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Steady state speed distribution analysis for a combined cellular automaton traffic model*

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Cellular Automaton (CA) based traffic flow models have been extensively studied due to their effectiveness and simplicity in recent years. This paper develops a discrete time Markov chain (DTMC) analytical framework for a Nagel–Schreckenberg and Fukui–Ishibashi combined CA model (W^2H traffic flow model) from microscopic point of view to capture the macroscopic steady state speed distributions. The inter-vehicle spacing Markov chain and the steady state speed Markov chain are proved to be irreducible and ergodic. The theoretical speed probability distributions depending on the traffic density and stochastic delay probability are in good accordance with numerical simulations. The derived fundamental diagram of the average speed from theoretical speed distributions is equivalent to the results in the previous work.

Keywords: cellular automaton, traffic flow model, speed distribution, discrete time Markov chain
PACC: 0560, 0250

1. Introduction

During the past few decades, cellular automaton (CA) traffic flow model has been widely employed as a powerful tool by physicists and traffic engineers to mimic, analyse and control high speed traffic flows from a microscopic point of view.[1−6] Even though it is simple conceptually, CA model can capture the complex dynamics (e.g. phase transitions) of the nonlinear characteristics of the traffic flow problem with merits of computational effectiveness and flexibility.

According to Wolfram's 184 cellular automata,[7] Nagel and Schreckenberg[5] first proposed a conservative one-dimensional (1D) CA model (NS model) to simulate freeway traffic. Biham *et al*[8] introduced a two-dimensional (2D) CA model (BML model) and studied the self organization effects in phase. After the successful introduction of CA into traffic flow modelling, dozens of CA based traffic models have been proposed thereafter, for example, Benjamin *et al*[9] presented the Benjamin–Johson–Hui (BJH) model which took into account the slow-to-start effect during the acceleration procedure, while authors of Ref.[10] considered the inter-car spacings and proposed a time-oriented cellular automata (TOCA), contrary to the conservative NS model, Fukui and Ishibashi[2] adopted the Fukui–Ishibashi (FI) traffic flow model with an aggressive acceleration rule. Other CA based models include Nagel's multi-lane traffic model[6,11] and Chowdhury's city transport network flow model (CS model).[12] See Ref.[13] for a complete review.

As a preliminary step to understand the diverse behaviours of traffic flow and facilitate the traffic prediction and management purposes, most current CA based models focus on the so-called fundamental diagram problem to reveal the relationship between the macroscopic average car speed in a steady state and traffic density. In addition to the numerical simulation method,[2,14] various mean-field based analytic approaches have been proposed theoretically for the qualitative description of fundamental diagram evolutions in one-dimensional and two-dimensional scenarios.[13,15] The macroscopic mean speed of cars and further the traffic flux are fundamental for traffic engineering. We believe that a thorough understanding of macroscopic steady state speed distributions is

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also meaningful for the better reflection of speed dynamics experienced by cars statistically. With the knowledge of steady state speed distributions, it is straightforward not only to calculate the fundamental metrics of average speed and traffic flux, but also to give pictures about speed dynamics which might further lead to analysing the potential traffic congestion occurrence and studying the blockage length with ease.[16,17]

In this paper, the discrete time Markov chain (DTMC) is first used to characterize the inter-car spacing evolutions in an asymptotic steady state. The synchronic update rules of CA model are represented by DTMC one-step transition probability matrix $\mathcal{P}$ describing the distance transition behaviour between consecutive vehicles. Then the DTMC is further employed to capture the speed transition of vehicles. The steady state speed distributions of cars are derived based on the stationary analysis of speed transition Markov chain.

The rest of the present paper is organized as follows. In Section 2, a cellular automaton traffic flow model W^{2}H combining the FI model with the NS model, proposed by Wang *et al*,[18] is presented. Studied in Section 3 are the inter-car spacing evolutions within the framework of discrete time Markov chain and the asymptotic steady state speed distribution of cars analytically. Theoretical investigations is evaluated and discussed through numerical simulations in Section 4. Finally, the conclusions drawn from the present study and our future work are given in Section 5.

2. Traffic flow model

As two popular and typical one-dimensional CA traffic flow models on highway, i.e. NS and FI models, have been extensively studied from different aspects. The NS model takes a conservative acceleration rule even in the condition of preceding car far ahead, while FI model adopts an over-circumspect stochastic delay rule for all cars without considering any inter-car spacings. In Ref.[18], Wang *et al* proposed a new one-dimensional traffic flow CA model (W^{2}H model), which combines the Fukui–Ishibashi-type aggressive acceleration rule for all cars with the Nagel–Schreckenberg-type stochastic delay only for the cars following the tail of the preceding car. The new model seems more realistic, and a car-oriented mean field method was used to study the fundamental diagrams analytically in Ref.[19].

In the paper, the steady state speed distribution analysis is based on the new model, i.e. W^{2}H model. As shown later, the DTMC based analysis can be applied to other CA models in the same way. To facilitate the following investigations, we first introduce some necessary terminologies and briefly recall the W^{2}H traffic flow model.

Consider a one-dimensional lattice of L sites with periodic boundary conditions. Let N denote the total number of vehicles and $\rho = \dfrac{N}{L}$ be the traffic density. Each site is either occupied by one vehicle or empty. Assume that $C_n(t), n = 1, \ldots, N$, is the number of empty sites ahead of the n-th vehicle, and $v_n(t)$ is an integer speed between 0 and the maximum value $v_{\max} = M$ representing the number of sites that the n-th vehicle can move at time t. The W^{2}H traffic model is composed of the following three rules synchronously updated for all vehicles at each time step t:[18]

(1) $v'_n(t) = \min\{C_n(t), M\}$;

(2) $v_n(t) = v'_n(t) - 1$ with a stochastic delay probability f if $v'_n(t) = C_n(t)$ and $v'_n(t) > 0$;

(3) The n-th vehicle moves $v_n(t)$ site (s) ahead.

From the traffic model, the speed of the n-th vehicle, $v_n(t+1)$, at time step $t+1$ is subject to both the inter-car spacing $C_n(t)$ and the delay probability f as shown in Eq.(1).

$$v_n(t+1) = F_M(C_n(t), f), \tag{1}$$

where function F_M is characterized by the following equation:

$$F_M(C, f) = \begin{cases} 0, & C = 0, \\ C - 1 \text{ with a probability } f, & 0 < C \le M, \\ C \text{ with a probability } (1-f), & 0 < C \le M, \\ M, & C > M. \end{cases} \tag{2}$$

3. Asymptotic steady state speed distribution analysis

3.1. Markov chain based inter-vehicle spacing evolution analysis

Let $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$ be a stochastic process, with state space $E = \{0, 1, \ldots, L-N\}$ denoting the inter-vehicle spacing evolutions of the n-th vehicle on a one-dimensional lattice, then $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$ will be a DTMC as the following Eq.(3) holds obviously for all vehicles:

$$\begin{aligned} &P\{\mathcal{C}_n(t) < x_n | \mathcal{C}_n(1) = x_1, \ldots, \mathcal{C}_n(t-1) = x_{n-1}\} \\ &= P\{\mathcal{C}_n(t) < x_n | \mathcal{C}_n(t-1) = x_{n-1}\}, \end{aligned} \tag{3}$$

where inter-vehicle spacing $x_i \in E$. In the traffic model, vehicles synchronously update their positions independently, thus $\{\mathcal{C}_n(t), n = 1, \ldots, N\}$ are identical and $\mathcal{C}_n(t)$ is expressed as $\mathcal{C}(t)$ for simplicity.

Let $p_{i,j}$ denote the inter-vehicle spacing transition probability from distance i to distance j during one time step, and assume p_i to be the steady state probability distribution of inter-vehicle space i, $\sum_{i=0}^{L-N} p_i = 1$, then the 3-rule W^{2}H model will be able to be strictly formulated by a one-step transition probability matrix $\mathcal{P} = (p_{i,j})_{i,j \in E}$.

Further exploration of the updated rules shows that the inter-vehicle spacing at time step $t+1$ is determined by the advances of two consecutive substeps as shown in Eq.(4), i.e. the n-th vehicle advance $v_n(t)$ of the first substep, subject to the inter-vehicle spacing at $C_n(t)$, and the $(n+1)$-th vehicle advance $v_{n+1}(t)$ of the second substep, independent of vehicle n.

$$C_n(t+1) = C_n(t) \underbrace{-v_n(t)}_{\text{step1}} \underbrace{+v_{n+1}(t)}_{\text{step2}} \tag{4}$$

Let $E' = \{0, 1, \ldots, L-N\}$ be the hidden state space of the inter-vehicle spacing after the first substep. And terms $p'_{i,j}$ and $p''_{i,j}$ denote the one-substep transition probabilities of the two substeps respectively. Then similar to the Chapman–Kolmogorov (C–K) equation,[20] transition probability $p_{i,j}$ can be rewritten as

$$p_{i,j} = \sum_{l \in E'} p'_{i,l} p''_{l,j}, \tag{5}$$

where $p'_{i,l}$ is

$$p'_{i,l} = \begin{cases} 1, & i = l = 0 \;||\; i > M,\ l = i - M, \\ 0, & i > M,\ l \neq i - M, \\ 1 - f, & 0 < i \leq M,\ l = 0, \\ f, & 0 < i \leq M,\ l = 1, \end{cases} \tag{6}$$

and $p''_{l,j}$ is determined by

$$p''_{l,j} = \begin{cases} 0, & j - l > M \;||\; j - l < 0, \\ p_0 + f p_1, & j = l = 0, \\ (1-f) p_M + \sum_{k=M+1}^{L-N} p_k, & j - l = M, \\ (1-f) p_{j-l} + f p_{j-l+1}, & \text{otherwise}. \end{cases} \tag{7}$$

In Eq.(6), the absorption state 0 represents the situation of moving 0 site ahead when the n-th vehicle follows the tail of its preceding car, and the $(L-N-M)$ rebound states show the occurrence of the situation that vehicles can move at their maximum speed M. $p''_{l,j}$ gives the probability that the $(n+1)$-th vehicle moves $(j-l)$ site(s) under a stationary state.

3.2. Steady state space reduction for inter-vehicle spacing Markov chain

In Ref.[18], authors grouped the inter-vehicle spacings into two types, i.e. long and short spacings, according to their comparisons with the speed $M+1$ and gave the primary property of the inter-vehicle spacing evolution,

$$C_n(t) \geq M+1 \quad \forall n,\ \rho \leq \frac{1}{M+2},$$
$$C_n(t) \leq M+1 \quad \forall n,\ \rho > \frac{1}{M+2}. \tag{8}$$

From Eq.(8), all vehicles will move at a maximum speed M for the low traffic density case $\left(\rho \leq \frac{1}{M+2}\right)$ as the inter-vehicle spacings are always larger than $M+1$. The steady state speed distribution is simple and straightforward.

For the heavy traffic density case $\left(\rho > \frac{1}{M+2}\right)$, the (hidden) inter-vehicle spacing state space (E') E can be shrunk into $\{0, 1, \ldots, M+1\}$. The third case of Eq.(7) is simplified into $(1-f)p_M + p_{M+1}$.

Let $\mathcal{P}_M$ denote a one-step transition probability matrix $\mathcal{P}$ at M and substitute Eqs.(6) and (7) into Eq.(5), then $\mathcal{P}_M$ will be a matrix in the following form:

$$\mathcal{P}_M = \begin{bmatrix} p_{0,0} & \cdots & p_{0,i} & \cdots & 0 \\ p_{1,0} & \cdots & p_{1,i} & \cdots & p_{1,M+1} \\ & \vdots & & \vdots & \\ 0 & \cdots & p_{M+1,i} & \cdots & p_{M+1,M+1} \end{bmatrix}_{(M+2)\times(M+2)}, \tag{9}$$

where

$$\begin{aligned} &p_{0,0} = p_0 + fp_1, \\ &p_{0,i} = (1-f)p_i + fp_{i+1}, 0 < i < M, \\ &p_{0,M} = (1-f)p_M + p_{M+1}, \\ &p_{0,M+1} = p_{M+1,0} = 0, \\ &p_{i,0} = (1-f)p_{0,0}, 1 < i < M, \\ &p_{i,j} = (1-f)p_{0,j} + fp_{0,j-1}, \\ &1 \leq i \leq M,\ 1 \leq j \leq M, \\ &p_{i,M+1} = fp_{0,M}, \\ &p_{M+1,j} = p_{0,j-1}, j > 1. \end{aligned} \tag{10}$$

Matrix (11) shows the transition probability matrix $\mathcal{P}_5$ as an example under heavy traffic density case.

$$\mathcal{P}_5 = \begin{bmatrix} p_0 + fp_1 & (1-f)p_1 + fp_2 & \cdots & 0 \\ (1-f)(p_0+fp_1) & (1-f)((1-f)p_1 + fp_2) + f(p_0+fp_1) & \cdots & f((1-f)p_5 + p_6) \\ (1-f)(p_0+fp_1) & (1-f)((1-f)p_1 + fp_2) + f(p_0+fp_1) & \cdots & f((1-f)p_5 + p_6) \\ \vdots & \vdots & \cdots & \\ 0 & p_0 + fp_1 & \cdots & (1-f)p_5 + p_6 \end{bmatrix}_{7\times 7} \tag{11}$$

3.3. Analysis of asymptotic steady state speed distributions

Theorem 1 The inter-vehicle spacing Markov chain $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$ is irreducible and ergodic under the heavy traffic density condition.

proof: In the probability matrix $\mathcal{P}_M$, we first prove that steady state distributions $p_i > 0$, for all $i \in E$. When the traffic is heavy, $0 < p_{M+1} < 1.0$ holds true according to Eq.(8). As p_{M+1} is less than 1.0, there must exist at least an additional inter-vehicle spacing (active state), implying that state i satisfies $0 < p_i = 1.0 - p_{M+1} < 1.0$. It follows that $p_j = 0$, for all $j \in E - \{i, M+1\}$ and the corresponding state transition diagram can be illustrated as Fig.1(a) based on the $\mathcal{P}_M$ definition of Eq.(9).

If states i and $M+1$ are the only two active states for Markov chain $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$, the one-step transition probabilities $p_{i,i}, p_{i,M+1}, p_{M+1,i}$ and $p_{M+1,M+1}$ should satisfy the following equation:

$$\begin{aligned} &p_{i,i} + p_{i,M+1} = p_i + fp_{M+1} = 1.0, \\ &p_{M+1,i} + p_{M+1,M+1} = fp_i + p_{M+1} = 1.0, \end{aligned} \tag{12}$$

which is in contradiction to the probability constraint $p_i + p_{M+1} = 1.0$ with $f < 1.0$. Therefore, there must exist at least one more active state j with $p_j > 0$ satisfying $p_i + p_j + p_{M+1} = 1.0$.

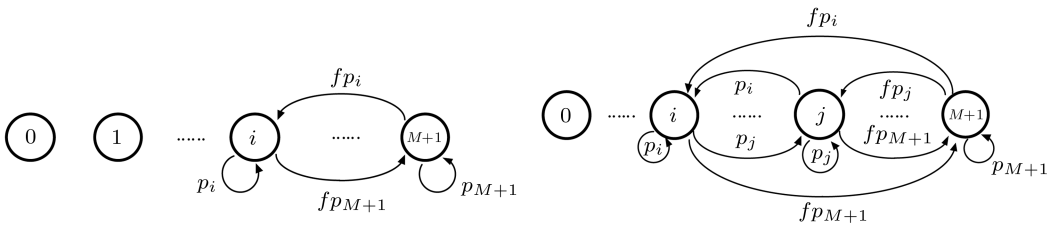
Figure 1(b) shows the state transition diagram for the case of three active states. Again the following equation:

$$\sum_{l\in\{i,j,M+1\}} p_{i,l} = p_i + p_j + fp_{M+1} = 1.0,$$

$$\sum_{l\in\{i,j,M+1\}} p_{j,l} = p_i + p_j + fp_{M+1} = 1.0,$$

$$\sum_{l\in\{i,j,M+1\}} p_{M+1,l} = fp_i + fp_j + p_{M+1} = 1.0 \quad (13)$$

is contrary to the assumption of only three active states, and there should be more active states. It should be noted that the one-step transition probabilities in Fig.1 are slightly different for different selections of i and j. We can prove that all states are active, i.e. $p_i > 0$ for all $i \in E$.



(a) two-state case ($p_i, p_{M+1} > 0$) (b) three-state case (p_i, p_j and $p_{M+1} > 0$)

Fig.1. The typical state transition diagram.

In transition probability matrix $\mathcal{P}_M$, there are only two 0 elements, i.e. $p_{0,M+1}$ and $p_{M+1,0}$ for $f > 0$, and $p_{0,M}$ and $p_{M,0}$ for $f = 0$. Any active states are mutually reachable as two-step transition probability $p_{i,j}(2) > 0,\ i, j \in E$.

Owing to the state mutual reachability, the inter-vehicle spacing Markov chain $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$ is irreducible and further ergodic.

Following the ergodic property of $\{\mathcal{C}_n(t), t \in \mathbb{N}\}$, it is easy to resolve the steady state spacing distribution $\boldsymbol{p} = \{p_i, i \in E\}$ from the following equation:

$$\boldsymbol{p} = \boldsymbol{p} \cdot \mathcal{P}_M,$$

$$p_i > 0,\ \sum_{i\in E} p_i = 1.0,\ \sum_{i\in E} ip_i = \frac{1}{\rho} - 1. \quad (14)$$

Let stochastic process $\{\mathcal{V}_n(t), t \in \mathbb{N}\}$ with state space $S = \{0, 1, \ldots, M\}$ denote the n-th vehicle speed evolution. Then $\{\mathcal{V}_n(t), t \in \mathbb{N}\}$ ($\mathcal{V}(t)$ for simplicity) is also a discrete time Markov chain. Suppose that vector $\boldsymbol{v} = \{v_i, i \in S\}$ is a steady state speed distribution (not an absolute speed vector) and assume that $v_{i,j}$ is a one-step speed transition probability from speed i to speed j during one time step. Then the one-step speed transition probability matrix $\mathcal{T}_M = (v_{i,j})_{i,j\in S}$ under the maximum speed M is written as the following form:

$$\mathcal{T}_M = \begin{bmatrix} p_0 + fp_1 & (1-f)p_1 + fp_2 & \ldots & (1-f)p_M + p_{M+1} \\ p_0 + fp_1 & (1-f)p_1 + fp_2 & \ldots & (1-f)p_M + p_{M+1} \\ & \vdots & & \vdots \\ p_0 + fp_1 & (1-f)p_1 + fp_2 & \ldots & (1-f)p_M + p_{M+1} \end{bmatrix}_{(M+1)\times(M+1)}, \quad (15)$$

which reveals that the speed of a vehicle at time step t is completely independent of its previous speed at $t-1$.

As $p_i > 0, i \in E$, stochastic process $\{\mathcal{V}_n(t), t \in \mathbb{N}\}$ is an ergodic Markov chain on state space S subject to the following equation

$$\boldsymbol{v} = \boldsymbol{v} \cdot \mathcal{T}_M,$$
$$v_i > 0,\ \sum_{v\in S} v_i = 1.0. \quad (16)$$

The steady state speed distribution $\boldsymbol{v}$ can be resolved as shown by the following equation:

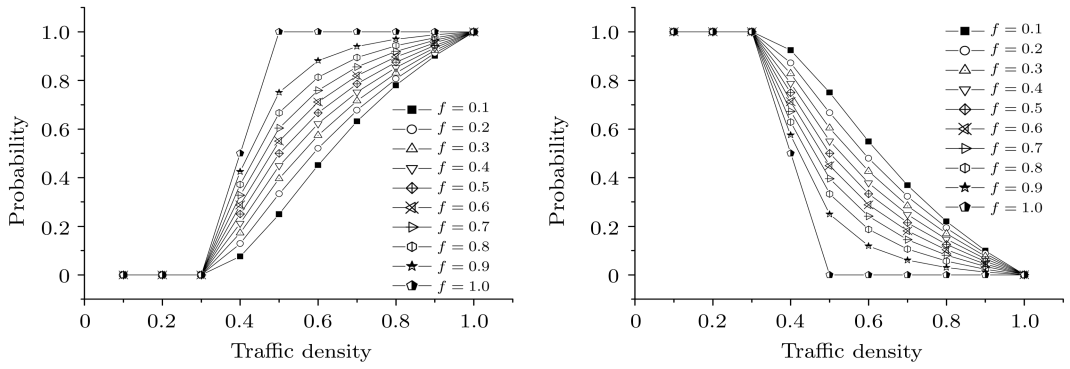
$$v_i = \begin{cases} p_0 + fp_1, & i = 0, \\ (1-f)p_i + fp_{i+1}, & 0 < i < M, \\ (1-f)p_M + p_{M+1}, & i = M. \end{cases} \quad (17)$$

4. Experimental evaluation

To validate our analysis in Section 3, numerical simulations are conducted for one-dimensional highway with a fixed number of vehicles $N = 1000$. The length of highway L is adjusted to generate a desired traffic density f between 0.1 and 1.0. The discretely empirical speed distributions are collected based in 80000 time step simulation for all vehicles, with additional initial 20000 time steps excluded to eliminate the transient effect.

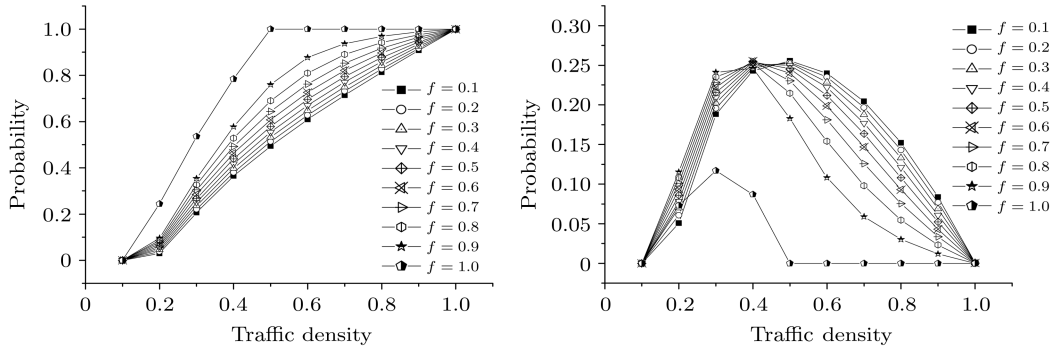
Figures 2 and 3 present the theoretical steady state speed distributions according to Eq.(17) and comparisons between these distributions and the numerical results for $M = 1, 5$ separately. From the figures, numerical results positively prove the correctness and the accuracy of our theoretical studies. Figure 2 shows that when traffic density is low enough i.e. $\rho < \dfrac{1}{3}$, all vehicles will run at a maximum speed of 1 and the probabilities of maximum speed decrease with the increase of traffic density. This decrease trend is also applicable to Fig.3. When the traffic density increases, the probability that vehicles run at higher speeds decreases correspondingly.

Figure 3 also illustrates an important role of stochastic delay probability f. Except for the two cases of Figs.3(a) and 3(f), when traffic density is lower than a critical value ρ_c, the speed distributions are directly proportional to f for the general case of $0 < f < 1.0$. The speed distribution trend turns to being inversely proportional to f in the situation of $\rho \geq \rho_c$. For the extreme case of $f = 1.0$, the probability distribution of speeds larger than 0 becomes zero when traffic density $\rho \geq 0.5$. This phenomenon happens due to the fact that the average inter-vehicle spacing is no larger than one and the traffic flow model will reduce the speed to zero deterministically.



(a) speed =0 (b) speed = 1

Fig.2. The asymptotic steady state speed distributions of the W^{2}H traffic flow model for $v_{\max} = 1$ with the values of stochastic delay f ranging from 0.1 to 1.0 in steps of 0.1, the solid curves denoting the theoretical results, points with different symbols representing the simulation results, and theoretical analysis being in good accordance with numerical simulations due to their overlapping at all discrete points in the figure.



(a) speed =0 (b) speed = 1

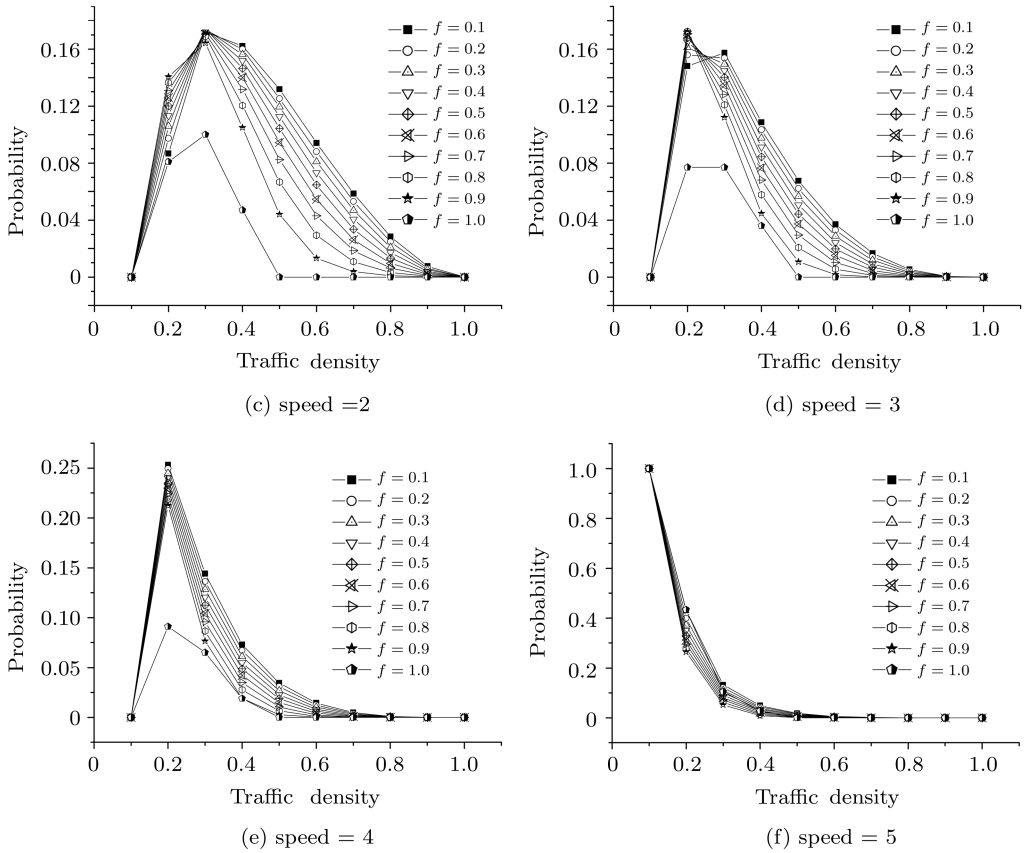


Fig.3. The asymptotic steady state speed distributions of the W^{2}H traffic flow model for $v_{\max} = 5$ with the values of stochastic delay f ranging from 0.1 to 1.0 in steps of 0.1, and theoretical analyses in solid curves being in good accordance with numerical simulations at all discrete points.

Figures 4(a) and 4(b) show the fundamental diagrams at the maximum speeds of 1 and 5 respectively. The results are obtained from numerical simulations and theory (calculated according to $\bar{v} = \sum_{i \in S} i v_i$) under different values of traffic density ρ. From the figure, theoretical analysis is in good agreement with numerical results at all discrete points.

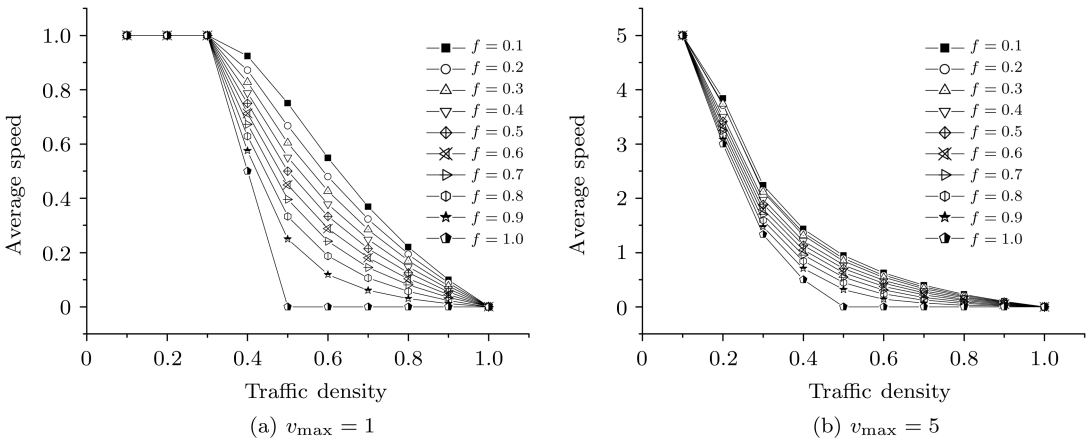


Fig.4. The fundamental diagram of the average speed of the W^{2}H traffic flow model with the values of stochastic delay f ranging from 0.1 to 1.0 in steps of 0.1, the solid curves denoting the theoretical results, and points with different symbols representing the simulation results.

In addition, the average speed when $v_{\max} = 2$ is given in Table 1. The theoretical results are in accordance with the simulation and the theoretical fundamental diagrams in Ref.[18].

Table 1. The theoretical steady state average speeds from the W^{2}H traffic flow model for $v_{\max} = 2$ with the values of stochastic delay ranging from 0.1 to 1.0.

f	traffic density $\rho = N/L$									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	2.00	2.00	1.90	1.34	0.91	0.62	0.39	0.22	0.11	0.00
0.2	2.00	2.00	1.78	1.24	0.83	0.57	0.36	0.20	0.11	0.00
0.3	2.00	2.00	1.70	1.15	0.76	0.50	0.32	0.18	0.08	0.00
0.4	2.00	2.00	1.66	1.07	0.73	0.48	0.29	0.16	0.08	0.00
0.5	2.00	2.00	1.57	1.00	0.67	0.43	0.24	0.14	0.05	0.00
0.6	2.00	2.00	1.55	0.93	0.57	0.35	0.22	0.11	0.04	0.00
0.7	2.00	2.00	1.50	0.84	0.48	0.35	0.16	0.10	0.03	0.00
0.8	2.00	2.00	1.44	0.79	0.42	0.23	0.12	0.07	0.02	0.00
0.9	2.00	2.00	1.39	0.66	0.30	0.16	0.07	0.03	0.01	0.00
1.0	2.00	2.00	1.33	0.50	0.00	0.00	0.00	0.00	0.00	0.00

5. Conclusions

We have studied the asymptotic steady state speed distributions for a combined one-dimensional traffic flow model, i.e. the W^{2}H model. The Markov chain based model rule characterization, steady state analysis of inter-vehicle spacings and vehicle speeds provide an alternative approach to a thorough understanding of traffic flow dynamics statistically. Simulations demonstrate the correctness and the accuracy of our analytical investigations. The derived fundamental diagrams from speed distributions also agree with the previous results in other literature.

The inver-vehicle spacing Markov chain can be extended to describing other CA models in a similar manner. Thus the distribution analysis approach can be used to explore different traffic flow models in principle. Furthermore, the knowledge of inter-vehicle spacings and speed distributions can facilitate our future study in the following aspects:

(1) For CA traffic flow models, the question of when and how the stationary state is approached is still an important open problem.[19] We often discard initial time steps to reduce the transient effect in simulation studies. To speed up simulation convergence, it is possible to distribute the vehicles on lattices with inter-vehicle spacings subject to $p_i,\ i \in E$ and configure the speed to the steady state distribution $v_i, i \in S$ in the initialization procedure, while current solution often initializes vehicle speeds and inter-vehicle spacings under uniform distribution;

(2) The inter-vehicle spacing and speed distributions help to calculate the traffic congestion probabilities at different levels of traffic density, derive the frequencies of congestion occurrence and average blockage length for different traffic models.

References

[1] Chen R X, Bai K Z and Liu M R 2006 *Chin. Phys.* **15** 1471

[2] Fukui M and Ishibashi Y 1996 *J. Phys. Soc. Japan* **65** 1868

[3] Guo S L, Wei Y F and Xue Y 2006 *Acta Phys. Sin.* **55** 3336 (in Chinese)

[4] Liu J and Hua W 2006 *Chin. Phys.* **15** 687

[5] Nagel K and Schreckenberg M 1992 *J. Phys.* I **2** 2221

[6] Nagel K, Wolf D E, Wagner P and Simon P 1998 *Phys. Rev.* E **58** 1425

[7] Wolfram S 1994 *Cellular automata and complexity:* Collected papers (New York: Addison-Wesley) p83

[8] Biham O, Middleton A A and Levine D 1992 *Phys. Rev.* A **46** R6124

[9] Benjamin S C, Johnson N F and Hui P M 1996 *J. Phys.* A **29** 3119

[10] Yang Z X, Wang C P and Mao B H 1998 *Proceedings of Traffic and Transportion Studies* (ICTTS'98) (Reston, VA: American Society of Civil Engineers) p935

[11] Rickert M, Nagel K, Schreckenberg M and Latour A 1996 *Phys.* A **31** 534

[12] Helbing D and Herrmann H J 1999 *Traffic and Granular Flow* 1999 *Social, Traffic, and Granular Dynamics* (Berlin: Springer) p155

[13] Sven M and Bart D M 2005 *J. Phys. Rep.* **419** 1

[14] D'Souza R M 2005 *Phys. Rev.* E **71** 066112

[15] Wang B H, Wang L, Hui P M and Hu B 1998 *Phys. Rev.* E **58** 2876

[16] Boccara N, Fuk's H and Zeng Q 1997 *J. Phys.* A **30** 3329

[17] Lee H K, Barlovic R, Schreckenberg M and Kim D 2004 *Phys. Rev. Lett.* **92** 238702

[18] Wang L, Wang B H and Hu B 2001 *Phys. Rev.* E **63** 056117

[19] Fu C J, Wang B H and Yin C Y 2007 *Chaos Sol.* F **31** 772

[20] Ross S M 2004 *Introduction to Probability and Statistics for Engineers and Scientists* (New York: Academic) p726