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Stabilization of spiral wave and turbulence in the excitable media using parameter perturbation scheme*

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This paper proposes a scheme of parameter perturbation to suppress the stable rotating spiral wave, meandering spiral wave and turbulence in the excitable media, which is described by the modified Fitzhugh–Nagumo (MFHN) model. The controllable parameter in the MFHN model is perturbed with a weak pulse and the pulse period is decided by the rotating period of the spiral wave approximatively. It is confirmed that the spiral wave and spiral turbulence can be suppressed greatly. Drift and instability of spiral wave can be observed in the numerical simulation tests before the whole media become homogeneous finally.

Keywords: spiral wave, spiral turbulence, parameter perturbation
PACC: 0545,8730C, 0565, 8220M

1. Introduction

The spiral wave is formed under nonequilibrium conditions, which is observed in the chemical, physical and biological systems.[1−3] It is confirmed that the appearance of spiral wave in cardiac tissue may link to one dangerous heart disease, tachycardia, and the instability of spiral wave will induce spiral turbulence. The breakup and instability of spiral wave in cardiac tissue is called ventricular fibrillation (VF), which causes a rapid death of heart.[4−7] Spiral wave has been observed in the excitable media and the oscillatory media as well.[1,8] Generally, the reaction–diffusion equation is often used to describe dynamics of the spatial extended system. For example, the complex Ginzburg–Landau equation (CGLE),[8] Fitzhugh–Nagumo (FHN) equation,[9−11] and Oregnator model,[12−15] are used to describe the Belousov–Zhabotinsky (BZ) reaction in light-sensitive media.[13−15] More relevant topics about spiral wave, antispiral wave and spiral turbulence have been carried out in experiments and theoretical discussion.[16−24] In Ref.[25,26] Margerit and Barkley proposed a formula to calculate the rotation frequency of spiral wave and scroll wave in the media, and local periodical signal perturbation, spatial perturbation, self-coupling, parameter perturbation and intermittent shock etc have been used to remove the spiral wave.[27−29] For some reaction–diffusion systems, some variable or parameters are controllable; for example, the external control parameters $P_{\rm CO}$, $P_{\rm O_2}$, T that define the partial pressures of CO and oxygen and the crystal temperature in the CO oxidation on Pt (110) surface are mapped into the model parameters a, b, ε,[30] and the model is similar to the modified Fitzhugh–Nagumo (MFHN) model; another example is the external illumination in BZ reaction.[14,15] In this paper, we focus on the MFHN model, which can generate spiral wave, meandering spiral wave and spiral turbulence within appropriate parameter region. Our aim is to suppress the spiral wave, meandering wave and spiral turbulence by perturbing the controllable parameter with a weak pulse.

2. Models and Scheme

The two-dimensional MFHN model is given by[31]

$$\partial_t u = \varepsilon^{-1} u(u-1)\left(u - \frac{v+b}{a}\right) + D\nabla^2 u, \partial_t v = f(u) - v,$$

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$$f(u)=\begin{cases}0, & 0\le u<1/3,\\ 1-6.75u(u-1)^2, & 1/3\le u\le 1,\\ 1, & 1<u,\end{cases} \quad (1)$$

where the activator and inhibitor can be defined with u, v, and diffusion coefficient D equals 1, Laplace operator $\nabla^2 = \partial_{xx} + \partial_{yy}$; for more information in detail please refer to Ref.[31]. With parameters a=0.84, b=0.07, stable rotating spiral wave is induced and developed for ε=0.04, meandering spiral wave for ε >0.06 and spiral turbulence for ε >0.07.

The controllable parameter b is perturbed as

$$b = 0.07 + k\mathrm{int}(t/T), \quad (2)$$

where the k is the intensity of perturbation, int$(\cdot)$ is integer function, T is the rotation frequency of spiral wave which can be calculated via the formula in Eq.(3) proposed by Margerit and Barkley[25,26]

$$\omega = \frac{2\pi}{T} = \frac{0.69\mu}{\varepsilon^{1/3}} - \frac{0.926}{a};$$
$$\mu \approx 2.7\left[\frac{v^s(1-v^s)}{a}\right]^{2/3}; \quad v^s = a/2 - b. \quad (3)$$

3. Numerical simulation and discussion

In our numerical simulation, the Euler forward algorithm with the non-flux boundary condition is imposed on this system, time step size h=0.02, the whole media are decentralized into 256×256 sites, and space step $\Delta x = \Delta y$=100/256 space units, the diffusion coefficient D=1. Different pulse intervals and intensity of stimulation are applied to investigate the evolution of stable rotating spiral wave, meandering spiral wave and spiral turbulence, and the numerical results are plotted in Figure1 to Figure 7, respectively.

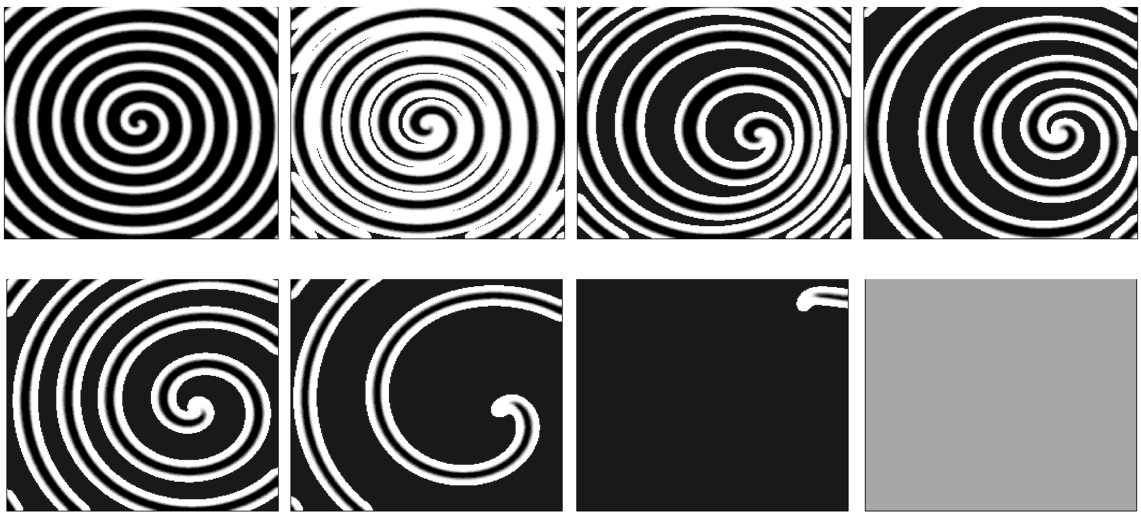


Fig.1. Evolution of stable rotating spiral wave, a=0.84, ε=0.04, ω=1.178, T=5.334, k=0.005292, b=0.07+k int(t/T) for t=0, 150, 180, 200, 220, 240, 260, 280 time units, respectively (the order is left to right, up to down, the snapshot is shown in grey).

It is confirmed that stable rotating spiral wave can be suppressed greatly within 300 time units. It is the intensity of stimulation and the pulse interval that determine the transient period to reach homogeneous state. The stronger intensity of pulse, the shorter transient period. Finally, the whole media becomes homogeneous when the tip of spiral wave drifts toward the border along an arbitrary orientation (left or right randomly).

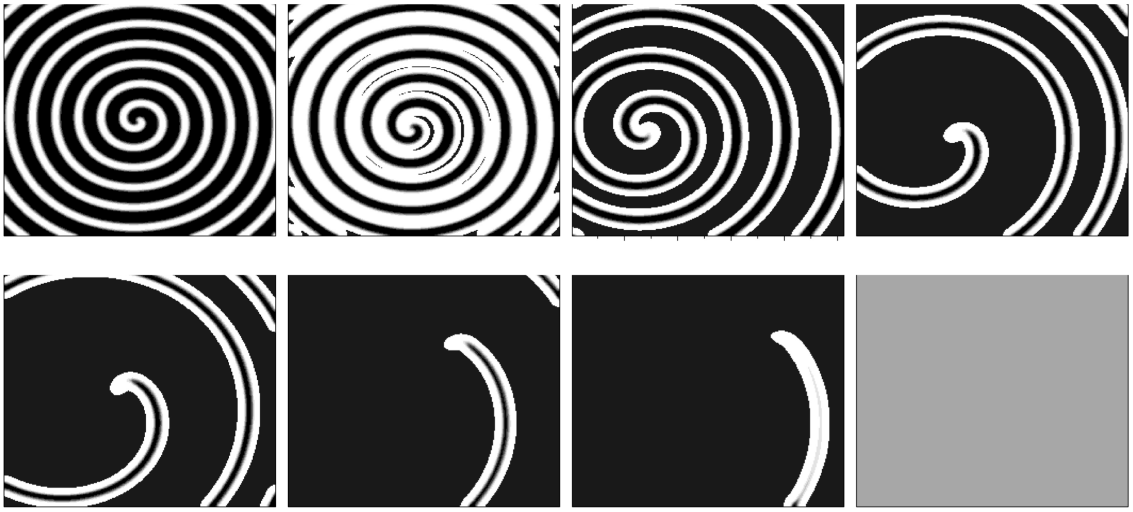


Fig.2. Evolution of stable rotating spiral wave, a=0.84, ε=0.04, ω=1.178, T=5.334, k=0.007056, b=0.07+k int(t/T) for t=0, 100, 150, 170, 175, 185, 190, 200 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

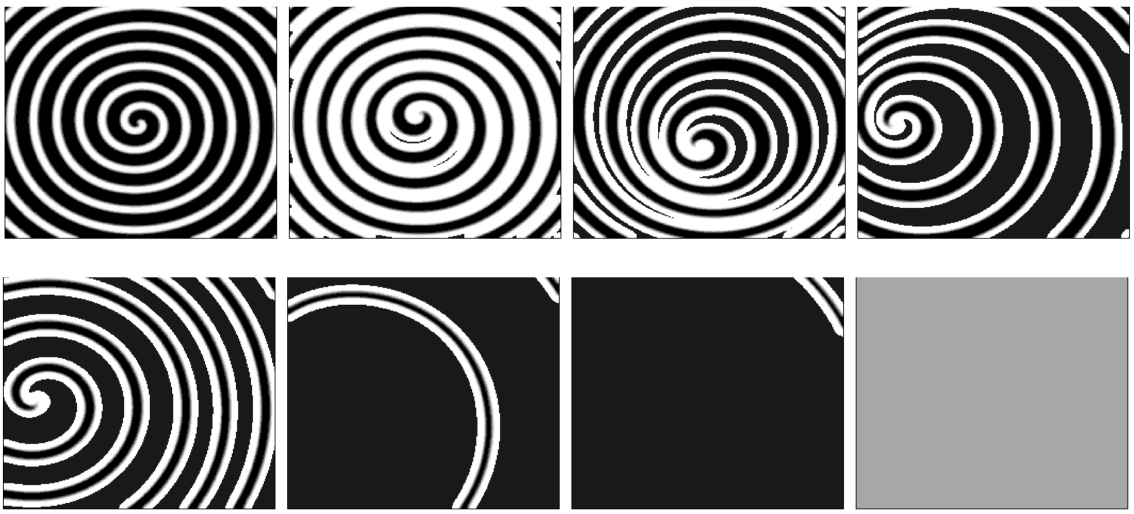


Fig.3. Evolution of stable rotating spiral wave, a=0.84, ε=0.04, ω=1.178, T=5.334, k=0.007056, b=0.07+k int($t/2T$) for t=0, 200, 250, 300, 350, 400, 415, 425 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

Compared with the above numerical results, we can conclude that smaller pulse interval can cause a shorter transient period to reach homogeneous state. The results in Fig.3 prove that bigger pulse interval will need a longer transient period before the tip of the spiral wave drifts to the border, the results in Fig.4 show that smaller pulse interval and stronger intensity of stimulation can induce breakup and instability of spiral wave before the whole media become homogeneous finally. The results in Fig.5 show that weaker intensity of pulse cannot induce breakup of spiral wave or instability before the whole media become homogeneous when the interval of pulse is not too small.

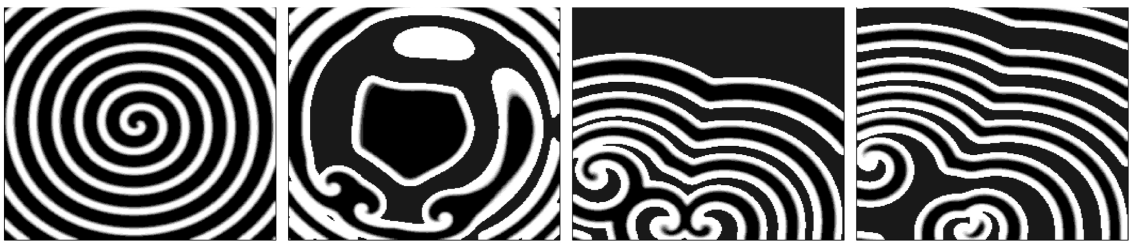




Fig.4. Evolution of stable rotating spiral wave, a=0.84, ε=0.04, ω=1.178, T=5.334, k=0.007056, b=0.07+k int(t/0.5T) for t=0, 10, 3 0, 40, 55, 65, 70, 75 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

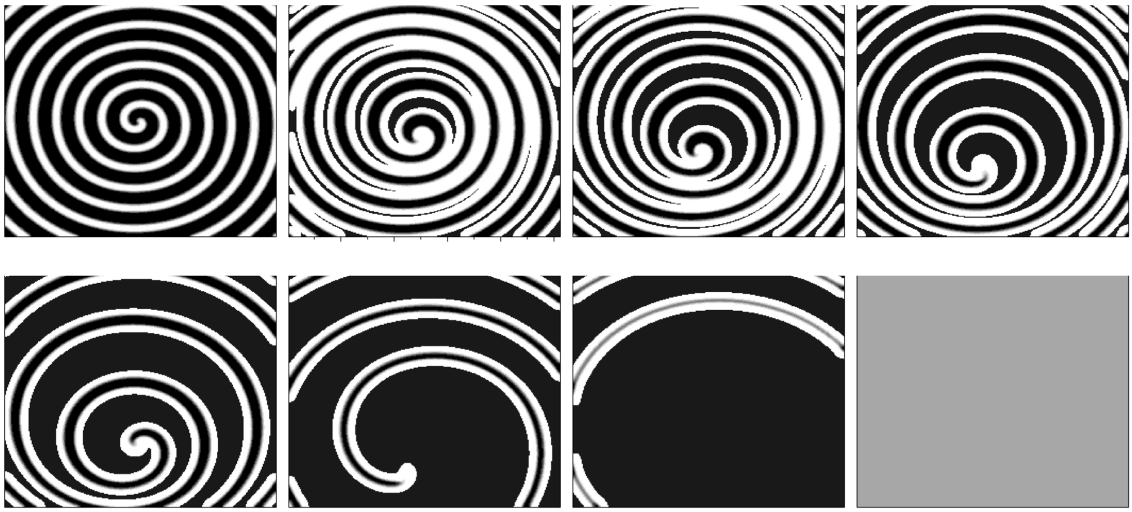


Fig.5. Evolution of stable rotating spiral wave, a=0.84, ε=0.04, ω=1.178, T=5.334, k=0.005292, b=0.07+k int(t/0.5T) for t=0, 50, 60, 70, 80, 100, 110, 115 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

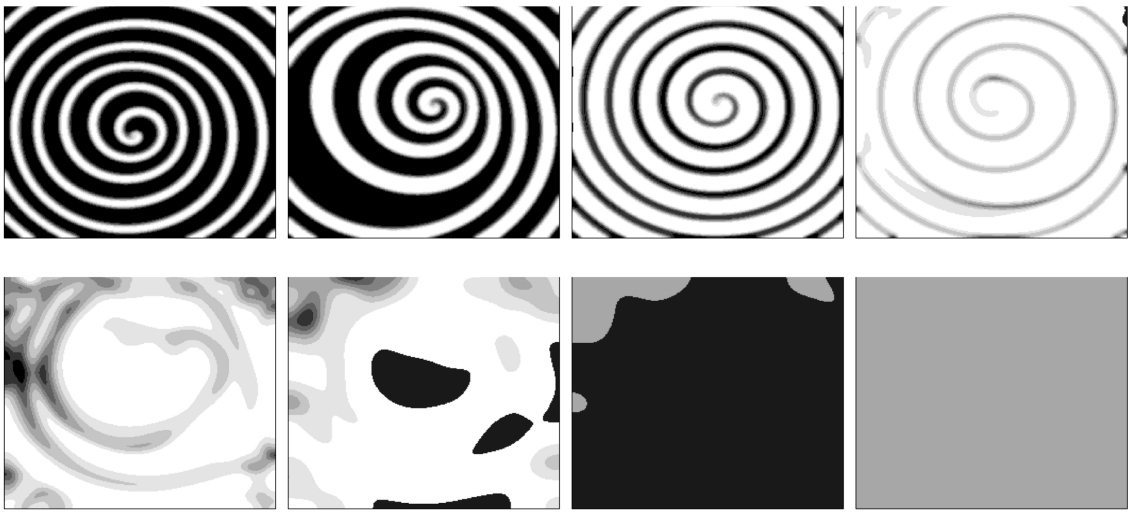


Fig.6. Evolution of meandering spiral wave, a=0.84, ε=0.066, ω=0.82736, T=7.5942, k=0.007056, b=0.07+k int(t/T) for t=0, 50, 150, 210, 230, 240, 245, 250 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

It is confirmed that the meandering spiral wave can be suppressed within about 250 time units, and the whole media become homogeneous. The meandering spiral wave begins to lose instability near the tip and then the media become homogeneous slowly. Now we will investigate its effectiveness on suppressing the spiral turbulence. The numerical results are plotted in Fig.7.

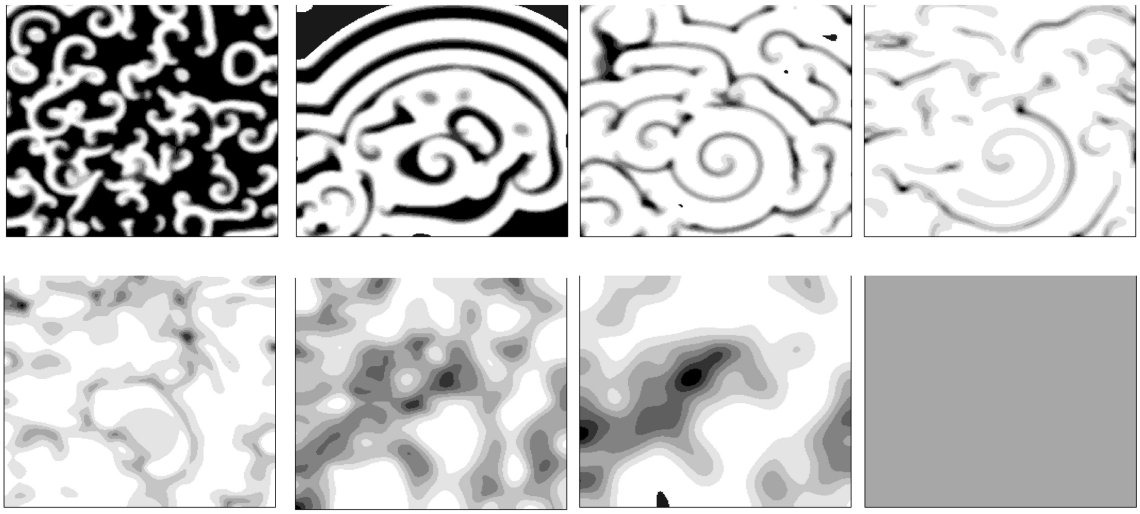


Fig.7. Evolution of spiral turbulence, a=0.84, ε=0.075, ω=0.74686, T=8.4128, k=0.007056, b=0.07+k int(t/T) for t=0, 50, 100, 130, 150, 160, 170, 180 time units respectively (the order is left to right, up to down, the snapshot is shown in grey).

The numerical simulation results confirm that the scheme is powerful to eliminate the spiral turbulence within about 200 time units. Furthermore, the transient period to reach homogeneous state proves to be shorter when the stimulating intensity (coefficient k) increases or the smaller pulse interval is imposed. Furthermore, other parameters region which can generate stable rotating spiral wave, meandering spiral wave and spiral turbulence or spatiotemporal chaos are checked under this scheme by selecting different pulse interval and intensity. It is also confirmed that the scheme in this paper is effective to suppress the stable rotating wave, meandering spiral wave and spiral turbulence as well. To our knowledge, it is the perturbation on the controllable parameter that the mutual coupling between the cells in the media can change greatly, thus the self-organization in the media is affected. On the other hand, it is the parameters region that determine the pattern formation and transition; the spiral wave and spiral turbulence cannot survive or resume its original state when the next pulse of parameter perturbation is imposed on the media. Therefore, the pattern transition is induced and new patterns are developed and formed.

4. Conclusion

In this paper, we propose a scheme of parameter perturbation to suppress and remove the spiral wave and turbulence in the excitable media by adjusting the controllable parameter with a weak pulse. The scheme is used to control the patterns in the MFHN model that can generate stable rotating spiral wave, meandering spiral wave and spiral turbulence within appropriate parameters. It is confirmed that the scheme is effective to eliminate the stable rotating spiral wave, meandering spiral wave and the spiral turbulence as well. A smaller pulse interval and/or stronger intensity of pulse will cost a shorter transient period for the media to reach homogeneous state. Instability and drift of spiral wave can be observed before the whole media become homogeneous. We hope that this scheme can be used to investigate other real surface reaction system, which contains some controllable parameter or variables.

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