Vol 17 No 8, August 2008
1674-1056/2008/17(08)/2829-08

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Circuit implementation of a new hyperchaos in fractional-order system

Liu Chong-Xin(刘崇新) and Liu Ling(刘 凌)

School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, *China*

State Key Laboratory of Electrical Insulation and Power Equipment, Xi'an 710049, *China*

(Received 9 January 2008; revised manuscript received 24 February 2008)

This paper introduces a new four-dimensional (4D) hyperchaotic system, which has only two quadratic nonlinearity parameters but with a complex topological structure. Some complicated dynamical properties are then investigated in detail by using bifurcations, Poincaré mapping, LE spectra. Furthermore, a simple fourth-order electronic circuit is designed for hardware implementation of the 4D hyperchaotic attractors. In particular, a remarkable fractional-order circuit diagram is designed for physically verifying the hyperchaotic attractors existing not only in the integer-order system but also in the fractional-order system with an order as low as 3.6.

Keywords: hyperchaotic system, fractional-order system, integer-order chaotic circuit, fractional-order circuit

PACC: 0545, 4265

1. Introduction

Recently, hyperchaotic systems with more than one positive Lyapunov exponent have been extensively studied due to its potential extensive application in mathematics, physics and engineering, and so on.[1−9] This paper reports on the finding of a new continuous time, four-dimensional (4D) autonomous hyperchaotic system, which exhibits more complex and abundant hyperchaotic dynamics behaviours. According to the detailed theoretical analysis and numerical simulations, some basic hyperchaotic dynamical properties, such as Lyapunov exponents, fractal dimensions, continuous spectrum, strange attractor and Poincaré mapping of the new hyperchaotic system are further investigated separately. Furthermore, the forming mechanism of its compound structure can be obtained by merging together two single-scroll attractors after performing one mirror operation. Based on circuit theory, an integer-order electronic circuit diagram is designed for physically realizing the new hyperchaotic system. In particular, the hyperchaotic attractors can be generated by a novel fractional-order electronic circuit.[10−16] Experimental results prove that hyperchaos exists not only in this integer-order 4D autonomous system but also in the corresponding fractional-order nonlinear dynamical system with an order as low as 3.6.[17,18]

It is believed that the application of the hyperchaotic systems will enhance the security of communication scheme in information processing and image manipulation.

2. The new 4D hyperchaotic system and its circuit design

Consider the following 4D quadratic autonomous system, which evolved from the proposed reverse structure form of the Liu system,[5]

$$\begin{aligned} \dot{x} &= a(y-x)+ew, \\ \dot{y} &= bx+kxz-mw, \\ \dot{z} &= -cz-hx^2, \\ \dot{w} &= \mathrm{d}x, \end{aligned} \tag{1}$$

where x, y, z, w are state variables and a, b, c, d, e, k, h, m are positive constant parameters. This system is found to be hyperchaotic for parameters a=10, b=40, $c = d$=2.5, $e = 1$, h=4, k=1, m=1.

2.1. Dynamical behaviours of the 4D hyperchaotic system

It is known that the characteristic of equilibria greatly influence the nonlinear dynamics of the system. We find that system (1) has a unique real equilibrium, which is described as $O(0, 0, 0, 0)$. These eigenvalues that correspond to equilibrium $O(0, 0, 0, 0)$ are

respectively obtained as $\lambda_1 = -25.6996$, $\lambda_2 = -2.5$, $\lambda_3 = 15.6374$, $\lambda_4 = 0.0622$. Here λ_1 and λ_2 are two negative real roots; while λ_3 and λ_4 are two positive real roots. Therefore, the equilibrium $O(0, 0, 0, 0)$ is an unstable saddle in this 4D autonomous nonlinear system.

When initial values of the system (1) are appointed to be (2.4, 2.2, 0.8, 0), hyperchaotic strange attractors are shown in Fig.1.

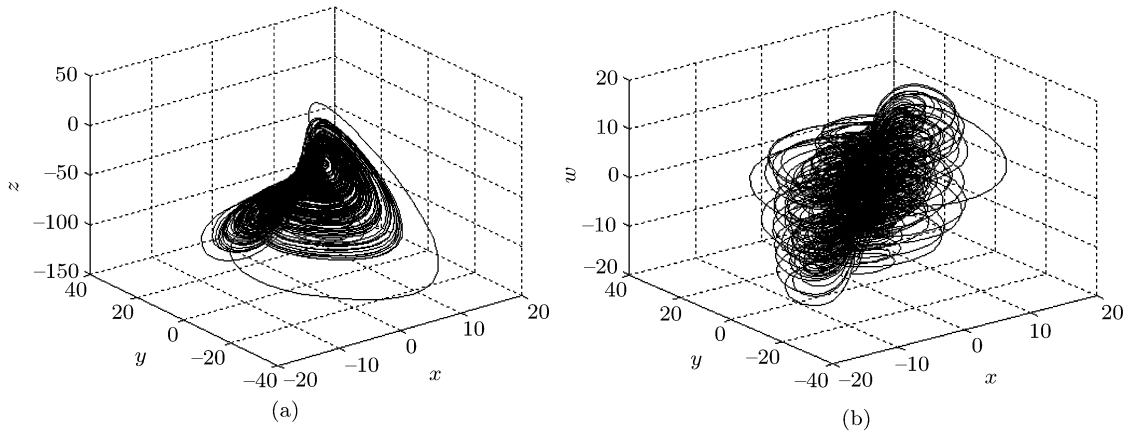


Fig.1. 3D view of the system (1) on (a) $x - y - z$ space and (b) $x - y - w$ space.

The waveforms of $x(t)$ in time domain are shown in Fig.2; apparently, the waveforms of $x(t)$ are non-periodic in the system (1). Spectrum of the system (1) is continuous as shown in Fig.3.

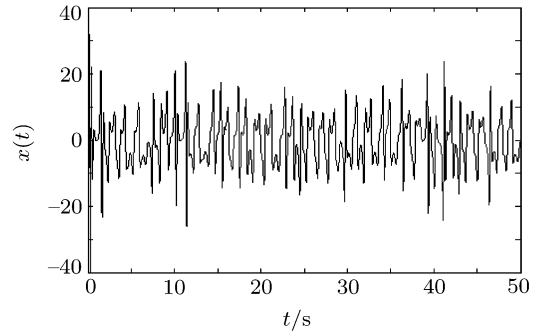


Fig.2. $x(t)$ waveform of the system (1).

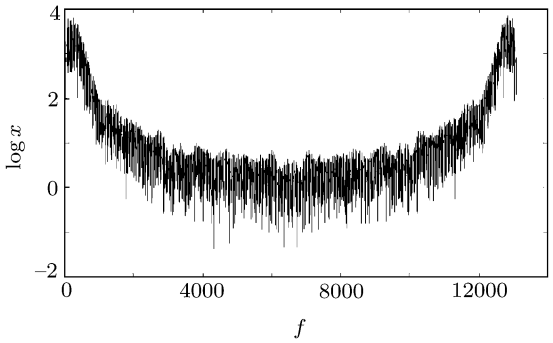


Fig.3. Spectrum of $|x|$ in the system (1).

The Poincaré mapping and bifurcation diagram of this nonlinear system are shown in Figs.4 and 5, respectively. It can be seen that the Poincaré mapping are the points of the confusion. The bifurcation diagram of x with increasing a demonstrates that the system can generate complex dynamics within a wide range of parameters with more complex dynamical behaviours.

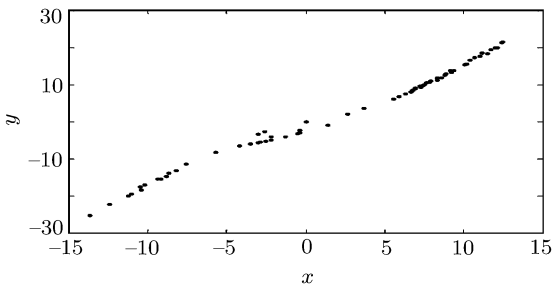


Fig.4. Poincaré mapping on $x - y$ plane of the system (1).

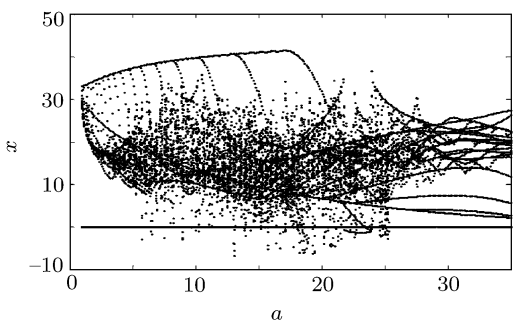


Fig.5. Bifurcation diagram for increasing a with b=40, $c = d$=2.5, e=1, h=4, k=1, m=1.

As is well known, the Lyapunov exponents (LEs) measure the exponential rates of divergence and convergence of nearby trajectories in phase space of the system (1). Hyperchaotic systems have multiple positive exponents in the Lyapunov spectrum of their data series. The two positive Lyapunov exponents of system (1) are obtained as $\mathrm{LE}_1 = 1.7213$ and $\mathrm{LE}_2 = 0.0216$. Another Lyapunov exponent is $\mathrm{LE}_3 = 0$; the negative Lyapunov exponent is $\mathrm{LE}_4 = -14.2303$. A plot of the four Lyapunov exponents versus a, is obtained, as shown in Figs.6(a) and 6(b).

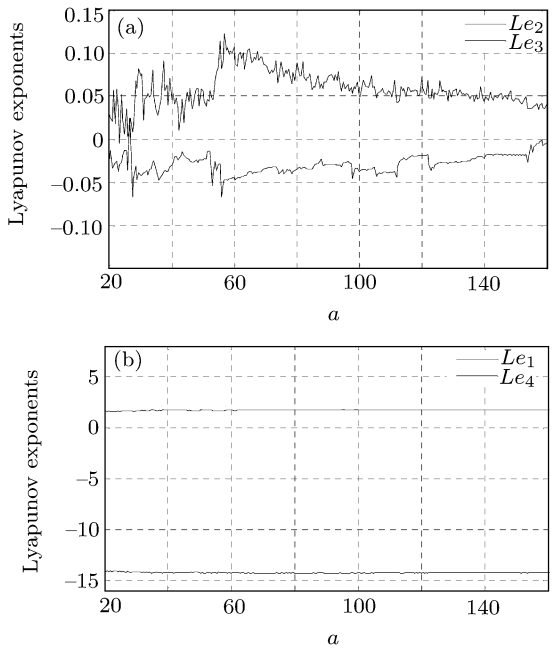


Fig.6. The LE spectra versus a with b=40, $c = d$=2.5, e=1, h=4, k=1, m=1.

The Lyapunov dimension of system (1) is

$$D_L = j + \frac{1}{|Le_{j+1}|}\sum_{i=1}^{j} Le_i = 3 + \frac{(Le_1 + Le_2 + Le_3)}{|Le_4|}$$
$$= 3 + \frac{1.7213 + 0.0216 + 0}{|-14.2303|} = 3.12.$$

For system (1), let Ω be a region in R^4 with smooth boundary $\partial\Omega$. In order to obtain $\Omega(t) = \{\Phi(t; x_0) : x_0 \in \Omega\}$, let $\Omega(t)$ be the region formed by flowing along for time t. Then, we let $V(t)$ be the volumes of $\Omega(t)$, and obtain $V(t) = V(0)\mathrm{e}^{(-a+c)t}$.

Apparently, the volume $V(t)$ will be decreased exponentially fast. If defining the basin of hyperattraction of a closed invariant set A with finite volume, then, the invariant set we see for the reverse structure hyperchaotic system (1), has zero volume. For system (1), the divergence is given by $\dot{V}(t) = \int_{\Omega(t)} \nabla \cdot F \mathrm{d}V$. Thus, we have

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a + c = -\beta.$$

Since $\nabla \cdot V = -\beta = -12.5$, its value is a negative constant; and the system (1) is a dissipative one. The invariant set A is contained in the region $\Omega(t)$ of a finite volume. Therefore, each volume containing this system trajectory shrinks to zero as $t \to \infty$ at an exponential rate $V(0)\mathrm{e}^{-\beta t}$. Thus, all orbits of system (1) are ultimately confined to a subset of zero volume, it is apparent that this nonlinear system asymptotic motion settles finally on to an attractor. According to the afore-mentioned analysis, apparently, it is a new hyperchaotic attractor. The sensitive dependence on the initial conditions is a prominent characteristic of dynamical behaviours, when the appointed initial values are changed; and the chaotic behaviour of this system will disappear soon.

2.2. Forming mechanism of the 4D hyperchaotic system (1)

In order to reveal the forming mechanism of this 4D hyperchaotic system, a controlled system has been constructed as follows:

$$\begin{aligned} \dot{x} &= a(y - x) + ew, \\ \dot{y} &= bx + kxz - mw, \\ \dot{z} &= -hx^2 - bz, \\ \dot{w} &= \mathrm{d}x + u, \end{aligned} \tag{2}$$

where u is a designed control parameter whose value can be changed within a certain range. When u is changed, the hyperchaotic behaviour of this system can effectively be controlled. We choose initial values as $(2.4, 2.2, 0.8, 0)$ to simulate the controller system.

(1) Let $u = 5$, the hyperchaotic attractor is shown in Fig.7(a). Moreover the hyperchaotic attractor which evolves into a partial one is still bounded at this time.

(2) Let $u = 9$, the hyperchaotic attractor is shown in Fig.7(b). Moreover the hyperchaotic attractor is evolved into a single left scroll hyperchaotic attractor and is only one half of the original hyperchaotic attractor at this time.

(3) Let $u = 9.25$, the corresponding hyperchaotic attractor which evolves into the period-doubling bifurcations is shown in Fig.7(c).

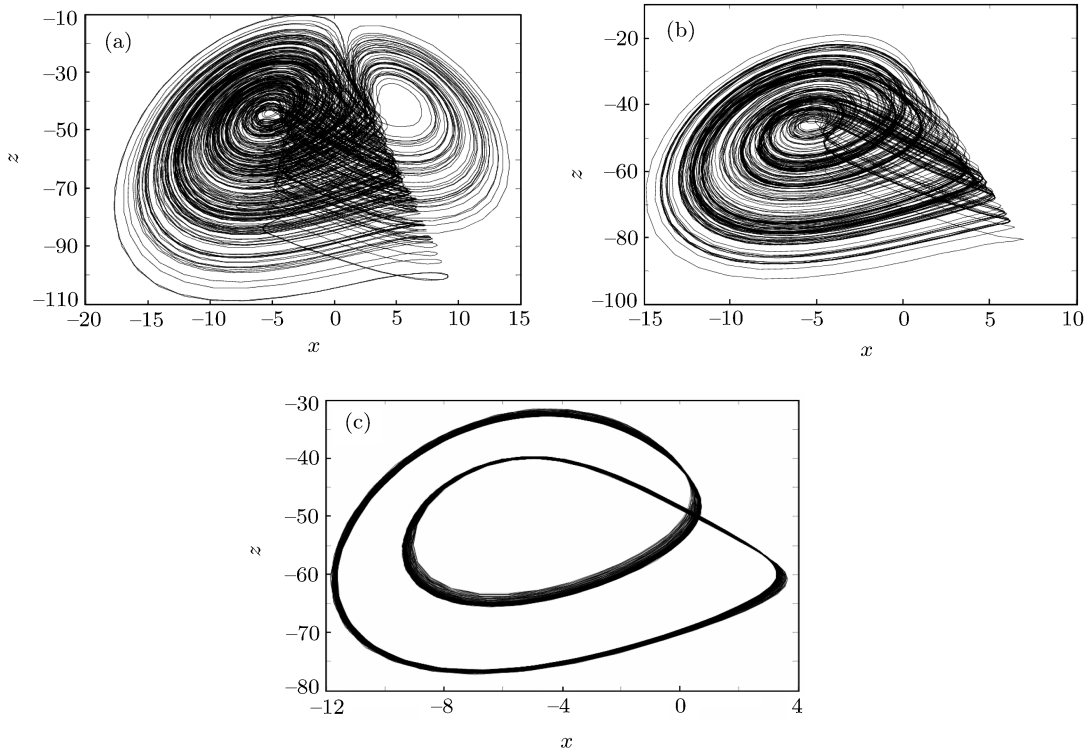


Fig.7. Phase portrait of hyperchaotic attractors on $x-z$ plane: (a) u=5, (b) u=9, (c) u=9.25.

When u has a negative value, the hyperchaotic behaviour of this system can be also affected.

(1) Let $u=-5$, the hyperchaotic attractor which also evolves into a partial one and is still bounded is shown as in Fig.8(a).

(2) Let $u=-9$, the hyperchaotic attractor is shown in Fig.8(b). Moreover, the hyperchaotic attractor evolves into a single right scroll hyperchaotic attractor; but it is only one half of the original hyperchaotic attractors at this time.

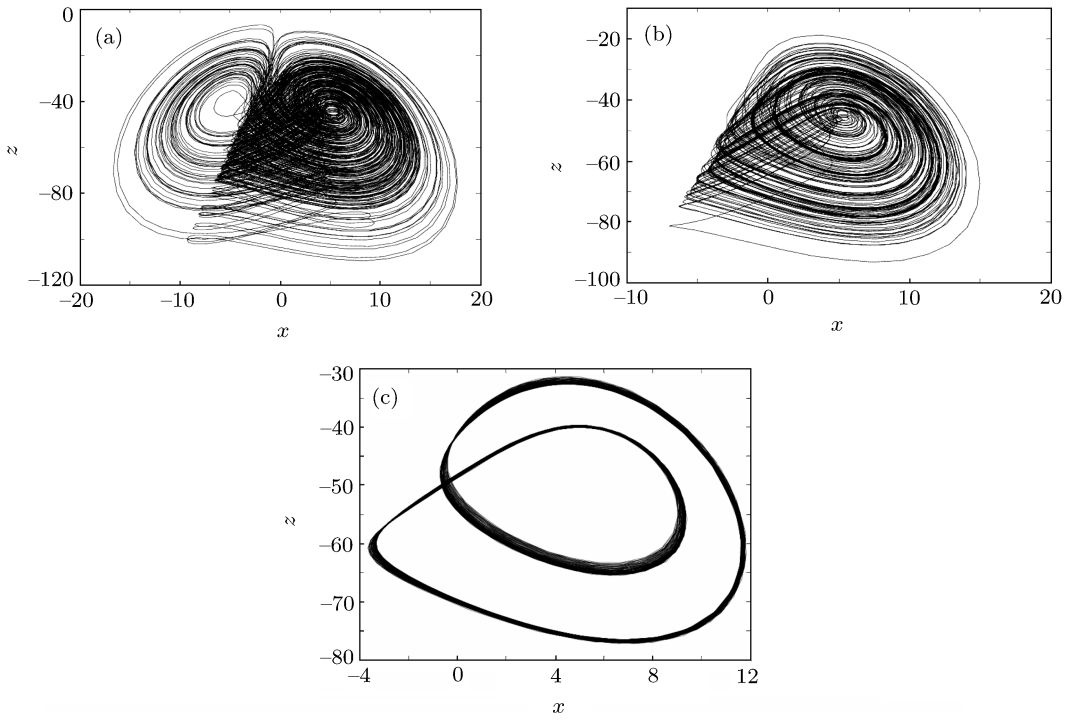


Fig.8. Phase portrait of hyperchaotic attractors on $x-z$ plane: (a) $u=-5$, (b) $u=-9$, (c) $u=-9.25$.

(3) Let $u = -9.25$, the corresponding hyperchaotic attractor which evolves into the period-doubling bifurcations is shown in Fig.8(c).

In the controller, one can see when $|u|$ is large enough, the hyperchaotic attractor disappears; when $|u|$ is small enough, a complete hyperchaotic attractor appears. So $|u|$ is an important parameter for controlling hyperchaos in this system. This means that this hyperchaotic attractor is also a compound structure obtained by merging together two single scroll attractors after performing one mirror operation.

2.3. Circuit implementation of the 4D hyperchaotic attractors

In this section, a fourth-order electronic circuit is designed to realize the previous hyperchaotic system (1). Figure 9 shows the diagram of the circuit which comprises linear resistors, linear capacitors, operational amplifiers (LM741 chip) and analogue multipliers (AD633 chip).In this electronic circuit the four state variables x, y, z, and w are obtained from the terminal outputs of u_{C_1}, u_{C_2}, u_{C_3} and u_{C_4}, respectively. Considering the error of circuit experiment, we select the circuit parameter as resistance: R_1, R_3, R_5,R_9, R_{12}, R_{14}, R_{15}, R_{17}, $R_{18} = 10\,\mathrm{k\Omega}$; R_2, $R_6 = 20\,\mathrm{k\Omega}$; $R_4 = 100\,\mathrm{k\Omega}$; $R_7 = 8\,\mathrm{k\Omega}$; R_8, $R_{16} = 40\,\mathrm{k\Omega}$; R_{10}, $R_{13} = 100\,\mathrm{k\Omega}$; R_{19}, R_{20}, $R_{21} = 800\,\mathrm{k\Omega}$; $R_{11} = 1\,\mathrm{k\Omega}$. Capacitance: C_1, C_2, C_3, $C_4 = 1\,\mu\mathrm{F}$. Figure 10 shows the experimental phase portraits observation of the 4D hyperchaotic attractor. The circuit experiment shows results in agreement with numerical simulations.

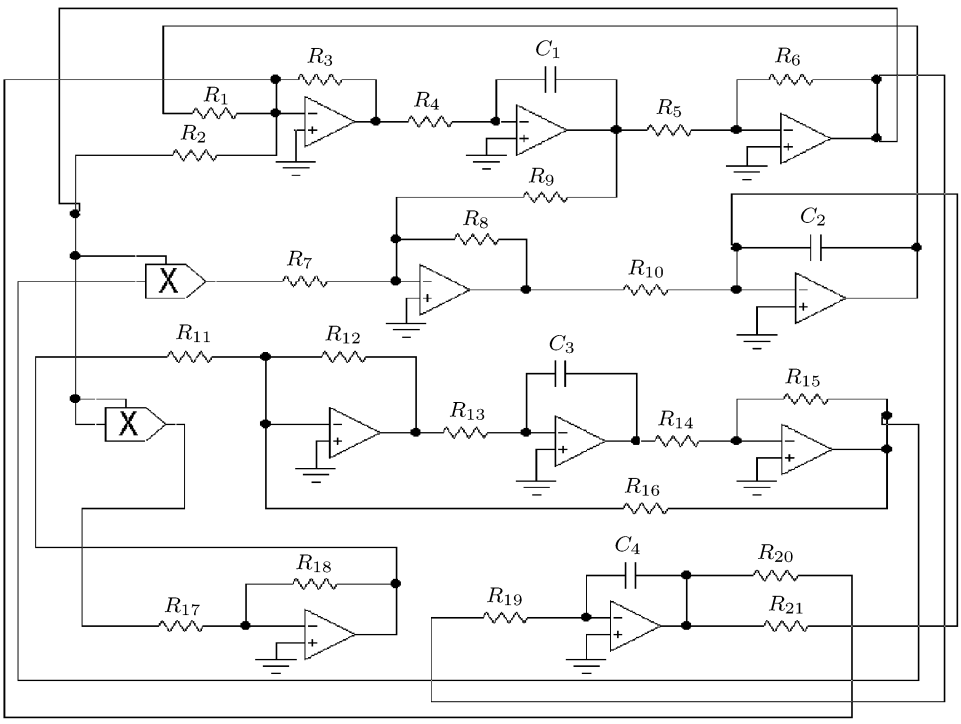
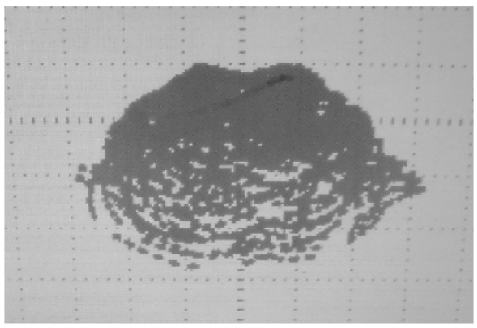
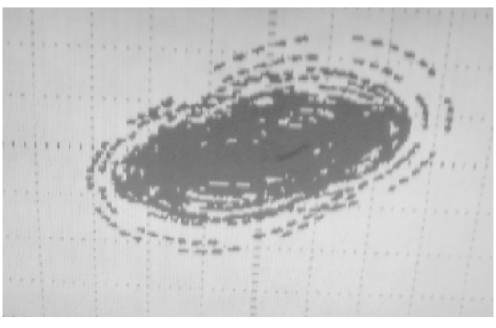


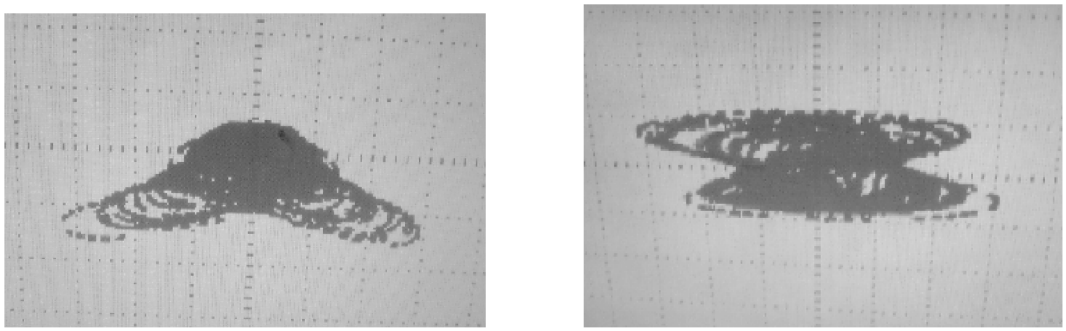
Fig.9. Circuit diagram for implementing the 4D hyperchaotic system (1).



(a)



(b)



(c) (d)

Fig.10. Experimental observations of system (1): (a) $x-z$ plane (1V/div, 1V/div); (b) $x-y$ plane (1V/div, 2V/div); (c) $y-z$ plane (2V/div, 2V/div); (d) $x-z$ plane (1V/div, 1V/div).

3. Circuit implementation of the hyperchaos in a fractional-order system

Recently, it has been found that many systems in physics and engineering can be described by the fractional differential equations; and nowadays, fractional-order system have attracted more and more attention. It is known that many fractional-order differential dynamical systems behave chaotically, such as , the fractional Lorenz system, the fractional Duffing system, the fractional Chua circuit, the fractional Chen system, the fractional hyperchaotic Rössler system, the fractional Liu chaotic system and the fractional unified chaotic system.

There are several definitions of fractional derivatives. Perhaps the best known is the Riemann-Liouville definition, which is given by

$$\frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^{n}}{\mathrm{d}t^{n}} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau, \quad (3)$$

where $\Gamma(\cdot)$ is the gamma function with $n-1 \leq \alpha \leq n$. Considering all the initial values to be zero, the Laplace transform of the Riemann-Liouville fractional derivative is written as

$$L\left\{\frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}}\right\} = s^{\alpha} L\left\{f(t)\right\}. \quad (4)$$

Thus, the fractional integral operator of order "α" can be represented by the transfer function $F(s) = 1/s^{\alpha}$ in the frequency domain. As is well known, the standard definition of fractional differ-integral does not allow direct implementation of the fractional operators in time-domain simulations. An efficient method of solving this problem is to approximate fractional operators by using standard integer-order operators. In Ref.[17], an effective algorithm was developed to approximate fractional-order transfer functions. In Table 1 of Ref.[18], approximations for $1/s^{\alpha}$ with $\alpha = 0.1 - 0.9$ in step size 0.1 were given with errors of approximately 2dB. We will apply these approximations to realize the fractional-order hyperchaotic system (1) by using a circuit.

In order to generate attractors from fractional-order hyperchaotic system (1), the standard differential equations of system (1) are replaced by a fractional derivative as follows:

$$\begin{aligned} \frac{\mathrm{d}^{\alpha} x}{\mathrm{d}t^{\alpha}} &= 10(y-x)+w, \\ \frac{\mathrm{d}^{\alpha} y}{\mathrm{d}t^{\alpha}} &= 40x + xz - w, \\ \frac{\mathrm{d}^{\alpha} z}{\mathrm{d}t^{\alpha}} &= -2.5z - 4x^{2}, \\ \frac{d^{\alpha} w}{dt^{\alpha}} &= 2.5x. \end{aligned} \quad (5)$$

Here, we choose $\alpha = 0.9$ to design the circuit for system (5). From Table 1 of Ref.[18], we have an approximation of $1/s^{0.9}$ with an error of about 2 dB as follows:

$$\frac{1}{S^{0.9}} \approx \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)}. \quad (6)$$

We utilize the circuit diagram shown in Fig.11 to implement the function of Eq.(6) in the Laplace domain.

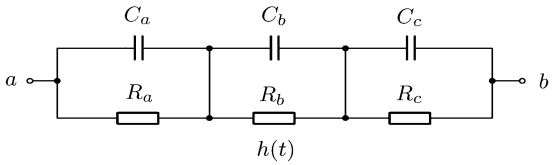


Fig.11. Circuit diagram of fractional-order $1/S^{0.9}$.

Thus, we can obtain the transfer function $H(s)$ between a and b as follows:

$$H(s) = \frac{1/C_a}{s+1/R_aC_a} + \frac{1/C_b}{s+1/R_bC_b} + \frac{1/C_c}{s+1/R_cC_c}$$
$$= \frac{\left(\frac{C_0}{C_a}+\frac{C_0}{C_b}+\frac{C_0}{C_c}\right)\left[s^2 + \frac{\left(\frac{C_b+C_c}{R_a}+\frac{C_a+C_c}{R_b}+\frac{C_a+C_b}{R_c}\right)s + \frac{(R_a+R_b+R_c)}{R_aR_bR_c}}{(C_aC_b+C_bC_c+C_aC_c)}\right]}{(s+1/R_aC_a)(s+1/R_bC_b)(s+1/R_cC_c)} = \frac{1}{C_0}. \quad (7)$$

Here C_0 is a unit parameter. Let $C_0 = 1\mu$F and $H(s) = F(s)/C_0$. Then let $F(s) = 1/s^{0.9}$; comparing Eq.(6) with Eq.(7) and we can work out the values of the resistances and capacitances as R_a=62.84MΩ, R_b=250kΩ, R_c=2.5kΩ; C_a=1.232μF, C_b=1.835μF, C_c=1.1μF. Then, we design the circuit of fractional-order system (5) with $\alpha = 0.9$, as shown in Fig.12. Here, the multiplier is still AD633 chip and the operation amplifier is still LM741 chip. For resistance, we have R_1, R_3, R_5, R_9, R_{12}, R_{14}, R_{15}, R_{17}, R_{18} = 10 kΩ; R_2, R_6 = 20 kΩ; R_4 = 100 kΩ; R_7 = 8 kΩ; R_8, R_{16} = 40 kΩ; R_{11} = 1 kΩ; R_{10}, R_{13} = 100 kΩ; R_{19}, R_{20}, R_{21} = 800 kΩ. The experimental results show that hyperchaos indeed exists in this fractional-order system with an order as low as 3.6. The circuit experiment phase portraits are shown in Fig.13. Apparently, there are some difference between the circuit experiment phase portraits of the fractional-order and the integer systems as a result of non-idealization factor.

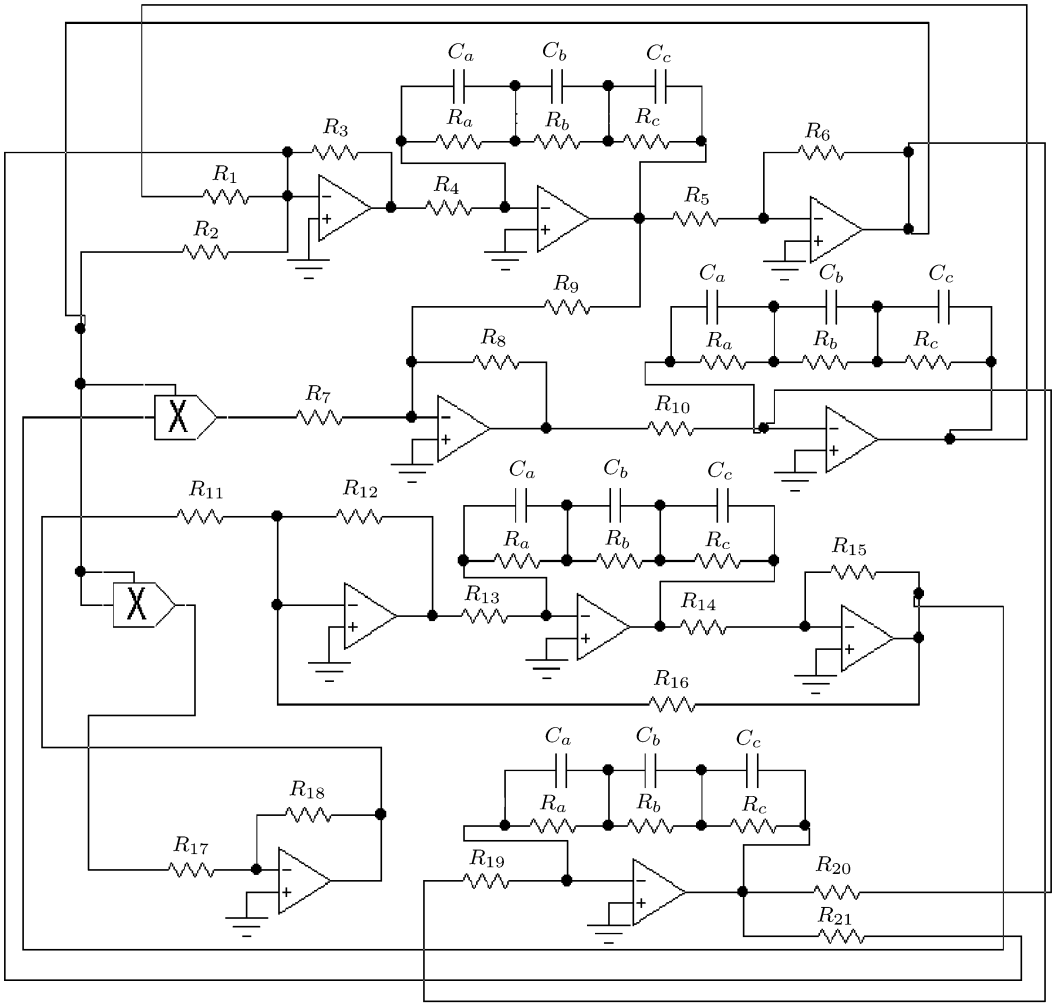


Fig.12. Fractional-order circuit for implementing the 3.6-order hyperchaotic system.

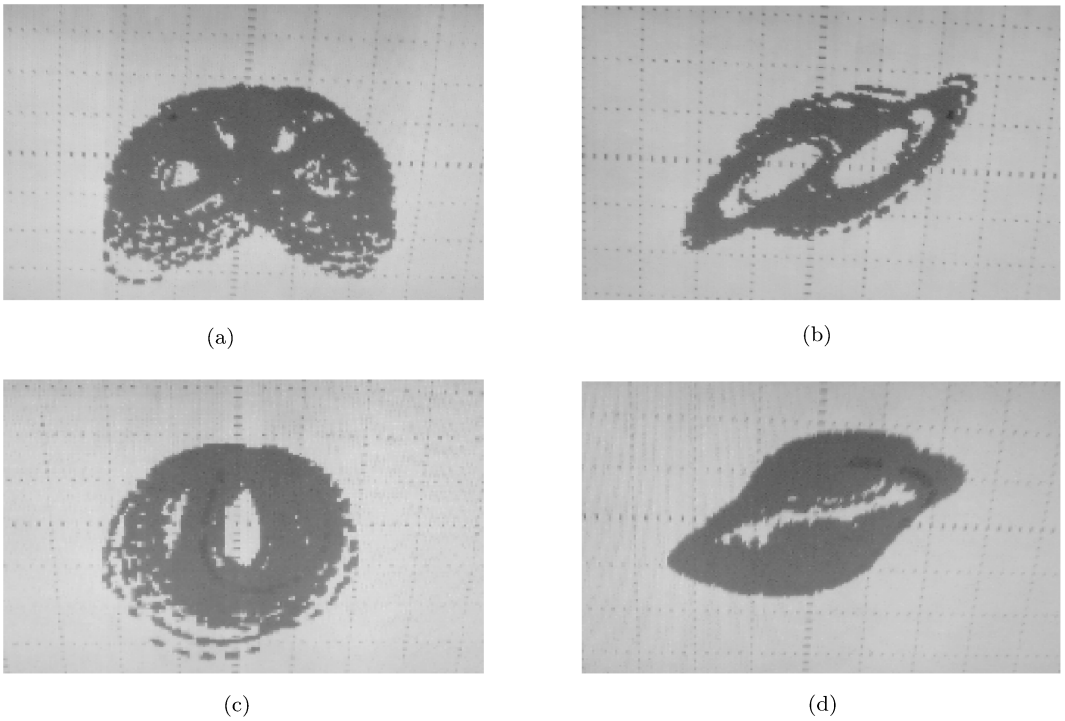


Fig.13. Experimental observations of system (5): (a) $x-z$ plane (1V/div, 1V/div); (b) $x-y$ plane (1V/div, 1V/div); (c) $y-z$ plane (1V/div, 0.5V/div); (d) $x-w$ plane (1V/div, 0.5V/div).

4. Conclusion

According to the detailed theoretical analysis, the system (1) is a novel reverse structure hyperchaotic system evolving from a proposed reverse 3-D chaotic Liu system. This nonlinear system has been confirmed including richer hyperchaotic dynamics and sophisticated topological structure. Furthermore, an integer-order circuit diagram and a fractional-order circuit diagram have been designed for hardware implementation of the new hyperchaotic attractors in integer-order and fractional-order systems. It should be pointed out that there are abundant and complex dynamical behaviours still unknown in these new reverse structure hyperchaotic systems. It is expected that various complex dynamic behaviours of this 4D hyperchaotic system will be further investigated in the near future. We believe that both of the circuits have potential applications in security communication, image manipulation and so forth.

References

[1] Rössler O E 1979 *Phys. Lett.* A **71** 155
[2] Lü J and Chen G 2002 *Int. J. Bifurcat. Chaos.* **12** 659
[3] Liu C X, Liu T, Liu L and Liu K 2004 *Chaos, Solitons and Fractals* **22** 1031
[4] Chen Z Q, Yang Y, Qi G Y and Yuan Z Z 2006 *Phys. Lett.* A **8** 85
[5] Liu C X 2007 *Chin. Phys* **16** 3279
[6] Liu L, Su Y C and Liu C X 2007 *Chin. Phys.* **16** 1897
[7] Liu L, Liu C X and Zhang Y B 2007 *Chin. Phys. Lett.* **24** 2756
[8] Liu C X 2007 *Acta Phys. Sin.* **56** 6865 (in Chinese)
[9] Podlubny I 1999 (New York: Academic)
[10] Hartley T T, Lorenzo C F and Qammer H K 1995 *IEEE Trans CAS-I* **42** 485
[11] Wu Z M, Lu J G and Xie J Y 2006 *Chin. Phys.* **15** 1201
[12] Wang X Y and Wang M J 2007 *Chaos* **17** 33106
[13] Ahmad W M and Harb W M 2003 *Chaos, Solitons and Fractals* **18** 693
[14] Li C G and Chen G 2004 *Physica* A **341** 55
[15] Wang FQ and Liu C X 2006 *Acta Phys. Sin.* **55** 3922 (in Chinese)
[16] Lu J J and Liu C X 2007 *Chin. Phys* **16** 1586
[17] Charef A, Sun H H, Tsao Y Y and Onaral B 1992 *IEEE Trans. Auto. Contr.* **37** 9
[18] Ahmad W M and Sprott J C 2003 *Chaos, Solitons and Fractals* **16** 339