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Modified de Broglie–Bohm approach to the Schwarzschild black hole*

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A modified de Broglie–Bohm approach is generalized to the Schwarzschild black hole. By using this method, the quantum potential and the quantum trajectories of the black hole are investigated. And we find that the linear combination of two particular solutions of the black hole wavefunction is not physical although each of them is physical, if we think that the quantum gravity should reduce into its corresponding classical counterpart in which the gravity vanishes. It seems to confirm the argument, given by Alwis and MacIntire, that a possible resolution on the quantum gravity is to give up the superposition principle.

Keywords: modified de Broglie–Bohm interpretation, black hole, quantum trajectory, quantum potential

PACC: 0460, 0470

1. Introduction

Quantum gravity is one of the most attractive subjects in Physics. There has been made a great effort to investigate the subject. The canonical formalism of gravity, as a theory of quantum gravities, has been formulated by Arnowitt *et al*[1] and also by Dirac.[2] In this formalism the Hamiltonian constraint leads to the well-known Wheeler–DeWitt (WD) equation.[3,4] However it is very difficult to solve the WD equation when the spacetime is inhomogeneous. In Refs.[5,6], they solved simultaneously the WD equation and the mass operator equation in a minisuperspace and obtained the wavefunction of the Schwarzschild black hole. In Ref.[5] Nakamura *et al* dealt with the wavefunction by using Wentzel–Kramers–Brillouin (WKB) approximate approach. But, in Ref.[6], Kenmoku *et al* introduced de Broglie–Bohm (dBB) interpretation into the wavefunction of the quantum black hole, and studied the quantum trajectories, quantum potential and the light ray on the quantum geometry. The latter method has been applied to the study of other black holes.[7−9]

The dBB interpretation provides a deterministic theory of motion of a quantum system.[10,11] In this approach, the wavefunction of a quantum system is expressed as $\Psi = R\mathrm{e}^{\mathrm{i}S/\hbar}$, where R and S are two real functions. However, this scheme has some difficulties. For example, for bound state problems, the time-independent part of the wavefunction is real, hence the speed of the particle turns out to be zero everywhere.[12] In Ref.[13], John provides a modified dBB approach by changing the expression $\Psi = R\mathrm{e}^{\mathrm{i}S/\hbar}$ into a new form $\Psi = N\mathrm{e}^{\mathrm{i}S/\hbar}$, where N is a normalization constant. Converting the new form of the wavefunction into the Schrödinger equation for a particle, one obtains a quantum-mechanical Hamilton–Jacobi (QMHJ) equation (i.e. Eq.(25)). John considered the case of $S = W - Et$, where W is the Hamilton characteristic function and E is the energy of the particle, and obtained the energy $E = \frac{1}{2}m\dot{x} + V + \frac{\hbar}{2\mathrm{i}}\frac{\partial \dot{x}}{\partial x}$, where V is the classical potential. So $V_q = \frac{\hbar}{2\mathrm{i}}\frac{\partial \dot{x}}{\partial x}$ is nothing but the quantum potential. We find that the quantum potential is the minus-signed term on the right-hand side of QMHJ equation (25). So, in general, the quantum potential can be extracted from the QMHJ equation. For quantum black hole the WD equation corresponds to the Schrödinger equation. Inserting $\Psi = N\mathrm{e}^{\mathrm{i}S/\hbar}$ into the WD equation, we have Eq.(31), which corresponds to the QMHJ equation. And from this equation we can extract the quantum potential of the quantum black hole. Although the modified dBB approach will bring some complex quantities, one can only take their real parts as the corresponding physical quantities.

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In this paper, we generalize the modified dBB approach to the Schwarzschild black hole. First, the wavefunction is given by solving WD equation and mass function simultaneously. Then by using the modified dBB approach, the quantum trajectoried and the quantum potential are obtained. We further investigate the behaviour of the quantum black hole in the asymptotic flat (where the gravitation vanishes) and find that the two particular solutions of the wavefunction can be reduced into their classical counterparts respectively, but their linear combination cannot. This is an interesting phenomenon, and it provides an evidence in favour of the point of view that a possible resolution is to give up the superposition principle for quantum gravity in Ref.[14].

The rest of the present paper is organized as follows. In Section 2, we first recapitulate the canonically quantum formalism of the Schwarzschild black hole. In Section 3, the modified dBB approach is applied to the black hole. Conclusions and discussion are given in Section 4. The units $G = c = 1$ are used in the paper.

2. Canonical quantum formalism in spherically symmetric spacetime

In this section we briefly give the canonical formalism for the Schwarzschild black hole according to the method in Ref.[6]. The vacuum Einstein action is

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-{}^{(4)}g}{}^{(4)}R, \tag{1}$$

where ${}^{(4)}R$ is the four-dimensional Riemann curvature scalar. The metric of the general spherically symmetric spacetime is

$$\mathrm{d}^2s = -u\left(r\right)\mathrm{d}t^2 + \frac{\alpha\left(r\right)^2}{u\left(r\right)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \tag{2}$$

where $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$ denotes the line element of a unit sphere. The Schwarzschild solution is the unique one of the vacuum Einstein equation for

$$\alpha\left(r\right) = 1, \quad \text{and} \quad u\left(r\right) = 1 - \frac{r_g}{r}. \tag{3}$$

We restrict our attention to the inside of the horizon, i.e. $0 < r < r_g = 2m$, where m denotes the mass of the black hole. In this case, $u\left(r\right) < 0$ and the roles of time and space coordinates are interchanged. And the interior metric can be denoted as

$$\mathrm{d}^2s = -\frac{\alpha\left(T\right)^2}{U\left(T\right)}\mathrm{d}T^2 + U\left(T\right)\mathrm{d}r^2 + V\left(T\right)\mathrm{d}\Omega^2, \tag{4}$$

where we have performed these transformations: $t \to R$, $r \to T$, $u\left(r\right) \to U\left(T\right)$ and $\alpha\left(r\right) \to \alpha\left(T\right)$. The classical solution of metric (4) is

$$\alpha = 1, U = -\left(1 - \frac{r_g}{V^{1/2}}\right), \quad \text{and} \quad V^{1/2} = T. \tag{5}$$

Substituting Eq.(4) into action (1), one can obtain the Arnowitt–Deser–Misner (ADM) hypersurface action

$$S_\Sigma = \int \mathrm{d}t \int \mathrm{d}r L, \tag{6}$$

where the Lagrangian is

$$L = \frac{1}{4}\left(-\frac{1}{\alpha}\left(\dot{U}\dot{V} + \frac{U\dot{V}^2}{2V}\right) + 2\alpha\right), \tag{7}$$

with dot denoting the first derivative with respect to time t. For convenience, two new variables z_+ and z_- are introduced

$$z_+ = U\sqrt{V} \quad \text{and} \quad z_- = \sqrt{V}. \tag{8}$$

Accordingly Lagrangian (7) changes into a simpler form

$$L = \frac{1}{2}\left(-\frac{1}{\alpha}\dot{z}_+\dot{z}_- + \alpha\right). \tag{9}$$

Three canonical momenta conjugated to z_+, z_- and α are

$$\Pi_+ \equiv \frac{\partial L}{\partial \dot{z}_+} = -\frac{1}{2\alpha}\dot{z}_-, \tag{10}$$

$$\Pi_- \equiv \frac{\partial L}{\partial \dot{z}_-} = -\frac{1}{2\alpha}\dot{z}_+, \tag{11}$$

$$\Pi_\alpha \equiv \frac{\partial L}{\partial \dot{\alpha}} = 0. \tag{12}$$

Here, the variable α plays a role of a lapse function because Lagrangian (9) does not include $\dot{\alpha}$. Its canonical conjugate momentum is zero and then provides a primary constraint. Using the Legendre transformation action (6) becomes

$$S_\Sigma = \int \mathrm{d}t \int \mathrm{d}r \alpha H, \tag{13}$$

where

$$H = -2\Pi_+\Pi_- - \frac{1}{2} \tag{14}$$

is the Hamiltonian, which is restricted to being weakly zero, i.e. $H \approx 0$, called the Hamiltonian constraint. On the other hand, following the Kuchar's method,[15] one can introduce a mass function M as a dynamical variable

$$M = 2\Pi_+ z_+ \Pi_+ + \frac{1}{2}z_-. \tag{15}$$

Now we start the process of the canonical quantization. In the Schrödinger representation, taking into account the operator ordering, the canonical momenta are expressed by operators as follows:

$$\hat{\Pi}_{+p} \equiv z_+^p \hat{\Pi}_+ z_+^{-p} = \hat{\Pi}_+ + \frac{\mathrm{i}\hbar p}{z_+}, \tag{16}$$

and

$$\begin{aligned} \hat{\Pi}_{-s} &\equiv (z_- - r_g)^s \hat{\Pi}_- (z_- - r_g)^{-s} \\ &= \hat{\Pi}_- + \frac{\mathrm{i}\hbar s}{z_- - r_g}, \end{aligned} \tag{17}$$

where

$$\hat{\Pi}_+ = -\mathrm{i}\hbar \frac{\partial}{\partial z_+}, \quad \hat{\Pi}_- = -\mathrm{i}\hbar \frac{\partial}{\partial z_-}, \tag{18}$$

and ordering parameters p and s are integers. According to the Dirac canonical quantization procedure, the Hamiltonian constraint ($H \approx 0$) becomes a operator equation as

$$\hat{H}\,\Psi = -\left(2\hat{\Pi}_{+r}\hat{\Pi}_{-s} + \frac{1}{2}\right)\Psi = 0, \tag{19}$$

which is a well known WD equation in the minisuperspace, and ordering parameters r and s are arbitrary integers. The wave vector Ψ can be written as the function of z_+ and z_-, i.e. $\Psi = \Psi(z_+, z_-)$.

In the process of dBB interpretation, the mass function also becomes an operator $\hat{M}$, which acts as a state vector with a mass eigenvalue m,

$$\begin{aligned} \left(\hat{M} - m\right)\Psi &= \left(2\hat{\Pi}_{+p} z_+ \hat{\Pi}_{+q} + \frac{1}{2} z_- - m\right)\Psi \\ &= 0, \end{aligned} \tag{20}$$

where ordering parameters p and q are arbitrary integers. The general solution of simultaneous equations of Eqs.(19) and (20) is[6]

$$\begin{aligned} &\Psi(z_+, z_-) \\ &= y^{p-s} z^{q+s} \left(c_1 H_{p-q}^{(1)}(z) + c_2 H_{p-q}^{(2)}(z)\right), \end{aligned} \tag{21}$$

with

$$y = \frac{1}{\hbar}\sqrt{-z_+/(z_- - r_g)}, \tag{22}$$

and

$$z = \frac{1}{\hbar}\sqrt{-z_+(z_- - r_g)}, \tag{23}$$

where c_1 and c_2 are two constants, and $H_{p-q}^{(1)}$ and $H_{p-q}^{(2)}$ are the Hankel functions, which are linearly independent of and complex conjugate to each other.

3. Modified de Broglie–Bohm interpretation to the Schwarzschild black hole

3.1. Modified de Broglie–Bohm approach in quantum mechanics

In this subsection, we briefly present the results in Ref.[12]. The one-dimensional Schrödinger equation for a particle is

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi, \tag{24}$$

where m is the mass of the particle, V a classical potential, and Ψ the wavefunction. By writing the wavefunction as a polar form $\Psi = N\mathrm{e}^{\mathrm{i}S/\hbar}$, where N is a normalization constant and S is a function of t and x, one can have a QMHJ equation from Eq.(24)[16]

$$\frac{\partial S}{\partial t} + \left[\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + V\right] = \frac{\mathrm{i}\hbar}{2m}\frac{\partial^2 S}{\partial x^2}. \tag{25}$$

According to the dBB approach, the momentum of the particle is assumed to be

$$m\dot{x} \equiv \frac{\partial S}{\partial x} = \frac{\hbar}{\mathrm{i}}\frac{1}{\Psi}\frac{\partial \Psi}{\partial x}. \tag{26}$$

The quantum trajectory can be obtained from Eq.(26). However the variable x is a complex, for the wavefunction is complex. So it is assumed that the physical coordinate of a particle is the real part of x, which gives

$$m\dot{x} = \frac{\hbar}{2\mathrm{i}}\frac{\left[\Psi^*\dfrac{\partial \Psi}{\partial x} - \Psi\left(\dfrac{\partial \Psi}{\partial x}\right)^*\right]}{\Psi^*\Psi}. \tag{27}$$

Note that x itself denotes the real part of x for simplification in Eq.(27).

Considering the case in which the phase can be expressed as $S = W - Et$, where W is the Hamilton's characteristic function and E is the energy of the particle. Here E and t are assumed to be real, the Schrödinger equation gives the energy[12]

$$E = \frac{1}{2}m\dot{x}^2 + V(x) + \frac{\hbar}{2\mathrm{i}}\frac{\partial \dot{x}}{\partial x}. \tag{28}$$

It is obvious that the last term on the right-hand side in the above equation is the quantum potential, i.e. $V_q = \dfrac{\hbar}{2\mathrm{i}}\dfrac{\partial \dot{x}}{\partial x}$. Using Eq.(26) we have

$$V_q = \frac{\hbar}{2\mathrm{i}}\frac{\partial \dot{x}}{\partial x} = -\frac{\mathrm{i}\hbar}{2m}\frac{\partial^2 S}{\partial x^2}. \tag{29}$$

Note that, in fact, the quantum potential is equal to the minus–signed term on the right–hand side of QMHJ equation (25). This means that we can directly extract the quantum potential from QMHJ equation (25). In the following, we will obtain a quantum potential in quantum black hole by using this method.

3.2. Modified de Broglie–Bohm approach to the black hole

According to the modified dBB approach, the wavefunction of the black hole should be expressed as

$$\Psi\left(z_+, z_-\right) = N\exp\left[\mathrm{i}S/\hbar\right], \tag{30}$$

where the amplitude N is a normalization constant and the phase S is a real function of z_+ and z_-. Substituting the expression into WD equation (19), we obtain

$$2\frac{\partial S}{\partial z_+}\frac{\partial S}{\partial z_-} + \frac{1}{2} = 2\mathrm{i}\hbar\frac{\partial^2 S}{\partial z_+\partial z_-}. \tag{31}$$

This equation corresponds to the QMHJ equation. Using the aforementioned method we can directly extract the quantum potential from Eq.(31)

$$V_q = -2\mathrm{i}\hbar\frac{\partial^2 S}{\partial z_+\partial z_-}. \tag{32}$$

In fact, as in the original dBB approach, by inserting $\Psi = R\exp(\mathrm{i}S/\hbar)$ into WD equation (19), where R and S each are a function of z_+ and z_-, one can obtain from the real part of it[6]

$$2\frac{\partial S}{\partial z_+}\frac{\partial S}{\partial z_-} + \frac{1}{2} + Q = 0, \tag{33}$$

where $Q = -\dfrac{2\hbar^2}{R}\left(\dfrac{\partial}{\partial z_+} - \dfrac{q}{z_+}\right)\left(\dfrac{\partial}{\partial z_-} - \dfrac{S}{z_- - r_g}\right)R$ is the (original) quantum potential. Here we have set $v_0 = G = 1$ in Ref.[6]. Comparing this equation with Eq.(31), we can find that the quantum potential expression (32) is reasonable.

The quantum trajectories $Z_+(T)$ and $Z_-(T)$ are introduced by assuming

$$\Pi_+ \equiv \left.\frac{\partial S}{\partial z_+}\right|_{z_+=Z_+, z_-=Z_-, A=A}, \tag{34}$$

and

$$\Pi_- \equiv \left.\frac{\partial S}{\partial z_-}\right|_{z_+=Z_+, z_-=Z_-, A=A}, \tag{35}$$

which give

$$-\frac{1}{2}\dot{Z}_- = \frac{\hbar}{\mathrm{i}}\frac{1}{\Psi}\frac{\partial\Psi}{\partial Z_+}, \tag{36}$$

and

$$-\frac{1}{2}\dot{Z}_+ = \frac{\hbar}{\mathrm{i}}\frac{1}{\Psi}\frac{\partial\Psi}{\partial Z_-}. \tag{37}$$

where a dot denotes the first derivative with respect to T, i.e. $\cdot = \partial/\partial T$. These quantities are complex, however we can take their real parts as the corresponding physical quantities. So the two equations can be modified, respectively, as

$$\dot{Z}_- = -\frac{\hbar}{\mathrm{i}}\frac{\left[\Psi^*\dfrac{\partial\Psi}{\partial Z_+} - \Psi\left(\dfrac{\partial\Psi}{\partial Z_+}\right)^*\right]}{\Psi^*\Psi}, \tag{38}$$

and

$$\dot{Z}_+ = -\frac{\hbar}{\mathrm{i}}\frac{\left[\Psi^*\dfrac{\partial\Psi}{\partial Z_-} - \Psi\left(\dfrac{\partial\Psi}{\partial Z_-}\right)^*\right]}{\Psi^*\Psi}. \tag{39}$$

Note that quantities Z_+, Z_- and T are real now.

From Eqs.(38) and (39), and noting that $\Psi^*\dfrac{\partial\Psi}{\partial y} - \Psi\left(\dfrac{\partial\Psi}{\partial y}\right)^* = 0$, we have $\mathrm{d}Z_-/\mathrm{d}Z_+ = (Z_- - r_g)/Z_+$. By integrating it we obtain $Z_+ = c\,(Z_- - r_g)$, where c is an integration constant. Without losing generality, we can set $c = -1$, so

$$Z_+ = -\left(Z_- - r_g\right). \tag{40}$$

This gives quantum U–V relation as

$$U = -\left(1 - \frac{r_g}{\sqrt{V}}\right), \tag{41}$$

which coincides with its corresponding classical relation (5).

Now we consider the quantum T–$V^{1/2}$ relation. Substituting Eq.(21) into Eq.(38) yields an integral form

$$T = \frac{\pi}{2(c_1^2 - c_2^2)}\int Z\left(c_1 H_\nu^{(1)}(Z) + c_2 H_\nu^{(2)}(Z)\right) \times\left(c_1 H_\nu^{(2)}(Z) + c_2 H_\nu^{(1)}(Z)\right)\mathrm{d}V^{1/2}, \tag{42}$$

where $Z = \dfrac{1}{\hbar}\left|V^{1/2} - r_g\right|$ and Eq.(40) have been used. We know that the region of $V^{1/2} \gg r_g$ is an asymptotic flat and its gravitation vanishes. So quantum behaviour should reduce into a classical one in the asymptotic flat. Considering the asymptotic forms of the Hankel function for $Z \gg 1$

$$H_\nu^{(1)}(Z) \to \sqrt{\frac{2}{\pi Z}}\mathrm{e}^{\mathrm{i}(Z - \nu\pi/2 - \pi/4)}, \tag{43}$$

and

$$H_\nu^{(2)}(Z) \to \sqrt{\frac{2}{\pi Z}}\mathrm{e}^{-\mathrm{i}(Z - \nu\pi/2 - \pi/4)}, \tag{44}$$

We obtain from Eq.(42)

$$T = \frac{c_1^2 + c_2^2}{c_1^2 - c_2^2} V^{1/2} + \frac{2c_1 c_2 \hbar}{c_1^2 - c_2^2} \times \sin\left(\frac{V^{1/2} - r_g}{\hbar} - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) + T_0, \quad (45)$$

with $V^{1/2} \gg r_g$ and T_0 being a constant. We can see that, in general, this quantum relation (45) cannot reduce into the classical one $T = V^{1/2}$ when $V^{1/2} \gg r_g$. However they can reduce into $T = V^{1/2}$ when $c_1 \neq 0$ and $c_2 = 0$, or $T = -V^{1/2}$ (physically equivalent to $T = V^{1/2}$ with a transformation $T \to -T$) when $c_1 = 0$ and $c_2 \neq 0$, with T_0 set as 0. This is an interesting result. If we think that a quantum gravity must reduce into the classical counterpart in the asymptotic flat, we seem to have to accept the fact that the superposition principle cannot be applied to the quantum black hole, that is, $\Psi_1 = c_1 y^{p-s} z^{(q+s)} H_\mu^{(1)}(Z)$ and $\Psi_2 = c_2 y^{p-s} z^{(q+s)} H_\mu^{(2)}(Z)$ are two physical solutions, but their linear combination is not a physical solution. This result provides an evidence to support the point of view in Ref.[14].

Shown in Fig.1 are the numerical estimations of T–$V^{1/2}$ relation with $c_1 \neq 0$ and $c_2 = 0$. In another case of $c_1 = 0$ and $c_2 \neq 0$, similar results can be obtained by just using a transformation $T \to -T$. The integration constant is fixed so that the quantum trajectories and the classical trajectories coincide with each other at infinity ($V^{1/2} \to \infty$). We can see that the quantum trajectory is a continuous curve across the horizon for $\nu = 0$, but it approaches negative infinity at the horizon for $\nu \neq 0$. It is noted that in Fig.1(b) we have fixed the integration constant by the point ($V^{1/2} = 0$, and $T = 0$) in order to portray the quantum trajectory inside the horizon for $\nu = 1$.

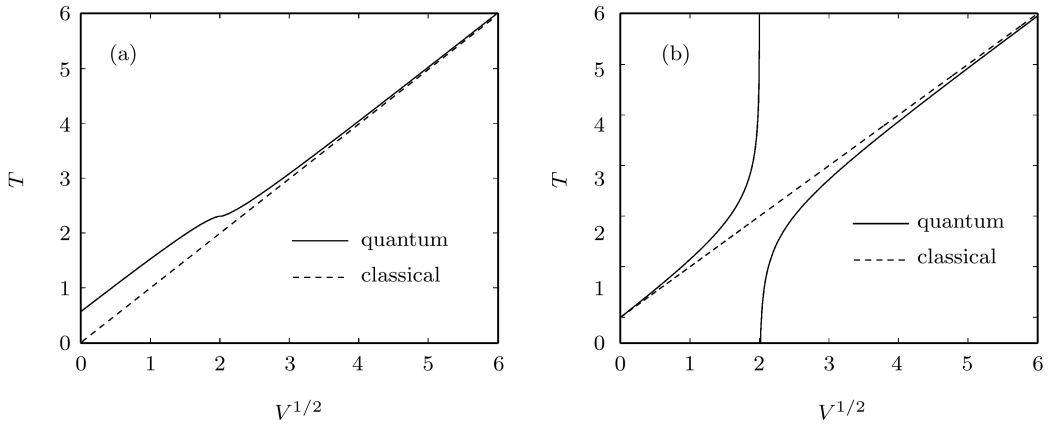


Fig.1. The T–$V^{1/2}$ relation with $c_1 \neq 0$ and $c_2 = 0$, in which the values of ordering parameter ν take 0 in (a) and 1 in (b); the classical relation is denoted by dashed line; the natural units $c = \hbar = G = 1$ and $r_g = 2$ are used.

A quantum potential V_q represents quantum nature. Substituting Eq.(30) into Eq.(32), we can obtain

$$V_q = -\frac{1}{2}\left[1 + \left(\frac{1}{\Psi}\frac{\partial \Psi}{\partial Z}\right)^2\right]. \quad (46)$$

This expression is complex, so the (physically) quantum potential should be its real part,

$$V_q = -\frac{1}{2}\left\{1 + \frac{1}{2}\left[\left(\left(\frac{1}{\Psi}\frac{\partial \Psi}{\partial Z}\right)^2\right)^* + \left(\frac{1}{\Psi}\frac{\partial \Psi}{\partial Z}\right)^2\right]\right\}. \quad (47)$$

By inserting Eq.(21) into the above, the quantum potential becomes

$$V_q = -\frac{1}{2} - \frac{(p+s)^2}{2Z^2} + \frac{p+s}{2Z}\left[\frac{c_1 H_{\nu+1}^{(1)}(Z) + c_2 H_{\nu+1}^{(2)}(Z)}{c_1 H_\nu^{(1)}(Z) + c_2 H_\nu^{(2)}(Z)} + \frac{c_1 H_{\nu+1}^{(2)}(Z) + c_2 H_{\nu+1}^{(1)}(Z)}{c_1 H_\nu^{(2)}(Z) + c_2 H_\nu^{(1)}(Z)}\right] - \frac{1}{4}\left[\frac{\left(c_1 H_{\nu+1}^{(1)}(Z) + c_2 H_{\nu+1}^{(2)}(Z)\right)^2}{\left(c_1 H_\nu^{(1)}(Z) + c_2 H_\nu^{(2)}(Z)\right)^2} + \frac{\left(c_1 H_{\nu+1}^{(2)}(Z) + c_2 H_{\nu+1}^{(1)}(Z)\right)^2}{\left(c_1 H_\nu^{(2)}(Z) + c_2 H_\nu^{(1)}(Z)\right)^2}\right], \quad (48)$$

where $Z = \frac{1}{\hbar}\left|V^{1/2} - r_g\right|$. We can see that the quantum potential depends on the ordering parameters p, s and ν. In an asymptotical flat, $Z \gg 1$, and the quantum potential becomes

$$V_q = -\frac{1}{2}\left(1 - \frac{\left(c_1^2 - c_2^2\right)^2 - 4c_1^2c_2^2\sin^2 2\left(Z - \nu\pi/2 - \pi/4\right)}{\left(c_1^2 + c_2^2 + 2c_1c_2\cos 2\left(Z - \nu\pi/2 - \pi/4\right)\right)^2}\right). \tag{49}$$

The quantum potential should be zero in the asymptotical flat. So we also see that either c_1 or c_2 has to be zero, otherwise the quantum potential cannot vanish. This conclusion is consistent with that from the quantum trajectories.

Numerical estimations of quantum potential are shown in Fig.2. The quantum potential diverges at horizon in Figs.2(a), 2(b) and 2(c), but it is finite in Fig.2(d). The quantum potentials in Ref.[6] have also been drawn with dashed lines for a comparison, which are independent of p and s.

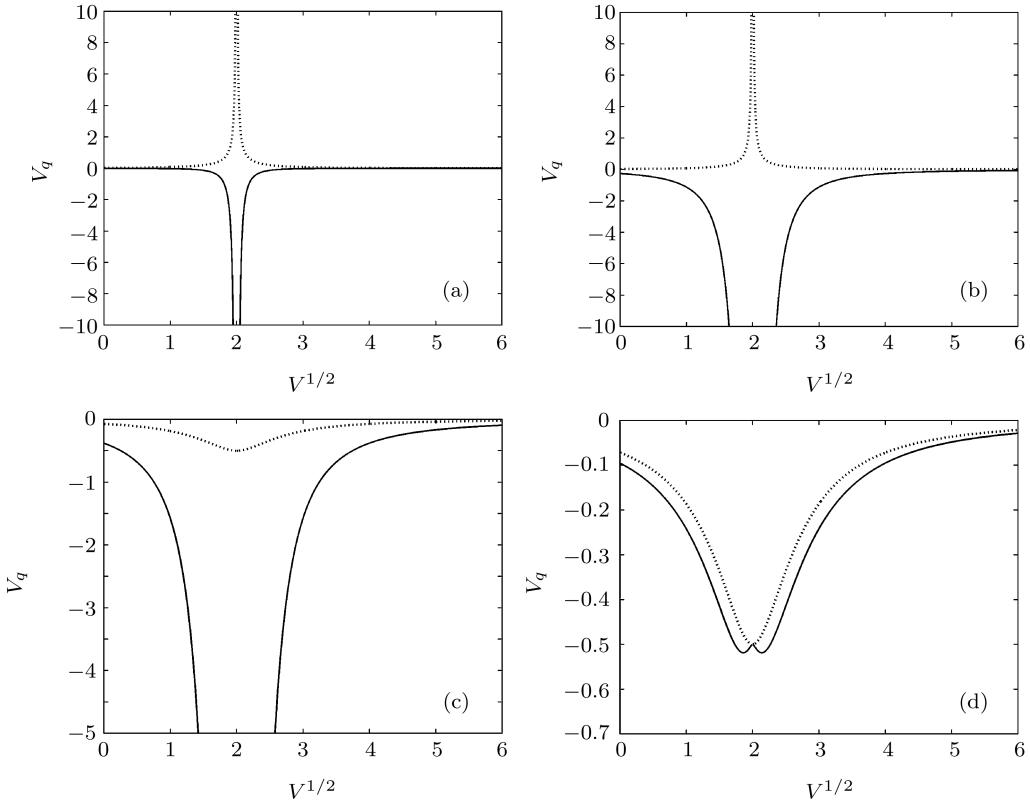


Fig.2. The quantum potential versus $V^{1/2}$ in the case of $c_1 \neq 0$ and $c_2 = 0$ or $c_1 = 0$ and $c_2 \neq 0$, in which $\nu = 0$, $s = p = 0$ in (a); $\nu = 0$, $s = p = 1$ in (b); $\nu = 1$, $s = p = 0$ in (c); $\nu = 1$, $s = p = 1$ in (d); the dot lines denote the results in Ref.[6] which are independent of p and s; the natural units $c = \hbar = G = 1$ and $r_g = 2$ are used.

The event horizon is located at the null surface $U = 0$, i.e. $V^{1/2} = r_g$. The apparent horizon is determined by[5,6]

$$\theta_-\theta_+\Psi = 0, \tag{50}$$

where

$$\theta_-\theta_+ = \frac{1}{V}\left(1 - \frac{r_g}{V^{1/2}}\right). \tag{51}$$

This leads the apparent horizon to be also located at $V^{1/2} = r_g$. So the two horizons coincide in the quantum black hole. The result is the same as that in Ref.[6] in which the ordinary dBB interpretation was used.

Now we consider the motion of the light ray on the geometry of the quantum black hole. The light is treated as a test particle. The null condition moving along the radial direction for metric (4) reads

$$\frac{\mathrm{d}R}{\mathrm{d}T} = \frac{1}{U} \quad \text{with} \quad U = \frac{r_g}{V^{1/2}} - 1. \tag{52}$$

Using Eq.(42), we have

$$R = \pm\frac{\pi}{2\hbar(c_1^2 - c_2^2)}\int V^{1/2}\left(c_1H_\nu^{(2)}(Z) + c_2H_\nu^{(1)}(Z)\right)$$

$$\times \left(c_1 H_\nu^{(1)}(Z) + c_2 H_\nu^{(2)}(Z)\right) \mathrm{d}V^{1/2}, \tag{53}$$

where $Z = \frac{1}{\hbar}\left|V^{1/2} - r_g\right|$. Note that the motion is independent of the ordering parameter p and s.

4. Conclusions and discussion

We have investigated the quantum theory of the Schwarzschild black hole by using the modified dBB approach. The quantum trajectories and the quantum potential are obtained. It is believed that a correct quantum gravitation theory should reduce into its corresponding classical counterpart in the asymptotical flat. By studying the properties of the quantum trajectories and the quantum potential, we find that the linear combination of two particular solutions of the wavefunction cannot reduce into the classical one, although each of them alone can do. It seems to hint that the superposition principle is not applicable for this quantum black hole, which provides an evidence to support the point of view, given by Alwis and MacIntire,[14] that a possible resolution on the quantum gravity is to give up the superposition principle.

Now we compare our results with those in Ref.[6] in which the original dBB approach was used. When $c_1 \neq 0$ and $c_2 = 0$, Eq.(42) becomes

$$T = \frac{\pi}{2} \int Z \left|H_\nu^{(2)}(Z)\right|^2 \mathrm{d}V^{1/2}, \tag{54}$$

which is the same as that in Ref.[6]. And when $c_1 \neq 0$ and $c_2 = 0$ or $c_1 = 0$ and $c_2 \neq 0$, expression (53) of motion of the light ray becomes

$$R = \pm\frac{\pi}{2\hbar} \int V^{1/2} \left|H_\nu^{(2)}(Z)\right|^2 \mathrm{d}V^{1/2}, \tag{55}$$

which is also the same as that in Ref.[6]. In addition, we have obtained the result that the event horizon and the apparent horizon coincide in the quantum clack hole by using the modified dBB interpretation, which is also the same as that in Ref.[6]. But the quantum potential (48) is different from that in Ref.[6]. Our results depend not only on ordering parameter ν but also on parameters p and s, while theirs depends only on ν. Their differences are shown in Fig.2.

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