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Purification of entangled states for two atoms via cavity decay*

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In this paper a scheme is proposed for the purification of entangled states for two atoms trapped in two distant cavities via cavity decay. In the scheme, the atoms have no probability of being populated in the excited state and thus the atomic spontaneous emission is suppressed. This scheme is valid no matter when the cavity decay rate is larger or smaller than the effective atom-cavity coupling strength. The fidelity of the final state is not affected by the imperfection of the photodetectors.

Keywords: purification, entangled state, cavity decay
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Entanglement, one of the most striking features of quantum mechanics, is fundamental to demonstrate quantum nonlocality.[1] Apart from the test of fundamental aspects of quantum mechanics, entangled states are useful for the implementation of quantum information, such as quantum cryptography[2] and teleportation.[3] The cavity QED system is a powerful tool for entanglement engineering. The atoms are ideal for storing quantum information, while the photons are ideal for transmitting information. Recently, schemes have been proposed for the generation of entangled states between two atoms and implementation of quantum computation by single-photon interference.[4−9] However, it is difficult to obtain a pure entangled state due to experimental imperfection. For example, due to the detection inefficiency the schemes of Refs.[4–6] can only be used to generate mixed entangled states of the type

$$\rho_{\pm} = a\left|g_1\right\rangle\left|g_2\right\rangle\left\langle g_1\right|\left\langle g_2\right| + b\left|\phi_{\pm}\right\rangle\left\langle\phi_{\pm}\right|, \qquad (1)$$

where $|\phi_{\pm}\rangle$ is the maximally entangled state

$$|\phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle \pm |g_1\rangle|e_2\rangle). \qquad (2)$$

In order to be useful for the test of quantum nonlocality and quantum communication, the nonmaximally entangled states should be purified. The process of turning a pure entangled state into a maximally entangled state is referred to as entanglement concentration,[10,11] while the process of turning a mixed entangled state into a maximally entangled state is referred to as entanglement purification.[12]

Usually, cavity loss is a main source of destroying entanglement, i.e. turning a pure entangled state into a mixed entangled state, as shown in previous cavity QED experiments.[13,14] In this paper, we propose an alternative scheme for purifying entanglement via cavity decay. In comparison with the scheme of Ref.[8], the advantage of the scheme is that the atomic spontaneous emission is suppressed since the atoms are always in ground states, which is important for the improvement of the success probability. Our scheme works no matter whether the cavity decay rate is smaller than the atom-cavity coupling strength or not. The fidelity of the final state is insensitive to the inefficiency of the photodetectors. Our scheme opens a prospect for long-distance entanglement purification of two atoms trapped in two separate cavities, which is essential for quantum cryptography and teleportation.

Assume that two distant atoms are trapped in two single-mode optical cavities, respectively. The atoms have one excited state $|r\rangle$ and three ground states $|e\rangle$, $|g\rangle$, and $|f\rangle$. Here $|f\rangle$ acts as an auxiliary state. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant g, while the transition $|e\rangle \rightarrow |r\rangle$ is driven by a classical laser field with a Rabi frequency Ω. Assume that the classical field and cavity mode are

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detuned from the respective transitions by the same amount Δ. The setup is shown in Fig.1. Photons are

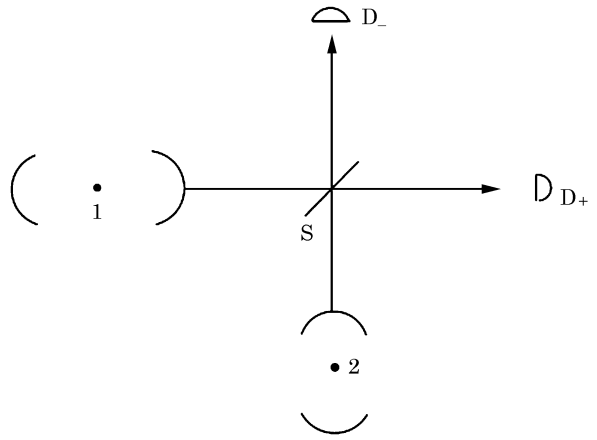


Fig.1. The experimental setup. Two distant atoms 1 and 2 are trapped in separate cavities. Photons leak through the sides of the cavities facing the beam-splitter S and then are detected by the photodetectors D_+ and D_-.

allowed to leak from both cavities and then mixed on a beam-splitter, which destroys the which-path information. The photons are then detected by two photodetectors. We here require that the cavities are one sided so that the only photon leakage occurs through the sides of the cavities facing the beam-splitter. Assume that the two atoms have been prepared in the mixed entangled state of Eq.(1). After the transformation $|g_j\rangle \rightarrow |f_j\rangle$ we have

$$\rho'_{\pm} = a\,|f_1\rangle\,|f_2\rangle\,\langle f_1|\,\langle f_2| + b\,|\phi'_{\pm}\rangle\,\langle\phi'_{\pm}|\,, \tag{3}$$

where

$$|\phi'_{\pm}\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle\,|f_2\rangle \pm |f_1\rangle\,|e_2\rangle). \tag{4}$$

Consider the case $\Delta \gg \Omega$, g, γ, where γ is the spontaneous decay rate. If each atom is initially populated in the ground states, the probability of the atom being populated in the upper level $|r\rangle$ is negligible and the atomic spontaneous emission is suppressed during the evolution. This leads to the effective Hamiltonian[4,15,16]

$$\begin{aligned} H_j =& \frac{\Omega^2}{\Delta}\,|e_j\rangle\,\langle e_j| + \frac{g^2}{\Delta}a^+a\,|g_j\rangle\,\langle g_j| \\ &+ \lambda(a_j S_j^+ + a_j^+ S_j^-), \end{aligned} \tag{5}$$

where a_j is the annihilation operator of the jth (j=1, 2) cavity mode, $\lambda = \dfrac{\Omega g}{\Delta}$, $S_j^+ = |e_j\rangle\,\langle g_j|$, and $S_j^- = |g_j\rangle\,\langle e_j|$. The first and second terms describe the Stark shifts for the states $|e_j\rangle$ and $|g_j\rangle$, induced by the classical field and cavity field, respectively. The third and fourth terms describe the Raman coupling between the two ground states $|e_j\rangle$ and $|g_j\rangle$.

We now include the cavity decay. Under the condition that no detection of photon has been registered the system is governed by the non-Hermitian Hamiltonian

$$H_{e,j} = H_j - \mathrm{i}\Gamma a_j^+ a_j, \tag{6}$$

where Γ is the cavity decay rate. Assume that $\Omega = g$ and $\lambda > \Gamma/2$, then we obtain the time evolution for the state $|e_j\rangle\,|0_j\rangle$

$$\begin{aligned} |e_j\rangle\,|0_j\rangle \rightarrow & \mathrm{e}^{-\Gamma t/2}\Big\{\Big[\cos(\alpha t/2) + \frac{\Gamma}{\alpha}\sin(\alpha t/2)\Big]\,|e_j\rangle\,|0_j\rangle \\ & - \mathrm{i}\frac{2\lambda}{\alpha}\sin(\alpha t/2)\,|g_j\rangle\,|1_j\rangle\Big\}, \end{aligned} \tag{7}$$

where

$$\alpha = \sqrt{4\lambda^2 - \Gamma^2}. \tag{8}$$

On the other hand, the state $|f_j\rangle\,|0_j\rangle$ undergoes no transition.

Then the evolution of whole system is

$$\begin{aligned} \rho''_{\pm} =& a\,|f_1\rangle\,|0_1\rangle\,|f_2\rangle\,|0_2\rangle\,\langle f_1|\,\langle 0_1|\,\langle f_2|\,\langle 0_2| \\ &+ b\,|\phi''_{\pm}\rangle\,\langle\phi''_{\pm}|\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} |\phi''_{\pm}\rangle =& \mathrm{e}^{-\Gamma t/2}\Big\{\Big[\cos(\alpha t/2) + \frac{\Gamma}{\alpha}\sin(\alpha t/2)\Big]\,|e_1\rangle\,|0_1\rangle - \mathrm{i}\frac{2\lambda}{\alpha}\sin(\alpha t/2)\,|g_1\rangle\,|1_1\rangle\Big\}\,|f_2\rangle\,|0_2\rangle \\ & \pm \mathrm{e}^{-\Gamma t/2}\,|f_1\rangle\,|0_1\rangle\Big\{\Big[\cos(\alpha t/2) + \frac{\Gamma}{\alpha}\sin(\alpha t/2)\Big]\,|e_2\rangle\,|0_2\rangle - \mathrm{i}\frac{2\lambda}{\alpha}\sin(\alpha t/2)\,|g_2\rangle\,|1_2\rangle\Big\}. \end{aligned} \tag{10}$$

The registering of a click at one of the photodetectors at the moment τ corresponds to the action of the jump operator $(a_1 \pm a_2)/\sqrt{2}$ on the state. The two atoms are then projected to the maximally entangled state

$$|\phi_{a,\pm}\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle\,|f_2\rangle \pm |f_1\rangle\,|g_2\rangle). \tag{11}$$

The cavity modes are now left in the vacuum states. As the coefficients of the two basis states $|g_1\rangle|f_2\rangle$ and $|f_1\rangle|g_2\rangle$ vary with the interaction time in the same way, i.e., the modes of the two coefficients are identical at any moment, the two atoms collapse to the maximally entangled state no matter when the photon is detected. After the transformations $|g_j\rangle \to |e_j\rangle$ and $|f_j\rangle \to |g_j\rangle$ the state $|\phi_{a,\pm}\rangle$ evolves to the state $|\phi_\pm\rangle$ of Eq.(2). As the success of the scheme depends upon the detection of a photon, the probability of successfully getting the maximally entangled state equals to the probability of detecting a photon. If one wishes to wait a time t_d, the probability of successfully getting the maximally entangled state is given by

$$P = b\left\{1 - \mathrm{e}^{-\Gamma t_\mathrm{d}}\left[\cos^2(\alpha t/2) + \frac{4\lambda^2 + \Gamma^2}{\alpha^2}\sin^2(\alpha t/2) + \frac{\Gamma}{\alpha}\sin(\alpha t)\right]\right\}. \tag{12}$$

If t_d is long enough so that $\mathrm{e}^{-\Gamma t_\mathrm{d}} \ll 1$ the success probability is about b.

We now consider the case that $\lambda < \Gamma/2$. Under the condition that no photon is detected the evolution of the state $|e_j\rangle|0_j\rangle$ is

$$|e_j\rangle|0_j\rangle \to \mathrm{e}^{-\Gamma t/2}\left[\left(\frac{\mathrm{e}^{\beta t/2} + \mathrm{e}^{-\beta t/2}}{2} - \frac{\Gamma}{\beta}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2}\right)|e_j\rangle|0_j\rangle - \frac{2\lambda}{\beta}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2\mathrm{i}}|g_j\rangle|1_j\rangle\right], \tag{13}$$

where

$$\beta = \sqrt{-4\lambda^2 + \Gamma^2}. \tag{14}$$

Thus evolution of whole system is

$$\rho''_\pm = a|f_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle\langle f_1|\langle 0_1|\langle f_2|\langle 0_2| + b\left|\phi''_\pm\right\rangle\left\langle\phi''_\pm\right|, \tag{15}$$

where

$$\begin{aligned}|\phi''\rangle =& \mathrm{e}^{-\Gamma t/2}\left\{\left[\left(\frac{\mathrm{e}^{\beta t/2} + \mathrm{e}^{-\beta t/2}}{2} - \frac{\Gamma}{\beta}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2}\right)|e_1\rangle|0_1\rangle\right.\right.\\ &\left.\left. - \frac{2\lambda}{\beta}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2\mathrm{i}}|g_1\rangle|1_1\rangle\right]|f_2\rangle|0_2\rangle\\ &\pm |f_1\rangle|0_1\rangle\left[\left(\frac{\mathrm{e}^{\beta t/2} + \mathrm{e}^{-\beta t/2}}{2} - \frac{\Gamma}{\alpha}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2}\right)|e_2\rangle|0_2\rangle\right.\\ &\left. - \frac{2\lambda}{\beta}\frac{\mathrm{e}^{-\beta t/2} - \mathrm{e}^{\beta t/2}}{2\mathrm{i}}|g_2\rangle|1_2\rangle\right]\Big\}.\end{aligned} \tag{16}$$

The registering of a click at one of the photodetectors at the moment τ also projects the two atoms to the state of Eq.(11). If one wishes to wait a time t_d the probability of success is given by

$$\begin{aligned}P =& b\left\{1 - \mathrm{e}^{-\Gamma t_\mathrm{d}}\left[\frac{1}{4}\mathrm{e}^{\beta t_\mathrm{d}}\left(1 + \frac{\Gamma}{\beta}\right)^2 + \frac{1}{4}\mathrm{e}^{-\beta t_\mathrm{d}}\left(1 - \frac{\Gamma}{\beta}\right)^2 + \frac{1}{2}\left(1 - \frac{\Gamma^2}{\beta^2}\right)\right.\right.\\ &\left.\left. + \frac{g^2}{\beta^2}\left(\mathrm{e}^{\beta t_\mathrm{d}} + \mathrm{e}^{-\beta t_\mathrm{d}} - 2\right)\right]\right\}.\end{aligned} \tag{17}$$

When $\mathrm{e}^{-(\Gamma-\beta)t_\mathrm{d}} \ll 1$ the success probability is also about b.

We now consider the imperfection of the photodetectors. There is at most one photon in the two cavities. Due to the imperfection of the photodetectors, one photon may leak out of the cavities, but the photodetectors fail to click. In this case, the procedure fails and thus the probability of success is given by $P' = \eta P$, where η is the detection efficiency. However, the fidelity of the entangled state is not affected.

In recent experiments,[17,18] a single and more Cs atoms were trapped in an optical cavity. We can use the hyperfine levels $|F=3,\ m=1\rangle$, $|F=3,\ m=0\rangle$, and $|F=3,\ m=-1\rangle$ of $6S_{1/2}$ as the states $|f\rangle$, $|g\rangle$, and $|e\rangle$, respectively. The level $|F=3,\ m=-1\rangle$ of $6P_{1/2}$ can act as the excited state $|r\rangle$. Suppose that the cavity mode is left-polarized. In this case the Raman transition between $|g\rangle$ and $|e\rangle$ can be induced by the cavity mode and a π-polarized classical field with the assistance of the virtual level $|r\rangle$.

In order to perform the transformation $|g\rangle \rightarrow |f\rangle$ we use another auxiliary sublevel $|F=1,\ m=1\rangle$ of $6P_{1/2}$ denoted by $|h\rangle$. We drive the transitions $|g\rangle \rightarrow |h\rangle$ and $|f\rangle \rightarrow |h\rangle$ with a pair of off-resonant classical fields, respectively. The two classical fields are detuned from the respective transitions by the same amount δ. In the case that the detuning δ is much larger than the respective Rabi frequencies, the upper level $|h\rangle$ can be adiabatically eliminated and the two classical fields just induce the Raman transition between the states $|g\rangle$ and $|f\rangle$.

In conclusion, we have proposed a scheme for the purification of entangled states for two atoms trapped in two separate cavities. In our scheme, the atoms have no probability of being populated in the excited states and thus the atomic spontaneous emission is suppressed, which is of importance for increasing the success probability. Our scheme works no matter when the cavity decay rate is larger than the effective atom-cavity coupling strength or not. The cavities cannot contain more than one photons and the success of the scheme depends upon the detection of a photon leaking out of the cavity. Therefore, the fidelity is not affected by the imperfection of the photodetectors.

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