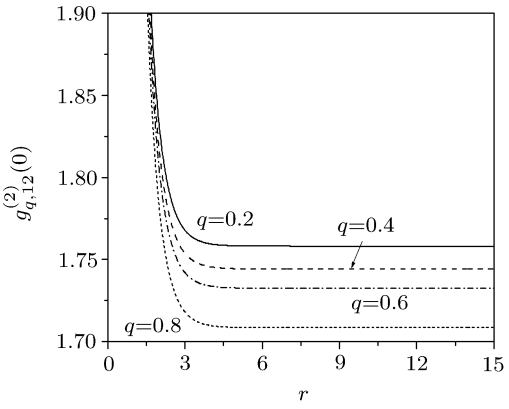


Fig.2. The variations of the second-order correlation function $g^{(2)}_{q,12}(0)$ between two modes with the squeezed parameter r.

For the q-deformed light fields, if

$$I_0 = \frac{\left(\left\langle a^{+2}_{q,1} a^{2}_{q,1} \right\rangle \left\langle a^{+2}_{q,2} a^{2}_{q,2} \right\rangle\right)^{1/2}}{\left\langle a^{+}_{q,1} a_{q,1} a^{+}_{q,2} a_{q,2} \right\rangle} - 1 < 0, \tag{45}$$

then the Cauchy–Schwartz inequality is violated, which means that the two modes in the q-deformed light fields are nonclassical corrections. By virtue of the numerical calculation results for I_0 shown in Fig.3, we find that the two modes in the $S'_2(r)\,|00\rangle$ are always nonclassical correlations for all of the deformed parameters q in the whole range of r.

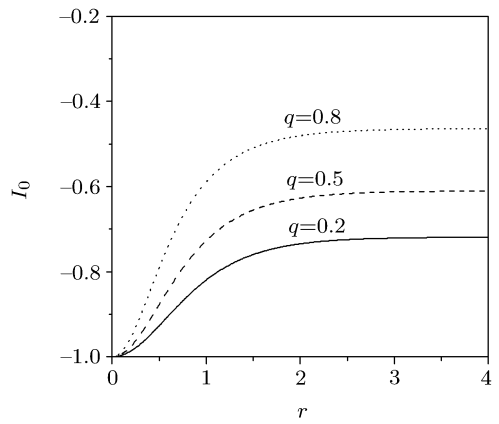


Fig.3. The variations of I_0 with the squeezed parameter r.

From the above discussion we also find that the ranges of the parameter r where the state shows antibunching effects, and the variations of the function $g^{(2)}_{q,i}(0)$ $(i = 1, 2)$ for the ith mode and $g^{(2)}_{q,12}(0)$ between two modes with the parameter r, as well as the nonclassical correlation of two modes are different from those of the two-mode nonlinear squeezed vacuum states.[10]

4. Conclusions

In the present paper, we have obtained the two-mode q-deformed squeezed states using the IWOP technique and the properties of the inverses of q-deformed annihilation and creation operators. This is a new and good way which is also suitable to construct other quantum states. Some new completeness relations composed of the bra and ket which are not mutually Hermitian conjugate are obtained. We can see that the introduction of the inverse operators provide a facility to set up the IWOP technique for operator functions of $a_{q,i}$ and $A^{+}_{q,i} = N_{q,i}a^{-1}_{q,i}$, $A^{=}_{q,i}(a^{+}_{q,i})^{-1}N_{q,i}$ and $a^{+}_{q,i}$. Finally, we have examined antibunching effects of the state $S'_2(r)\,|00\rangle$. It is found that, in different ranges of r, both modes exhibit the antibunching effects and two modes in the state $S'_2(r)\,|00\rangle$ are always nonclassical correlation. The results differ greatly from those of the two-mode nonlinear squeezed vacuum states.

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