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Thermal radiation and nonthermal radiation of the slowly changing dynamic Kerr–Newman black hole*

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Using the related formula of dynamic black hole, we have calculated the instantaneous radiation energy density of the slowly changing dynamic Kerr–Newman black hole. It is found that the instantaneous radiation energy density of a black hole is always proportional to the quartic of the temperature of the event horizon in the same direction. By using the Hamilton–Jacobin equation of scalar particles in the curved spacetime, the spontaneous radiation of the slowly changing dynamic Kerr–Newman black hole is studied. The energy condition for the occurrence of the spontaneous radiation is obtained.

Keywords: event horizon, thin film model, instantaneous radiation energy density, nonthermal radiation
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1. Introduction

Since Hawking (in 1974) proved that the black hole has thermal radiation,[1] much work has been done in research of the thermal radiation of black holes.[2,3] Starobinsky and Unruh have proved, for the charged black hole or the rotating black hole, that there is not only the thermal radiation, but also nonthermal radiation.[4,5] There has also been much work done on the nonthermal radiation of black holes.[6,7] So research on the radiation of black holes is always an important research field. Black holes are actually all dynamic because of the Hawking evaporation and accretion. Therefore, the research on the dynamic black hole is of particular significance. In recent years, the brick-wall model[8] has been improved and changed into the thin-film model.[9,10] Many papers adopted the thin-film model to study the entropy of black holes and obtained that the entropy of black holes is proportional to the area of the event horizon.[11−14] Where the event horizon exists, there should be entropy of black holes and Hawing radiation. Recently, research on the quantum tunnelling of black holes has shown that the quantum tunnelling rate is related to the change of entropy of black holes.[15−19] It is shown that there must be an internal relation between the entropy and the thermal radiation of black holes. It is particular important to further study the relation between the entropy and the thermal radiation of black holes. Reference [20] has studied the radiation energy density using the statistic entropy of Dirac field in the static spherically symmetric black holes and obtained that the radiation energy density of black holes is always proportional to quartic of the temperature of the event horizon. Reference [21] has obtained the instantaneous radiation energy density of dynamic black holes by using the entropy density near the event horizon of black holes:

$$M = M_0 - \alpha s r_{hv} T, \tag{1}$$

where M_0 is the radiation energy density of black holes under the condition of thermodynamic equilibrium, α is a coefficient related to both radiation and absorption coefficients, s is the entropy density in thin film near the event horizon, $r_{hv} = \left(\dfrac{\mathrm{d}r}{\mathrm{d}v}\right)_{r=r_h}$ is the changing rate of the event horizon, and T is the temperature of the event horizon. Equation (1) provides a new method to study the thermal radiation of dynamic black holes. In this paper, the thermal radiation of the slowly changing dynamic Kerr–Newman black hole will be investigated by adopting the above method.

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2. The instantaneous radiation energy density of the slowly changing dynamic Kerr–Newman black hole

The line element of the dynamic Kerr–Newman black hole has been given by Jing and Wang,[22] expressed by using the advanced Eddington coordinate

$$\begin{aligned}\mathrm{d}s^2 =&[1-(2mr-Q^2)\rho\bar{\rho}]\mathrm{d}\upsilon^2\\&-2\mathrm{d}\upsilon\mathrm{d}r+2a(2mr-Q^2)\rho\bar{\rho}\sin^2\theta\mathrm{d}\upsilon\mathrm{d}\phi\\&-\frac{1}{\rho\bar{\rho}}\mathrm{d}\theta^2+2a\sin^2\theta\mathrm{d}r\mathrm{d}\phi-[(r^2+a^2)\\&+(2mr-Q^2)a^2\rho\bar{\rho}\sin^2\theta]\sin^2\theta\mathrm{d}\phi^2,\end{aligned}\tag{2}$$

where $m=m(\upsilon)$, $Q=Q(\upsilon)$ are the mass and the charge of the black hole, respectively, $am(\upsilon)$ is the total angle momentum of the black hole, a is a constant, $\rho=-\dfrac{1}{r-\mathrm{i}a\cos\theta}$ and $\bar{\rho}$ is the complex conjugate of ρ. The corresponding nonzero contravariant components of the metric of Eq.(2) are

$$\begin{aligned}&g^{00}=-a^2\sin^2\theta\rho\bar{\rho},\\&g^{11}=-\Delta\rho\bar{\rho},\\&g^{22}=-\rho\bar{\rho},\\&g^{33}=-\frac{1}{\sin^2\theta}\rho\bar{\rho},\\&g^{01}=g^{10}=-(r^2+a^2)\rho\bar{\rho},\\&g^{03}=g^{30}=g^{13}=g^{31}=-a\rho\bar{\rho},\end{aligned}\tag{3}$$

where $\Delta=r^2+a^2-2mr+Q^2$.

Because of the axial symmetry of spacetime, the curved surface equation of the event horizon is set to be

$$F(\upsilon,r,\theta)=0\quad\text{or}\quad r=r(\upsilon,\theta).\tag{4}$$

This surface equation satisfies the null surface qualification

$$g^{\mu\nu}\frac{\partial F}{\partial x^\mu}\frac{\partial F}{\partial x^\nu}=0.\tag{5}$$

Substituting Eqs.(3) and (4) into Eq.(5), one can obtain

$$\begin{aligned}&a^2\sin^2\theta r_{h\upsilon}^2-(2mr_h-Q^2-r_h^2-a^2)\\&+r_{h\theta}^2-2(r_h^2+a^2)r_{h\upsilon}=0,\end{aligned}\tag{6}$$

where $r_{h\upsilon}=\left(\dfrac{\partial r}{\partial\upsilon}\right)_{r=r_h}$, $r_{h\theta}=\left(\dfrac{\partial r}{\partial\theta}\right)_{r=r_h}$, and r_h is the radius of the event horizon. Equation (6) can be regarded as the horizon equation of the slowly changing dynamic Kerr–Newman black hole. Reference [11] adopted the thin film model of black hole and gave the entropy area density on the horizon of the slowly changing dynamic Kerr–Newman black hole

$$\sigma_s=\frac{2\pi^2}{45\beta_h^3P_h^2(\upsilon,r_h,\theta)}\frac{\delta}{\varepsilon(\varepsilon+\delta)},\tag{7}$$

where

$$P_h(\upsilon,r_h,\theta)=\frac{2(r_h-m-2r_hr_{h\upsilon})}{r_h^2+a^2-r_{h\upsilon}a^2\sin^2\theta},\tag{8}$$

and $\beta_h=\dfrac{1}{k_\mathrm{B}T}$, k_B is the Boltzmann constant, T is the temperature of event horizon, ε is the distance between the thin film and the event horizon, δ is the thickness of the thin film. Because both ε and δ are much smaller than r_h, the entropy density near the event horizon of the black hole can be obtained from Eq.(7)

$$s=\frac{2\pi^2k_\mathrm{B}^3}{45P_h^2(\upsilon,r_h,\theta)}\frac{}{\varepsilon(\varepsilon+\delta)}T^3.\tag{9}$$

In order to obtain the instantaneous radiation energy density M, the term M_0 must be first given. M_0 is the radiation energy density of black holes under the condition of thermodynamic equilibrium. The following process is used to obtain this term M_0. For the slowly changing dynamic Kerr–Newman black hole, when its mass and charge approach to constant values, and the angle momentum approaches to zero, the whole state of this black hole approaches to a thermodynamic equilibrium. So the Reissner-Nondström black hole can be used as a reference background when the state of the slowly changing dynamic Kerr–Newman black hole tends to be in the global thermodynamic equilibrium. The spacetime line element of the corresponding Reissner–Nondström black hole is

$$\begin{aligned}\mathrm{d}s^2=&-\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2\\&+\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2\\&+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2).\end{aligned}\tag{10}$$

Because $\hat{g}'_{00}=-\left(1-\dfrac{2m}{r}+\dfrac{Q^2}{r^2}\right)=0$ is the horizon equation of the black hole, $\hat{g}'_{00}$ can be written as

$$\begin{aligned}\hat{g'}_{00}=&-\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\\=&-\frac{(r-r_-)}{r^2}(r-r_h)\\=&f(r)(r-r_h),\end{aligned}\tag{11}$$

where $f(r) = -\dfrac{(r-r_-)}{r^2}$, $r_h = M + \sqrt{M^2-Q^2}$, $r_- = M - \sqrt{M^2-Q^2}$, which is the radius of the inner event horizon.

In Ref.[20], the radiation energy density of the Dirac field of the static spherically symmetry black hole is given by

$$M_0^{(\mathrm{D})} = \frac{21\pi^2 c k_{\mathrm{B}}^3}{90\varepsilon(\varepsilon+\delta)f_h^2}T^4, \tag{12}$$

where c is the speed of light. Reference [23] has obtained the conclusion, when the same cut-off factor is obtained, that the statistical entropy of Dirac field of the static spherically symmetric black hole is $\dfrac{7}{2}$ times that of the scalar field. Then the radiation energy density of the scalar field of the static spherically symmetric black holes is

$$M_0^{(\mathrm{B})} = \frac{\pi^2 c k_{\mathrm{B}}^3}{15\varepsilon(\varepsilon+\delta)f_h^2}T^4. \tag{13}$$

Substituting Eqs.(9) and (13) into Eq.(1), one can obtain

$$M^{(\mathrm{B})} = \frac{\pi^2 k_{\mathrm{B}}^3}{15\varepsilon(\varepsilon+\delta)}\left(\frac{c}{f_h^2} - \frac{2\alpha r_{h\upsilon}}{3P_h^2}\right)T^4, \tag{14}$$

where

$$\begin{aligned} f_h =& f(r)\,|_{r=r_h} = -\frac{r_h - r_-}{r_h^2} \\ =& -\frac{2\sqrt{M^2-Q^2}}{(M+\sqrt{M^2-Q^2})^2}. \end{aligned} \tag{15}$$

Let

$$\sigma(\upsilon,\theta) = \frac{\pi^2 k_{\mathrm{B}}^3}{15\varepsilon(\varepsilon+\delta)}\left(\frac{c}{f_h^2} - \frac{2\alpha r_{h\upsilon}}{3P_h^2}\right). \tag{16}$$

Equation (14) can be rewritten as

$$M^{(\mathrm{B})} = \sigma(\upsilon,\theta)T^4. \tag{17}$$

It can be easily seen through Eqs.(8), (15) and (16) that $\sigma(\upsilon,\theta)$ has a fixed value for a fixed υ_0 at an orientation angle θ_0 for the event horizon. So the instantaneous radiation energy density of the slowly changing dynamic Kerr–Newman black hole is always proportional to quartic power of the temperature of the event horizon in the same direction. This result is similar to the Stefan–Bolzmann law of blackbody radiation in flat spacetime. Therefore, Eq.(17) can be called the generalized Stefan–Bolzmann law of the slowly changing dynamic Kerr–Newman black hole. Equation (16) is called the corresponding generalized Stefan–Bolzmann dynamic proportionate coefficient. Then σ is no longer a constant compared to the blackbody radiation in flat spacetime, but a dynamic proportionate coefficient related to the changing rate of event horizon, the spacetime metric near the event horizon, the absorption and radiation factors of the black hole. It is obvious that the thermal radiation of the curved spacetime is different from that of flat spacetime, and the gravitational field around the black hole will affect the thermal radiation.

3. Nonthermal radiation of the slowly changing dynamic Kerr–Newman black hole

The motion of the scalar particles, with charge e and mass μ, in the curved spacetime, obeys the Hamilton-Jacobin equation[24]

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu} - eA_\mu\right)\left(\frac{\partial S}{\partial x^\nu} - eA_\nu\right) - \mu^2 = 0, \tag{18}$$

where $S = \int L\mathrm{d}\tau$ is the main Hamilton function.

From Eq.(2) the four-dimensional electromagnetic potential in the spacetime can be expressed as

$$A_\mu = (A_0, A_1, A_2, A_3). \tag{19}$$

When the slowly changing dynamic Kerr–Newman black hole approaches to a stationary state, its four-dimensional electromagnetic potential[25] is

$$\begin{aligned} A_0 &= -\frac{Qr}{r^2 + a^2\cos^2\theta}, \quad A_1 = A_2 = 0, \\ A_3 &= \frac{Qr\sin^2\theta}{r^2 + a^2\cos^2\theta}. \end{aligned} \tag{20}$$

Substituting Eqs.(3) and (19) into Eq.(18), one can obtain

$$\begin{aligned} &g^{00}\left(\frac{\partial S}{\partial \upsilon} - eA_0\right)^2 + 2g^{01}\left(\frac{\partial S}{\partial \upsilon}\frac{\partial S}{\partial r} - eA_0\frac{\partial S}{\partial r} - eA_1\frac{\partial S}{\partial \upsilon} + e^2A_0A_1\right) \\ &+ 2g^{03}\left(\frac{\partial S}{\partial \upsilon} - eA_0\right)\left(\frac{\partial S}{\partial \phi} - eA_3\right) + g^{11}\left(\frac{\partial S}{\partial r} - eA_1\right)^2 + 2g^{13}\left(\frac{\partial S}{\partial r} - eA_1\right)\left(\frac{\partial S}{\partial \phi} - eA_3\right) \\ &+ g^{22}\left(\frac{\partial S}{\partial \theta} - eA_2\right)^2 + g^{33}\left(\frac{\partial S}{\partial \phi} - eA_3\right)^2 - \mu^2 = 0. \end{aligned} \tag{21}$$

Make a general Tortoise coordinate transformation[6,24]

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa}\ln[r - r_h(\upsilon,\theta)], \\ \upsilon_* &= \upsilon - \upsilon_0, \\ \theta_* &= \theta - \theta_0. \end{aligned} \tag{22}$$

Here κ is an adjustable parameter, and κ, υ_0 and θ_0 are all constants under the general Tortoise coordinate transformation. The differential operators can be expressed under the general Tortoise coordinate transformation as

$$\begin{aligned} \frac{\partial}{\partial r} &= \frac{2\kappa(r-r_h)+1}{2\kappa(r-r_h)}\frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial \upsilon} &= \frac{\partial}{\partial \upsilon_*} - \frac{r_{h\upsilon}}{2\kappa(r-r_h)}\frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial \theta} &= \frac{\partial}{\partial \theta_*} - \frac{r_{h\theta}}{2\kappa(r-r_h)}\frac{\partial}{\partial r_*}. \end{aligned} \tag{23}$$

Through the general Tortoise coordinate transformation, Eq.(21) can be rewritten as

$$\begin{aligned} &g^{00}\left(\frac{\partial S}{\partial \upsilon_*} - \frac{r_{h\upsilon}}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*} - eA_0\right)^2 + 2g^{01}\left\{\left[\frac{\partial S}{\partial \upsilon_*} - \frac{r_{h\upsilon}}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*}\right]\left[\frac{2\kappa(r-r_h)+1}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*}\right]\right. \\ &\left. - eA_0\frac{2\kappa(r-r_h)+1}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*} - eA_1\left[\frac{\partial S}{\partial \upsilon_*} - \frac{r_{h\upsilon}}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*}\right] + e^2A_0A_1\right\} \\ &+ 2g^{03}\left\{\left[\frac{\partial S}{\partial \upsilon_*} - \frac{r_{h\upsilon}}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*}\right]\left(\frac{\partial S}{\partial \phi} - eA_3\right) - eA_0\frac{\partial S}{\partial \phi} + e^2A_0A_3\right\} \\ &+ g^{11}\left[\frac{2\kappa(r-r_h)+1}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*} - eA_1\right]^2 + 2g^{13}\left[\frac{2\kappa(r-r_h)+1}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*}\left(\frac{\partial S}{\partial \phi} - eA_3\right) - eA_1\frac{\partial S}{\partial \phi} + e^2A_1A_3\right] \\ &+ g^{22}\left[\frac{\partial S}{\partial \theta_*} - \frac{r_{h\theta}}{2\kappa(r-r_h)}\frac{\partial S}{\partial r_*} - eA_2\right]^2 + g^{33}\left(\frac{\partial S}{\partial \phi} - eA_3\right)^2 - \mu^2 = 0. \end{aligned} \tag{24}$$

There is a killing vector because this spacetime is axisymmetric, so

$$\frac{\partial S}{\partial \phi} = m \quad (\text{const}). \tag{25}$$

Define

$$\omega = -\frac{\partial S}{\partial \upsilon_*}, \quad l = \frac{\partial S}{\partial \theta_*}, \tag{26}$$

where ω and l are the energy and the angle momentum of a particle respectively.[6,26] Substituting Eqs.(25), (26) and (18) into Eq.(24), one can obtain

$$\begin{aligned} &U\left(\frac{\partial S}{\partial r_*}\right)^2 - 4\kappa(r-r_h)V\frac{\partial S}{\partial r_*} \\ &+ 4\kappa^2(r-r_h)^2W = 0, \end{aligned} \tag{27}$$

where

$$U = g^{00}r_{h\upsilon}^2 - 2g^{01}[2\kappa(r-r_h)+1]r_{h\upsilon} + g^{11}[2\kappa(r-r_h)+1]^2 + g^{22}r_{h\theta}^2, \tag{28}$$

$$\begin{aligned} V =& g^{01}\{[2\kappa(r-r_h)+1](\omega + eA_0) - eA_1r_{h\upsilon}\} - g^{00}r_{h\upsilon}(\omega + eA_0) + g^{03}(m - eA_3)r_{h\upsilon} \\ &+ g^{11}eA_1[2\kappa(r-r_h)+1] - g^{13}(m - eA_3)[2\kappa(r-r_h)+1] \\ &+ g^{22}r_{h\theta}[l - eA_2], \end{aligned} \tag{29}$$

$$\begin{aligned} W =& g^{00}(\omega^2 + e^2A_0^2 + 2\omega eA_0) + 2g^{01}(\omega eA_1 + e^2A_0A_1) + g^{11}e^2A_1^2 \\ &+ 2g^{13}eA_1(eA_3 - m) + g^{22}(l^2 + e^2A_2^2 - 2leA_2) + g^{33}(m - eA_3)^2 \\ &- 2g^{03}[\omega(m - eA_3) - eA_0m + e^2A_0A_3] - \mu^2. \end{aligned} \tag{30}$$

The solution of Eq.(27) is

$$\frac{\partial S}{\partial r_*}=\frac{2\kappa(r-r_h)}{U}(V\pm\sqrt{V^2-UW}). \tag{31}$$

Because both S and $\dfrac{\partial S}{\partial r_*}$ must be real numbers, we have

$$V^2-UW\geq 0. \tag{32}$$

This is the energy equation of a particle in the spacetime of the slowly changing dynamic Kerr–Newman black hole. When we set $V^2-UW=0$, we have

$$\omega^{\pm}=\frac{-B\pm\sqrt{B^2-4AC}}{2A}, \tag{33}$$

where

$$\begin{aligned}A=&\{g^{01}[2\kappa(r-r_h)+1]-g^{00}r_{hv}\}^2-Ug^{00}, &(34)\\ B=&2\{g^{01}[2\kappa(r-r_h)+1]-g^{00}r_{hv}\}\{g^{01}[2\kappa(r-r_h)+1]eA_0-g^{01}eA_1r_{hv}\\ &-g^{00}r_{hv}eA_0+g^{03}(m-eA_3)r_{hv}+g^{11}eA_1[2\kappa(r-r_h)+1]\\ &-g^{13}(m-eA_3)[2\kappa(r-r_h)+1]+g^{22}r_{h\theta}[l-eA_2]\}+U\{2g^{01}eA_1\\ &-2g^{03}(m-eA_3)+2g^{00}eA_0\}, &(35)\\ C=&\{g^{01}[2\kappa(r-r_h)+1]eA_0-g^{01}eA_1r_{hv}-g^{00}eA_0r_{hv}+g^{03}(m-eA_3)r_{hv}\\ &+g^{11}eA_1[2\kappa(r-r_h)+1]-g^{13}(m-eA_3)[2\kappa(r-r_h)+1]+g^{22}r_{h\theta}(l-eA_2)\}^2\\ &+U\{g^{00}e^2A_0^2+2g^{01}e^2A_0A_1+2g^{03}eA_0(m-eA_3)+g^{11}e^2A_1^2\\ &+2g^{13}eA_1(eA_1-m)+g^{22}(l-eA_2)^2+g^{33}(m-eA_3)^2-\mu^2\}. &(36)\end{aligned}$$

From Eq.(33), we can obtain the positive energy states $\omega\geq\omega^+$, the negative energy states $\omega\leq\omega^-$, and the energy restricted zone $\omega^-<\omega<\omega^+$. The width of the forbidden region is

$$\Delta\omega=\omega^+-\omega^-=\frac{\sqrt{B^2-4AC}}{2A}. \tag{37}$$

When the radius r is very large, the electromagnetic field can be ignored and the spacetime approaches to the flat spacetime. From Eqs.(33), (34), (35), (36) and (28), one can obtain

$$\lim_{r\to\infty}\omega^{\pm}\to\pm\mu. \tag{38}$$

This result is just the Dirac energy spectrum in the Minkowski spacetime, and its energy width of forbidden region $\Delta\omega=2\Delta\mu$.

When the radius $r\to r_h$, near the event horizon, from Eqs.(28) and (6), one can obtain

$$\lim_{r\to r_h}U=0. \tag{39}$$

The following result can be obtained from Eqs.(34), (35), (36) and (39)

$$\lim_{r\to r_h}\sqrt{B^2-4AC}=0. \tag{40}$$

The energy of a particle on the event horizon can be obtained from Eq.(33)

$$\begin{aligned}\omega_0=&\lim_{r\to r_h}\omega^+=\lim_{r\to r_h}\omega^-=\lim_{r\to r_h}\frac{-B}{2A}\\ =&-eA_0-\frac{[(m-eA_3)(g^{03}r_{hv}-g^{13})+eA_1(g^{11}-g^{01}r_{hv})+g^{22}r_{h\theta}(l-eA_2)]}{g^{01}-g^{00}r_{hv}}.\end{aligned} \tag{41}$$

It can be seen from Eq.(41) that ω_0 is related not only to the mass, charge, and angle momentum of the black hole, but also to the change of the event horizon, and the four-dimensional electromagnetic potential near the event horizon. On the event horizon, the energy width of the forbidden region of a parti-

cle is $\Delta\omega = 0$. When $\omega_0 > \mu$ near the event horizon, both the positive energy levels and the negative energy levels will show a cross phenomenon. For a part of negative energy particles, their energy can be larger than μ, and also larger than the energy of the nearby positive energy levels. When the energy satisfies $\mu < \omega \leq \omega_0$, the negative energy particle can travel across the forbidden region because of the quantum tunnelling effect and becomes a positive energy particle that can move out. This is a spontaneous radiation not related to the temperature, which can be called the Starobinsky–Unruh process.

For a slowly changing dynamic Kerr–Newman black hole, $r_{hv} \to 0$, $r_{h\theta} \to 0$, $\lim\limits_{r\to r_h} g^{00} = 0$ can be obtained from Eq.(6), so Eq.(41) can be expressed as

$$\omega_0 = -eA_0 + \frac{a(m - eA_3)}{r_h^2 + a^2}. \tag{42}$$

It is shown that ω_0 is not only related to the parameters (m, Q, a) of a black hole, the radius of the event horizon, but also related to A_0 and A_3 of the four-dimensional electromagnetic potential in the spacetime around the black hole.

4. Discussion

We have studied the law of the thermal and the nonthermal radiation of the slowly changing dynamic Kerr–Newman black holes. The results show that the thermal radiation of the slowly changing dynamic Kerr–Newman black holes satisfies the generalized Stefan-Boltzmann law. However, σ is no longer a constant compared to Stefan–Bolzmann's law of blackbody radiation in flat spacetime, it will be a dynamic proportional coefficient that is related to spacetime structure near the event horizon. It is obvious that the gravitational field around black holes will affect the thermal radiation, and thus there must be an internal relation between them. Let $r_{hv} = 0$, then the instantaneous radiation energy density of the dynamic black holes will naturally becomes that of the static or stationary black holes. From the above results, we see that there is nonthermal radiation in the spacetime of the slowly changing dynamic Kerr–Newman black holes.

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