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Hawking radiation from Kerr–Newman de Sitter black hole via anomalies*

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In this paper, Hawking radiation from the Kerr–Newman de Sitter black hole is studied via gauge anomaly and gravitational anomaly. The obtained results of Hawking radiation from the event horizon and the cosmological horizon accord with those by other methods.

Keywords: gauge anomaly, gravitational anomaly, Hawking radiation, Kerr–Newman de Sitter black hole

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1. Introduction

Black hole is a deduction of Einsteinian gravitational theory. General relativity predicts that old massive fixed stars will collapse and become compact stars. If the mass of compact star is heavier than $3M_{\odot}$, it would become a black hole. Classically, black hole is completely black. But considering quantum gravity, Stephen Hawking first proved the existence of Hawking radiation from black hole in 1974.[1] In the following several decades, physicists have researched into Hawking radiation from different kinds of black holes by many means.[2−17] Recently, Robinson and Wilczek proved Hawking radiation of the spherical symmetry black hole via anomalous method.[18] In their paper, the effects of gauge anomaly and gravitational anomaly were mostly considered. This is an interesting work. Based on this paper, Iso S, Murata K, Wu S Q and Jiang Q Q *et al* studied some stationary black holes using this theory[19−26] and got many significant results. Hawking radiation of Kerr–Newman de Sitter black hole has been studied by several other ways.[27] But nobody has studied black holes via the anomalous viewpoint. In this paper, we consider the Kerr–Newman de Sitter black hole, and study its Hawking radiation by the following procedure. First of all, we introduce a dimension reduction technique and change the dimensions of black hole from 4 to 2. Then, we just consider the outgoing modes at the event horizon (EH) (and similarly just consider the ingoing modes at the cosmological horizon (CH)), and find the effective field theory. Because of the quantum effect, anomalies (gravitational anomaly and gauge anomaly) will appear. We introduce two kinds of currents to cancel the anomalies. Finally, we prove that the currents correspond to those of Hawking radiation.

2. Kerr–Newman de Sitter black hole and 2-dimension theory

The Kerr–Newman de Sitter black hole is a rotational, charged, stationary and axial symmetry black hole. In the Boyer–Lindquist coordinate system, its metric is[28]

$$\begin{aligned} &\mathrm{d}s^2 \\ &= -\frac{\Delta_r^2}{\rho^2}\left(\mathrm{d}t_{kNS} - \frac{a}{\Xi}\sin^2\theta\mathrm{d}\varphi\right)^2 + \frac{\rho^2}{\Delta_r^2}\mathrm{d}r^2 \\ &\quad + \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\rho^2}\left(a\mathrm{d}t_{kNS} - \frac{\left(r^2+a^2\right)}{\Xi}\mathrm{d}\varphi\right)^2, \end{aligned} \tag{1}$$

where t_{KNS} is the time coordinate of Kerr-Newman de Sitter black hole, and

$$\Delta_r^2 = \left(r^2+a^2\right)\left(1-\frac{r^2}{l^2}\right) - 2Mr + Q^2,$$

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$$\Delta_\theta = 1 + \frac{a^2}{l^2}\cos^2\theta,$$
$$\Xi = 1 + \frac{a^2}{l^2}, \quad \rho^2 = r^2 + a^2\cos^2\theta. \tag{2}$$

Here, M, a and Q are mass, angular momentum and charge of the black hole respectively.

Considering the null hypersurface equation

$$g^{\mu v}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^v} = 0, \tag{3}$$

we can get a negative root r_- (since it is meaningless, we will abandon it) and three positive roots r_0 (the inner horizon (IH)), r_h (the event horizon (EH)) and r_c (the cosmological horizon (CH)). The idiographic results are

$$r_\mathrm{ih} = r_0 = -\sqrt{Z_1} + \sqrt{Z_2} + \sqrt{Z_3},$$
$$r_\mathrm{eh} = r_\mathrm{h} = \sqrt{Z_1} - \sqrt{Z_2} + \sqrt{Z_3},$$
$$r_\mathrm{ch} = r_\mathrm{c} = \sqrt{Z_1} + \sqrt{Z_2} - \sqrt{Z_3}, \tag{4}$$

where

$$\sqrt{Z_1} = \sqrt{\frac{l^2-a^2}{6}}\left[1 + \sqrt{1 - \frac{12l^2\left(Q^2+a^2\right)}{\left(l^2-a^2\right)^2}}\cos\frac{\alpha}{3}\right]^{1/2},$$
$$\sqrt{Z_2} = \sqrt{\frac{l^2-a^2}{6}}\left[1 - \sqrt{1 - \frac{12l^2\left(Q^2+a^2\right)}{\left(l^2-a^2\right)^2}}\cos\left(\frac{\alpha}{3}+\frac{\pi}{3}\right)\right]^{1/2},$$
$$\sqrt{Z_3} = \sqrt{\frac{l^2-a^2}{6}}\left[1 - \sqrt{1 - \frac{12l^2\left(Q^2+a^2\right)}{\left(l^2-a^2\right)^2}}\cos\left(\frac{\alpha}{3}-\frac{\pi}{3}\right)\right]^{1/2},$$
$$\alpha = \arccos\left\{-\frac{\left(l^2-a^2\right)\left[\left(l^2-a^2\right)^2 + 36l^2\left(Q^2+a^2\right)\right] - 54m^2l^4}{\left[\left(l^2-a^2\right)^2 - 12l^2\left(Q^2+a^2\right)\right]^{3/2}}\right\}.$$

We can see that the roots are not coincident with each other.

The electromagnetic potential can be expressed as

$$A_\mu = \left(A_t, 0, 0, A_\varphi\right), \tag{5}$$

where $A_t = -\dfrac{Qr}{\rho^2\Xi}$, $A_\varphi = \dfrac{Qra\sin^2\theta}{\rho^2\Xi}$. So, we can get the action of the scalar field from the line element Eq.(1)

$$\begin{aligned} S[\Phi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\Phi\nabla^2\Phi = \frac{1}{2}\int \mathrm{d}t\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi\frac{\sin\theta}{\Xi}\Phi\left[\frac{\Delta_r^2a^2\sin^2\theta - \Delta_\theta\left(r^2+a^2\right)^2}{\Delta_r^2\Delta_\theta}\left(\frac{\partial}{\partial t} + \frac{\mathrm{i}eQr}{\rho^2\Xi}\right)^2\right. \\ +2a\Xi\frac{\Delta_r^2 - \Delta_\theta\left(r^2+a^2\right)}{\Delta_\theta\Delta_r^2}\left(\frac{\partial}{\partial t} + \frac{\mathrm{i}eQr}{\rho^2\Xi}\right)\left(\frac{\partial}{\partial\varphi} - \frac{\mathrm{i}eQra\sin^2\theta}{\rho^2\Xi}\right) \\ \left.+\frac{\Delta_r^2 - \Delta_\theta a^2\sin^2\theta}{\Delta_\theta\Delta_r^2\sin^2\theta}\Xi^2\left(\frac{\partial}{\partial\varphi} - \frac{\mathrm{i}eQra\sin^2\theta}{\rho^2\Xi}\right)^2 + \frac{\partial}{\partial r}\left(\Delta_r^2\frac{\partial}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\Delta_\theta\sin\theta\frac{\partial}{\partial\theta}\right)\right]\Phi. \end{aligned} \tag{6}$$

We reduce the dimensions and establish an effective (1+1) dimensional field theory. Considering line element Eq.(1), we can decompose Φ to $\Phi = \Sigma_{l,m}\phi_{l,m}(t,r)Y_{lm}(\theta,\varphi)$. Transforming to the tortoise coordinate defined by $\mathrm{d}r_*/\mathrm{d}r = \left(r^2+a^2\right)/\Delta_r^2 \equiv f^{-1}(r)$, we find that the simplified action is[19−23]

$$S[\Phi] = \frac{\left(r^2+a^2\right)}{2\Xi}\Sigma_{l,m}\int \mathrm{d}t\mathrm{d}r\phi_{l,m}^*\left(-f^{-1}(r)\left(\partial_t + \frac{\mathrm{i}m\Xi a}{\left(r^2+a^2\right)} + \frac{\mathrm{i}eQr}{\left(r^2+a^2\right)\Xi}\right)^2 + \partial_r\left(f(r)\,\partial_r\right)\right)\phi_{l,m}, \tag{7}$$

let

$$A_t(r) = -m\Omega - e\,\Psi, \tag{8}$$

$$\Omega = \frac{a\Xi}{\left(r^2+a^2\right)}, \quad \Psi = \frac{Qr}{\left(r^2+a^2\right)\Xi},$$

Up to now, we have established a (1+1) dimensional

effective field theory near the horizons. What we are interested in is just the radial Hawking radiation of Kerr–Newman de Sitter black hole, so the technique of dimension reduction greatly simplifies our study, and it is the basis of our next work.

3. Research on the Hawking radiation at EH via anomalies

According to the theory of black hole radiation, there is quantum vacuum fluctuation effect near the EH of the black hole, so virtual particle pairs may come into being, and the negative energy particles would enter the EH and become ingoing modes, but the positive energy particle would emit and become outgoing modes. The latter is what we want to study, so we just consider the outgoing modes at EH. Near the EH $r_{\mathrm{h}} < r < r_{\mathrm{h}} + \varepsilon$, the outgoing mode should satisfy two anomalous equations

$$\nabla_\mu J^\mu_{(\mathrm{h})e} = e\frac{\varepsilon^{\mu\upsilon}}{4\pi\sqrt{-g}}\partial_\mu A_\upsilon \tag{9}$$

$$\nabla_\mu J^\mu_{(\mathrm{h})m} = m\frac{\varepsilon^{\mu\upsilon}}{4\pi\sqrt{-g}}\partial_\mu A_\upsilon, \tag{10}$$

where $\varepsilon^{10} = 1$. If we let

$$J^\mu_{(\mathrm{h})} \equiv \frac{J^\mu_{(\mathrm{h})e}}{e} \equiv \frac{J^\mu_{(\mathrm{h})m}}{m}, \tag{11}$$

the unitive form is

$$\nabla_\mu J^\mu_{(\mathrm{h})} = \frac{\varepsilon^{\mu\upsilon}}{4\pi\sqrt{-g}}\partial_\mu A_\upsilon. \tag{12}$$

So

$$\partial_r J^r_{(\mathrm{h})} = \frac{1}{4\pi}\partial_r A_t. \tag{13}$$

On the other hand, at $r_{\mathrm{h}} + \varepsilon < r < r_{\mathrm{c}}$, $J^r_{(\mathrm{o})}$ satisfies

$$\partial_r J^r_{(\mathrm{o})} = 0, \tag{14}$$

where

$$J^r_{(\mathrm{o})} \equiv \frac{J^r_{(\mathrm{o})e}}{e} \equiv \frac{J^r_{(\mathrm{o})m}}{m}. \tag{15}$$

from Eqs.(13) and (14)we can get

$$J^r_{(\mathrm{o})} = c_0, \quad r_{\mathrm{h}} + \varepsilon < r < r_{\mathrm{c}}, \tag{16}$$

$$J^r_{(\mathrm{h})} = c_{\mathrm{h}} + \frac{1}{4\pi}\left(A_t(r) - A_t(r_{\mathrm{h}})\right) \quad r_{\mathrm{h}} < r < r_{\mathrm{h}} + \varepsilon, \tag{17}$$

where

$$A_t(r_{\mathrm{h}}) = -\frac{am\Xi}{(r^2_{\mathrm{h}} + a^2)} - \frac{Qer_{\mathrm{h}}}{(r^2_{\mathrm{h}} + a^2)\Xi}. \tag{18}$$

We need work out the values of c_{o} and c_{h} via the effective action and get the gauge current, so we set

$$J^r = J^r_{(\mathrm{o})}\Theta_+(r) + J^r_{(\mathrm{h})}H(r), \tag{19}$$

where

$$\Theta_+(r) = \Theta(r - r_{\mathrm{h}} - \varepsilon), \quad H(r) = 1 - \Theta_+(r), \tag{20}$$

and

$$\Theta(r - r_{\mathrm{h}} - \varepsilon) = \begin{cases} 1 & r_{\mathrm{h}} + \varepsilon < r < r_{\mathrm{c}}, \\ 0 & r_{\mathrm{h}} < r < r_{\mathrm{h}} + \varepsilon. \end{cases} \tag{21}$$

Then the variation of effective action can be written as

$$\begin{aligned} & -\delta W \\ &= \int \mathrm{d}t\mathrm{d}r\lambda\nabla_\mu J^\upsilon \\ &= \int \mathrm{d}t\mathrm{d}r\lambda\left[\left(J^r_{(\mathrm{o})} - J^r_{(\mathrm{h})} + \frac{A_t}{4\pi}\right)\delta(r - r_{\mathrm{h}} - \varepsilon) \right. \\ &\quad \left. + \partial_r\left(\frac{A_t}{4\pi}H\right)\right], \end{aligned} \tag{22}$$

where λ is the general coordinate transformation parameter, and from the integral expression, we can get

$$J^r_{(\mathrm{o})} = J^r_{(\mathrm{h})} - \frac{A_t(r)}{4\pi}. \tag{23}$$

So, at the EH

$$c_{\mathrm{o}} = c_{\mathrm{h}} - \frac{A_t(r_{\mathrm{h}})}{4\pi}. \tag{24}$$

The covariant current tensor is given as follows

$$\tilde{J}^\mu = J^\mu \mp \frac{\varepsilon^{\mu\upsilon}}{4\pi\sqrt{-g}}A_\upsilon, \tag{25}$$

which satisfies

$$\nabla_\mu\tilde{J}^\mu = \pm\frac{\varepsilon_{\mu\upsilon}}{4\pi}F^{\mu\upsilon}. \tag{26}$$

In the (1+1) dimensional anomalous theory, the covariant current that we introduced to cancel anomalies should be expressed as

$$\tilde{J}^r = J^r + \frac{1}{4\pi}A_t(r_{\mathrm{h}})H(r). \tag{27}$$

From Eqs.(16), (17), (18), (24) and (27), we can get

$$c_{\mathrm{o}} = -\frac{A_t(r_{\mathrm{h}})}{2\pi}. \tag{28}$$

This is just the gauge current.

Now, let us research into the current flux of the energy–momentum tensor near EH via gravitational

anomaly. As has been mentioned above, we just consider the outgoing modes. Near the EH, the total energy–momentum tensor flux can be written as

$$T_{\upsilon}^{\mu}=T_{\upsilon(\mathrm{o})}^{\mu}\Theta_{+}\left(r\right)+T_{\upsilon(\mathrm{h})}^{\mu}H\left(r\right). \tag{29}$$

The Ward identical equation is

$$\nabla_{\mu}T_{\upsilon}^{\mu}=F_{\mu\upsilon}J^{\mu}+A_{\upsilon}\nabla_{\mu}J^{\mu}+\nabla_{\mu}N_{\upsilon}^{\mu}, \tag{30}$$

where

$$N_{\upsilon}^{\mu}=\frac{1}{96\pi}\varepsilon^{\beta\mu}\partial_{\alpha}\Gamma_{\upsilon\beta}^{\alpha}. \tag{31}$$

The effective 2-dimensional Levi-Civita tensor is $\varepsilon^{01}=1$. We just consider the case $\nu=t$ here, so

$$N_{t}^{r}=\frac{1}{192\pi}\left(f'^{2}+ff''\right). \tag{32}$$

In $r_{\mathrm{h}}+\varepsilon<r<r_{\mathrm{c}}$, $T_{t(\mathrm{o})}^{r}$ satisfies

$$\partial_{r}T_{t(\mathrm{o})}^{r}=c_{\mathrm{o}}\partial_{r}A_{t}. \tag{33}$$

In the region $r_{\mathrm{h}}<r<r_{\mathrm{h}}+\varepsilon$, $T_{t(\mathrm{h})}^{r}$ should satisfy the anomalous equation

$$\partial_{r}T_{t(\mathrm{h})}^{r}=J_{(\mathrm{h})}^{r}\partial_{r}A_{t}+A_{t}\partial_{r}J_{(\mathrm{h})}^{r}+\partial_{r}N_{t}^{r}. \tag{34}$$

Variation of the action is

$$\begin{aligned}&-\delta W\\&=\int\mathrm{d}t\mathrm{d}r\lambda^{\upsilon}\nabla_{\mu}T_{\upsilon}^{\mu}\\&=\int\mathrm{d}t\mathrm{d}r\lambda^{t}\left[\left(T_{t(\mathrm{o})}^{r}-T_{t(\mathrm{h})}^{r}+\frac{A_{t}^{2}}{4\pi}+N_{t}^{r}\right)\right.\\&\quad\times\delta\left(r-r_{\mathrm{h}}-\varepsilon\right)+\partial_{r}\left[\left(\frac{A_{t}^{2}}{4\pi}+N_{t}^{r}\right)H\right]\\&\quad\left.+c_{\mathrm{o}}\partial_{r}A_{t}\right].\end{aligned} \tag{35}$$

As we know, the second term is the quantum boundary effect, and the third term is the classical effect of the background electromagnetic field for current flow, so near EH we can set the covariant energy–momentum tensor to be

$$\tilde{T}_{t(\mathrm{h})}^{r}=T_{t(\mathrm{h})}^{r}+\frac{1}{192\pi}\left(ff''-2\left(f'\right)^{2}\right), \tag{36}$$

and it[29] satisfies

$$\nabla_{\mu}\tilde{T}_{\upsilon}^{\mu}=\frac{1}{96\pi\sqrt{-g}}\varepsilon_{\mu\upsilon}\partial^{\mu}R. \tag{37}$$

So from Eqs.(33) and (34), at the EH, we can obtain

$$k_{\mathrm{o}}=k_{\mathrm{h}}+\frac{A_{t}^{2}\left(r_{\mathrm{h}}\right)}{4\pi}-N_{t}^{r}\left(r_{\mathrm{h}}\right), \tag{38}$$

where

$$k_{\mathrm{o}}=T_{t(\mathrm{o})}^{r}-c_{\mathrm{o}}A_{t}\left(r_{\mathrm{h}}\right), \tag{39}$$

$$k_{\mathrm{h}}=T_{t(\mathrm{h})}^{r}-\int_{r_{\mathrm{h}}}^{r}\mathrm{d}r\partial_{r}\left[c_{\mathrm{o}}A_{t}\left(r_{\mathrm{h}}\right)+\frac{A_{t}^{2}}{4\pi}+N_{t}^{r}\right], \tag{40}$$

the energy–momentum tensor current is

$$k_{(\mathrm{o})}=\frac{A_{t}^{2}\left(r_{\mathrm{h}}\right)}{4\pi}+\frac{\pi}{12}T_{\mathrm{h}}^{2}, \tag{41}$$

where $T_{\mathrm{h}}=\partial_{r}f(r)/(4\pi)|_{r=r_{\mathrm{h}}}$ is the temperature of the black hole.

We have obtained the gauge current and the energy–momentum tensor current fluxes, which accord with Hawking radiation of the black hole. For fermions, the Hawking radiation spectrum at the EH of the black hole is

$$N_{\pm m}\left(\omega\right)=1\Big/\left[\exp\left(\frac{\omega\pm A_{t}\left(r_{\mathrm{h}}\right)}{T_{\mathrm{h}}}\right)+1\right]. \tag{42}$$

We can get the gauge current and the energy–momentum tensor current fluxes from the formulae of quantum field theory

$$F_{J}=\frac{1}{2\pi}\int_{0}^{\infty}\left[N_{+}\left(\omega\right)-N_{-}\left(\omega\right)\right]\mathrm{d}\omega=-\frac{A_{t}\left(r_{\mathrm{h}}\right)}{2\pi}. \tag{43}$$

Corresponding to Eq.(15),

$$F_{J}\equiv\frac{F_{J}^{m}}{m}\equiv\frac{F_{J}^{e}}{e}, \tag{44}$$

where

$$\begin{aligned}F_{J}^{e}&=\frac{e}{2\pi}\int_{0}^{\infty}\left[N_{+}\left(\omega\right)-N_{-}\left(\omega\right)\right]\mathrm{d}\omega\\&=-e\frac{A_{t}\left(r_{\mathrm{h}}\right)}{2\pi},\end{aligned} \tag{45}$$

$$\begin{aligned}F_{J}^{m}&=\frac{m}{2\pi}\int_{0}^{\infty}\left[N_{+}\left(\omega\right)-N_{-}\left(\omega\right)\right]\mathrm{d}\omega\\&=-m\frac{A_{t}\left(r_{\mathrm{h}}\right)}{2\pi}.\end{aligned} \tag{46}$$

And

$$\begin{aligned}F_{T}&=\frac{1}{2\pi}\int_{0}^{\infty}\omega\left[N_{+}\left(\omega\right)+N_{-}\left(\omega\right)\right]\mathrm{d}\omega\\&=\frac{A_{t}^{2}\left(r_{\mathrm{h}}\right)}{4\pi}+\frac{\pi}{12}T_{\mathrm{h}}^{2}.\end{aligned} \tag{47}$$

From the so-called Hawking radiation spectrum and corresponding Hawking radiation current, we get Eq.(43), which accords with Eq.(28). Next, we obtain Eq.(47) from the known Hawking radiation conclusion, and it accords with Eq.(41). This shows that we have successfully accomplished the research of pure thermal spectrum.

4. Research on the Hawking radiation from CH via anomalies

Near the CH, there is quantum vacuum fluctuation effect, and there form ingoing modes and outgoing modes too. But other than the case at EH, we pay attention to ingoing modes. So we should introduce anomalous currents inner CH. When we just consider the ingoing modes in $r_{\rm c}-\varepsilon<r<r_{\rm c}$, it should satisfy the anomalous equation

$$\partial_r J^r_{\rm (c)}=\frac{-1}{4\pi}\partial_r A_t, \tag{48}$$

where

$$J^r_{\rm (c)}\equiv\frac{J^r_{\rm (c)e}}{e}\equiv\frac{J^r_{\rm (c)m}}{m}. \tag{49}$$

The following two equations should be satisfied

$$\partial_r J^r_{\rm (c)e}=\frac{-e}{4\pi}\partial_r A_t, \tag{50}$$

$$\partial_r J^r_{\rm (c)m}=\frac{-m}{4\pi}\partial_r A_t. \tag{51}$$

But, in $r_{\rm h}<r<r_{\rm c}-\varepsilon$, there is no anomaly, the equation is

$$\partial_r J^r_{\rm (o)}=0, \tag{52}$$

where

$$J^r_{\rm (o)}\equiv\frac{J^r_{\rm (o)e}}{e}\equiv\frac{J^r_{\rm (o)m}}{m}. \tag{53}$$

Now, from the above formulae we can work out

$$J^r_{\rm (o)}=h_0,\quad r_{\rm h}<r<r_{\rm c}-\varepsilon \tag{54}$$

$$J^r_{\rm (c)}=h_{\rm c}-\frac{1}{4\pi}\left(A_t\left(r\right)-A_t\left(r_{\rm c}\right)\right),\quad r_{\rm c}-\varepsilon<r<r_{\rm c}, \tag{55}$$

where

$$A_t\left(r_{\rm c}\right)=-\frac{am\Xi}{\left(r_{\rm c}^2+a^2\right)}-\frac{Qer_{\rm c}}{\left(r_{\rm c}^2+a^2\right)\Xi}. \tag{56}$$

Similarly, we set

$$J^r=J^r_{\rm (c)}C+J^r_{\rm (o)}\Theta_-, \tag{57}$$

where

$$\Theta_-\left(r\right)=\Theta\left(r_{\rm c}-r-\varepsilon\right),\quad C\left(r\right)=1-\Theta_-\left(r\right). \tag{58}$$

So the simplified effective action variation is

$$\begin{aligned}&-\delta W\\&=\int{\rm d}t{\rm d}r\lambda\nabla_\mu J^\upsilon\\&=\int{\rm d}t{\rm d}r\lambda\left[\left(J^r_{\rm (c)}-J^r_{\rm (o)}+\frac{A_t}{4\pi}\right)\delta\left(r-r_{\rm c}+\varepsilon\right)\right.\\&\left.-\partial_r\left(\frac{A_t}{4\pi}C\right)\right],\end{aligned} \tag{59}$$

where $\dfrac{A_t}{4\pi}C$ is the boundary term of quantum effect, and we get

$$J^r_{\rm (o)}=J^r_{\rm (c)}+\frac{A_t}{4\pi}. \tag{60}$$

We introduce a covariant flux to cancel anomaly near CH

$$\tilde{J}^r=J^r-\frac{1}{4\pi}A_t\left(r_{\rm c}\right)C\left(r\right). \tag{61}$$

Corresponding to Eq.(28), from Eqs.(54), (55), (60) and (61), we can work out

$$h_{\rm o}=\frac{A_t\left(r_{\rm c}\right)}{2\pi}. \tag{62}$$

Researching into the energy–momentum tensor current and considering the ingoing modes near the CH of black hole, we define

$$T^\mu_\upsilon=T^\mu_{\upsilon\rm (o)}\Theta_-\left(r\right)+T^\mu_{\upsilon\rm (c)}C\left(r\right). \tag{63}$$

In $r_{\rm H}+\varepsilon<r<r_{\rm c}$, $T^r_{t\rm (o)}$ satisfies

$$\partial_r T^r_{t\rm (o)}=h_{\rm o}\partial_r A_t. \tag{64}$$

In the region $r_{\rm c}-\varepsilon<r<r_{\rm c}$, the anomalous equation of $T^r_{t\rm (c)}$ is

$$\partial_r T^r_{t\rm (c)}=J^r_{\rm (c)}\partial_r A_t+A_t\partial_r J^r_{\rm (c)}-\partial_r N^r_t. \tag{65}$$

We get the variation of effective action

$$\begin{aligned}&-\delta W\\&=\int{\rm d}t{\rm d}r\lambda^\upsilon\nabla_\mu T^\mu_\upsilon\\&=\int{\rm d}t{\rm d}r\lambda^t\left[\left(T^r_{t\rm (c)}-T^r_{t\rm (o)}+\frac{A_t^2}{4\pi}+N^r_t\right)\right.\\&\times\delta\left(r-r_{\rm c}+\varepsilon\right)-\partial_r\left[\left(\frac{A_t^2}{4\pi}+N^r_t\right)C\right]\\&\left.+h_{\rm o}\partial_r A_t\right].\end{aligned} \tag{66}$$

The second term is the boundary quantum effect, and the third term is the classical effect of the background electromagnet field for the current. Finally, at CH we can set the covariant energy–momentum tensor current to be

$$\tilde{T}^r_{t\rm (c)}=T^r_{t\rm (c)}-\frac{1}{192\pi}\left(ff''-2\left(f'\right)^2\right), \tag{67}$$

taking notice of Eqs.(64), (65), (66) and (67), we know

$$n_{\rm o}=n_{\rm c}-\frac{A_t^2\left(r_{\rm c}\right)}{4\pi}+N^r_t\left(r_{\rm c}\right), \tag{68}$$

where

$$n_{\rm o} = T^r_{t({\rm o})} - h_{\rm o} A_t\left(r_{\rm c}\right), \tag{69}$$

$$n_{\rm c} = T^r_{t({\rm c})} - \int_{r_{\rm c}}^{r} {\rm d}r \partial_r \left[h_{\rm o} A_t - \frac{A_t^2}{4\pi} - N_t^r\right]. \tag{70}$$

So, the energy–momentum tensor current flux is

$$n_{({\rm o})} = -\frac{A_t^2\left(r_{\rm c}\right)}{4\pi} - \frac{\pi}{12} T_{\rm c}^2, \tag{71}$$

where $T_{\rm c} = -\partial_r f(r)/(4\pi)|_{r=r_{\rm c}}$ is the temperature of black hole.

For fermions, the Planckian distribution of the Hawking radiation from the CH of Kerr–Newman de Sitter black hole is

$$N_{\pm m}\left(\omega\right) = -1 \Bigg/ \left[\exp\left(\frac{\omega \pm A_t\left(r_{\rm c}\right)}{T_{\rm c}}\right) + 1\right]. \tag{72}$$

We get the gauge current from the formulae of quantum field theory

$$F_J = \frac{1}{2\pi}\int_0^\infty \left[N_+\left(\omega\right) - N_-\left(\omega\right)\right]{\rm d}\omega = \frac{A_t\left(r_{\rm c}\right)}{2\pi}, \tag{73}$$

where

$$F_J \equiv \frac{F_J^m}{m} \equiv \frac{F_J^e}{e}. \tag{74}$$

Corresponding to Eqs.(45) and (46), we get

$$\begin{aligned} F_J^e &= \frac{e}{2\pi}\int_0^\infty \left[N_+\left(\omega\right) - N_-\left(\omega\right)\right]{\rm d}\omega \\ &= e\frac{A_t\left(r_{\rm c}\right)}{2\pi}, \end{aligned} \tag{75}$$

$$\begin{aligned} F_J^m &= \frac{m}{2\pi}\int_0^\infty \left[N_+\left(\omega\right) - N_-\left(\omega\right)\right]{\rm d}\omega \\ &= m\frac{A_t\left(r_{\rm c}\right)}{2\pi}. \end{aligned} \tag{76}$$

And the energy–momentum tensor current

$$\begin{aligned} F_T &= \frac{1}{2\pi}\int_0^\infty \omega\left[N_+\left(\omega\right) + N_-\left(\omega\right)\right]{\rm d}\omega \\ &= -\frac{A_t^2\left(r_{\rm c}\right)}{4\pi} - \frac{\pi}{12} T_{\rm c}^2. \end{aligned} \tag{77}$$

Focusing on Eqs.(73) and (77), we can see that Eqs.(62) and (71) are the results of Hawking radiation.

5. Conclusions

In the above discussions, we have researched into one type of black hole which is rotational, charged, stationary and axially symmetrical. Above all, We simplify the dimension of Kerr–Newman de Sitter black hole to (1+1) dimensions, and then just pay attention to the outgoing modes at EH (of course, the ingoing mode is considered near CH). After the effective field theory is set up, we introduce two currents to cancel the anomalies. We proved that the currents are Hawking radiation in the end. When $Q = 0$, the results of this paper can become the conclusions of Hawking radiation from Kerr–de Sitter black hole. If $a = 0$, we would get the results of Hawking radiation from R-N de Sitter black hole. Finally, let $\Xi = 1$, and we get the results of Kerr–Newman black hole. The results that we have obtained in this paper are rational, but not one and only. Similarly, we can do the work in a dragging coordinates frame. The problem can also be dealt with by using a covariant anomalous point of view.

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