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Thermal entanglement in a two-qubit Heisenberg XY chain with the Dzyaloshinskii–Moriya interaction

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This paper investigates thermal entanglements of a two-qubit Heisenberg XY chain in the presence of the Dzyaloshinskii–Moriya anisotropic antisymmetric interaction. By the concept of concurrence, it is found that the effects of spin–orbit coupling on the entanglement are different from those of spin–spin model. The analytical expressions of concurrence are obtained for this model.

Keywords: thermal entanglement, Heisenberg XY chain, Dzyaloshinskii–Moriya interaction
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1. Introduction

One of the amusing properties of quantum mechanics is entanglement. Entangled states of component systems and their unique properties have drawn a lot of attention since the early days. Entanglement is responsible for the non-local correlations, which can exist between spatially separated quantum systems.[1−3]

The quantum entanglement in solid state systems such as spin chains is an important emerging field. Spin chains are natural candidates for the realization of entanglement in comparison with other physics systems. In recent years, quantum Heisenberg models with anisotropic interactions have attracted a great of theoretical and experimental interest.[4−14]

But only spin–spin interaction is considered in those studies, the effects of spin–orbit coupling on the entanglement is rarely included. In this paper we investigate the effects of the Dzyaloshinskii–Moriya (DM) interaction on the entanglement properties of quantum spin chains. This interaction has a number of important consequences and may cause a number of unconventional phenomena. Interestingly, the DM interaction may appear while the non-equilibrium steady states of quantum spin chains with currents[15,16] are analysed.

In our study we restrict ourselves to a simpler model, i.e., two qubit Heisenberg anisotropic XY chain, whose entanglement properties are amenable to detailed analytical analysis.

2. General formalism

To be specific, we consider N spins, one-half is governed by the following Hamiltonian:

$$H = \sum_n J\left((1+\gamma)s_n^x s_{n+1}^x + (1-\gamma)s_n^y s_{n+1}^y\right) + \sum_n D\left(s_n^x s_{n+1}^y - s_n^y s_{n+1}^x\right) + \sum_n B s_n^z. \quad (1)$$

Here γ $(-1 \leq \gamma \leq 1)$ is the anisotropic parameter and equals 0 for the isotropic XX model and ± 1 for the Ising model respectively. $J(1+\gamma)$ and $J(1-\gamma)$ are the anisotropic XY exchange interaction constants, D is the z component of the DM interaction and B is the transverse magnetic field. We assume periodic boundary conditions, i.e., $N+1 \equiv 1$, and $J > 0$, $B > 0$.

The state of a typical condensed-matter system at thermal equilibrium (temperature T) is $\rho = \exp(-\beta H)/Z$, where H is the Hamiltonian, $Z = \mathrm{tr}(\exp(-\beta H))$ is the partition function, and $\beta = 1/(kT)$, where k is the Boltzmann's constant. The entanglement associated with the thermal state ρ is referred to as thermal entanglement.[17]

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To quantify the amount of entanglement associated with ρ, we consider the concurrence, which is defined as[18,19]

$$C = \max\left[0, 2\max(\lambda_i) - \sum \lambda_i\right], \tag{2}$$

where λ_i are the square roots of the eigenvalues of the matrix $R = \rho S \rho^* S$, in which ρ is the density matrix $S = \sigma^y \otimes \sigma^y$, and the asterisk stands for the complex conjugate.

It is also easy to check that the commutator $[H, Sz] = 0$ (rotation symmetry about the z axis) which guarantees that the reduced density matrix for the subspace of any two qubits has the form[20−23] in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$,

$$\rho_{ij} = \begin{pmatrix} a_{ij} & 0 & 0 & c_{ij} \\ 0 & x_{ij} & z_{ij} & 0 \\ 0 & z_{ij}^* & y_{ij} & 0 \\ c_{ij}^* & 0 & 0 & b_{ij} \end{pmatrix} \tag{3}$$

with real a, b, x, y and complex c, z. For translational invariant systems: $x = y$; for real Hamiltonians and stationary states: $c, z \in R$. Then we can get a simple form for Eq.(3)

$$\rho_{ij} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & x & z & 0 \\ 0 & \bar{z} & x & 0 \\ \bar{c} & 0 & 0 & b \end{pmatrix}. \tag{4}$$

These non-zero matrix elements in Eq.(3) can be calculated through

$$\begin{aligned} a_{ij} &= \mathrm{Tr}(|00\rangle\langle 00|\rho), \\ b_{ij} &= \mathrm{Tr}(|11\rangle\langle 11|\rho), \\ c_{ij} &= \mathrm{Tr}(|00\rangle\langle 11|\rho), \\ x_{ij} &= \mathrm{Tr}(|01\rangle\langle 01|\rho), \\ z_{ij} &= \mathrm{Tr}(|01\rangle\langle 10|\rho). \end{aligned} \tag{5}$$

Then by the standard procedure mentioned in Refs.[18] and [19], the corresponding concurrence quantifying the entanglement of two qubits is readily obtained as[20]

$$C_{ij} = 2\max\left\{0, |c_{ij}| - \sqrt{x_{ij}y_{ij}}, |z_{ij}| - \sqrt{a_{ij}b_{ij}}\right\} \tag{6}$$

As usual for entanglement measures, C ranges from 0 (no entanglement) to 1, when the two qubits are maximally entangled.

3. Thermal entanglement

For the two-qubit case, after a straightforward calculation, the eigenvalues and eigenvectors of the system are analytically obtained as

$$|\psi_1\rangle = |+\rangle, \quad |\psi_2\rangle = |-\rangle,$$

$$|\psi_3\rangle = \frac{J\gamma|11\rangle + (B+\Delta)|00\rangle}{\sqrt{M}} \quad |\psi_4\rangle = \frac{J\gamma|11\rangle + (B-\Delta)|00\rangle}{\sqrt{N}}, \tag{7}$$

$$E_1 = \sqrt{J^2+D^2}, \quad E_2 = -\sqrt{J^2+D^2}, \quad E_3 = \Delta, \quad E_4 = -\Delta, \tag{8}$$

where $|\pm> = (1/\sqrt{2})(|10> \pm e^{i\alpha}|01>)$ in which $\alpha = \arctan(D/J)$ and $\Delta = \sqrt{J^2\gamma^2 + B^2}$, $M = 2B^2 + 2J^2\gamma^2 + 2B\Delta$, $N = 2B^2 + 2J^2\gamma^2 - 2B\Delta$. We see that the four eigenvalues are independent of the angel α. The state at thermal equilibrium is described by the density matrix $\rho = \frac{1}{Z}\exp(-\beta H) = \frac{1}{Z}\sum_{k=0}^{4}\exp(-\beta E_i)|\psi_k\rangle\langle\psi_k|$.

$$\rho = \frac{1}{Z}\begin{bmatrix} \frac{e^{\frac{-\Delta}{T}}}{M}(B+\Delta)^2 + \frac{e^{\frac{\Delta}{T}}}{N}(B-\Delta)^2 & & & \frac{J\gamma\sinh\left(\frac{\Delta}{T}\right)}{\Delta} \\ & \cosh\frac{\sqrt{J^2+D^2}}{T} & -\sinh\frac{\sqrt{J^2+D^2}}{T}e^{-i\alpha} & \\ & -\sinh\frac{\sqrt{J^2+D^2}}{T}e^{i\alpha} & \cosh\frac{\sqrt{J^2+D^2}}{T} & \\ \frac{J\gamma\sinh\left(\frac{\Delta}{T}\right)}{\Delta} & & & J^2\gamma^2\left(\frac{e^{\frac{-\Delta}{T}}}{M} + \frac{e^{\frac{\Delta}{T}}}{N}\right) \end{bmatrix}. \tag{9}$$

The square roots of the eigenvalues of the matrix R are

$$\lambda_{1,2} = |x \pm z|, \quad \lambda_{3,4} = |\sqrt{ab} \pm c|. \tag{10}$$

Based on the definition of concurrence, we can obtain the concurrence at finite temperature

$$C = \frac{2}{Z} \max \left(0, \left(\left| \sinh \frac{\sqrt{J^2 + D^2}}{T} \right| - \frac{\sqrt{2J^2\gamma^2 \left(\cosh \left(\frac{2\Delta}{T} \right) + 1 \right) + 4B^2}}{2\Delta}, \right. \right.$$
$$\left. \left. \left| \frac{J\gamma \sinh \left(\frac{\Delta}{T} \right)}{\Delta} \right| - \cosh \left(\frac{\sqrt{J^2 + D^2}}{T} \right) \right) \right), \tag{11}$$

where $Z = \mathrm{e}^{-\beta\Delta} + e^{\beta\Delta} + \mathrm{e}^{-\beta\sqrt{J^2+D^2}} + \mathrm{e}^{\beta\sqrt{J^2+D^2}}$. The concurrence $C = 0$ indicates vanishing entanglement. From the analytical expressions of C, we also see that entanglement does not depend on the sign of the anisotropic parameter D. The critical temperature T_c above which the concurrence is zero is determined by the equation of C.

Figure 1 demonstrates the dependence of thermal entanglement on B and T. Figure 1(a) shows the concurrence versus magnetic-field for different temperatures. The parameters $J = 1$, $\gamma = 0.5$, $D = 1$. From the figure we see that there is evidence of phase transition for low temperature by increasing magnetic field. And the value of entanglement decreases with the increase of temperature T. Figure 1(b) shows the concurrence versus temperature for different magnetic fields. The parameters $J = 1$, $\gamma = 0.6$, $D = 0.5$. As T increases, the concurrence decreases as seen from Fig.1(b) due to the mixing of other states with maximally entangled state. The critical temperature T_c is independent of magnetic field B. We can find that Fig.1 has different features in comparison with the two qubits XY model without DM interaction.[8]

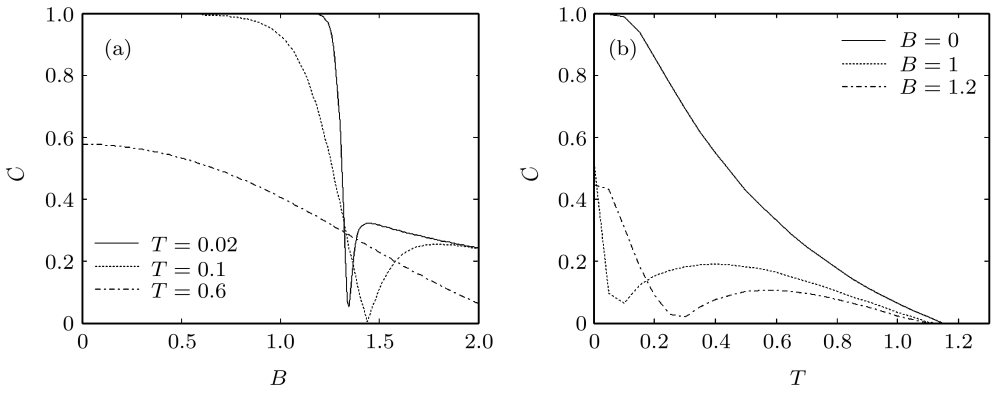


Fig.1. The concurrence is plotted as a function of the magnetic fields (a) and temperatures (b).

Figure 2 demonstrates the dependence of thermal entanglement on T for different D (a) and γ (b). The parameters of Fig.1(a) are $J = 1$, $\gamma = 0.6$, $B = 1$. Apparently, the critical temperature increases with the increase of the absolute value of the anisotropic parameter D, which can be seen in Fig.2(a). When the temperature is higher than the critical temperature, the entanglement disappears. The parameters of Fig.1(b) are $J = 1$, $D = 0.6$, $B = 1$. The figure shows that the critical temperature decreases as the anisotropic parameter increases form 0 to 1. And they have fluctuation respectively in the Fig.2(a) when $D = 0$ and in the Fig.2(b) when γ increases. It also can be found that the concurrence is 1 when $T = 0$.

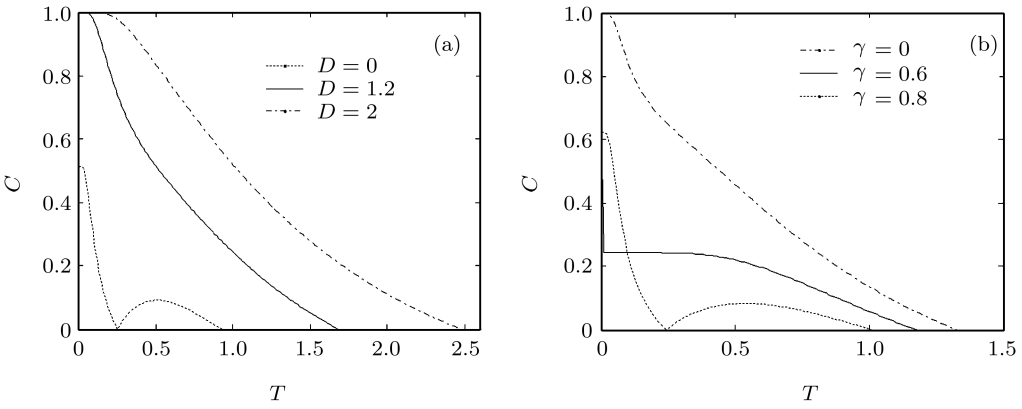


Fig.2. The concurrence is plotted as a function of the temperature T for different D (a) and γ (b).

4. Conclusion

In this paper, we have studied the pairwise entanglement in a two-qubit Heisenberg XY chain with the DM interaction. We obtain the analytical expressions of the concurrence C. Since thermal entanglement is a natural type of entanglement for a system embedded in a thermal environment, we expect that the present results could be useful for solid-state applications. Our results show some evidences that the quantum phase transition appears in this model. The entanglement is not completely determined by the partition function, i.e., by the usual quantum statistical physics. The entanglement is the same as the antiferromagnetic and ferromagnetic Heisenberg XY model with DM interaction. In this paper, we've only studied the two-qubit case. It is also worthwhile to study thermal entanglement in the multi-qubit models with DM interactions.

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