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Complex wave excitations and chaotic patterns for a general (2+1)-dimensional Korteweg–de Vries system*

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Starting from an improved mapping approach and a linear variable separation approach, a new family of exact solutions (including solitary wave solutions, periodic wave solutions and rational function solutions) with arbitrary functions for a general (2+1)-dimensional Korteweg de Vries system (GKdV) is derived. According to the derived solutions, we obtain some novel dromion-lattice solitons, complex wave excitations and chaotic patterns for the GKdV system.

Keywords: improved mapping approach, GKdV system, complex wave excitations, chaotic patterns
PACC: 0340, 0200, 0365

1. Introduction

As is well known, many dynamical problems in physics and other natural fields are usually characterized by nonlinear evolution of partial differential equations known as governing equations. Searching for an analytical exact solution to a nonlinear system has long been an important and interesting topic in nonlinear science both for physicists and mathematicians, and various methods of obtaining exact solutions of nonlinear system have been proposed, for example, the bilinear method,[1] the standard Painlevé truncated expansion,[2] the method of 'coalescence of eigenvalue' or 'wavenumbers',[3] the homogeneous balance method,[4] and the mapping method,[5−7] etc. In the past, we have solved the exact solutions of some nonlinear systems via improved mapping method, such as (1+1)-dimensional related Schrödinger equation, (2+1)-dimensional Broer–Kaup–Kupershmidt system, (2+1)-dimensional Generalized Breor–Kaup system, (3+1)-dimensional Burgers system, (3+1)-dimensional Jimbo–Miwa system, (2+1)-dimensional modified dispersive water-wave system, (2+1)-dimensional Boiti–Leon–Pempinelli system, etc.[8−15]

In addition, solitons and chaos are two most important notions of nonlinear science, which have been widely applied in natural sciences[16−18] such as chemistry, biology, mathematics, communication, and particularly in almost all branches of physics like fluid dynamics, plasma physics, field theory, optics, condensed matter physics, etc.[19,20] Usually, these two aspects are treated independently since one often believes that solitons are the basic excitations of an integrable model while chaos is the elementary behaviour of a non-integrable system. In other words, one does not analyse the possibility of existence of chaos in a soliton system. However, the above consideration may not be complete especially in higher dimensions. In the recent study of soliton systems, scientists[21−27] have found that some characteristic lower-dimensional arbitrary functions exist in the exact excitation of some two-dimensional integrable models. This means that any lower-dimensional chaotic solutions can be used to construct the exact solution of a higher-dimensional integrable model, which also implies that any exotic behaviour may propagate along this characteristic. To verify the above viewpoint, we take a (2+1)-dimensional GKdV system as a concrete example, which reads

$$u_t - u_{xxy} - auu_y - bu_x\partial_x^{-1}u_y = 0, \qquad (1)$$

where a and b are arbitrary constants. Many researchers have investigated some interesting properties of Eq.(1). For instance, the GKdV system has

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been proved to be integrable by Calogero,[28] and the Painlevé property test for the equation has been proved to be completely integrable only when $a = 2b$ by Clarkson and Manshield.[29] Some variable separation solutions for the special model for $a = b$ have also been obtained by Lou and Ruan.[30] Zheng and Chen found some semifolded localized coherent structures by using a multilinear variable separation method.[31]

In the following parts of the paper, we will discuss its new exact solutions and the dromion-lattice solitons, the complex wave excitations and the chaotic patterns for the integrable GKdV system ($a = 2b$). For simplicity, we first introduce a transformation $v = \partial_x^{-1} u_y$ and change the GKdV system into a set of two coupled nonlinear partial differential equations

$$u_t - u_{xxy} - 2buv_x - bvu_x = 0,$$

and

$$u_y - v_x = 0.$$

As is well known, in order to search for the solitary wave solutions for a nonlinear physical model, we can adopt different approaches. One of the most efficient methods of finding soliton excitations of a physical model is the so-called modified mapping approach with variable coefficients. The basic ideal of the algorithm is as follows. For a given nonlinear partial differential equation (NPDE) with the independent variables $x = (x_0 = t, x_1, x_2, \ldots, x_m)$, and the dependent variable u, in the form

$$P(u, u_t, u_{x_i}, u_{x_i x_j}, \ldots) = 0, \tag{3}$$

where P is, in general, a polynomial function of its arguments, and the subscripts denote the partial derivatives, the solution can be assumed to be in the form

$$\begin{aligned} u =& A(x) + \sum_{i=1}^{n} \{B_i(x)\phi^i[q(x)] \\ &+ C_i(x)\phi^{i-1}[q(x)]\sqrt{\sigma + \phi^2[q(x)]}\} \end{aligned} \tag{4}$$

with

$$\phi' = \sigma + \phi^2, \tag{5}$$

where σ is a constant and the prime denotes the first derivative with respect to q. To determine u explicitly, one may substitute expression (4) and Eq.(5) into the given NPDE and collect coefficients of polynomials of ϕ, then eliminate each coefficient to derive a set of partial differential equations of A, B_i, C_i, and q, and solve the system of partial differential equations to obtain A, B_i, C_i, and q. Finally, Eq.(5) possesses the general solution (the tanh-type and tan-type solutions are neglected here) as follows:

$$\begin{aligned} &\text{(a)} \quad \phi = -\sqrt{-\sigma}\coth(\sqrt{-\sigma}q), \quad \text{when} \quad \sigma < 0; \\ &\text{(b)} \quad \phi = -\sqrt{\sigma}\cot(\sqrt{\sigma}q), \quad \text{when} \quad \sigma > 0; \\ &\text{(c)} \quad \phi = \frac{-1}{q}, \quad \text{when} \quad \sigma = 0. \end{aligned} \tag{6}$$

Substituting A, B_i, C_i, q and expression (6) and into expression (4), one can obtain the exact solutions of the given NPDE.

2. New exact solutions of the (2+1)-dimensional GKdV system

First, we make a transformation for Eq.(2): $u = Q_x$ and $v = Q_y$. Substituting the transformation into Eq.(2) yields

$$Q_{xt} - Q_{xxxy} - 2bQ_xQ_{xy} - bQ_{xx}Q_y = 0. \tag{7}$$

Now we apply the improved mapping approach to Eq.(7). Through the balancing procedure, the ansatz (4) becomes

$$Q = f + g\phi + h\sqrt{\sigma + \phi^2}, \tag{8}$$

where f, g, h, and q are functions of (x, y, t) to be determined. Substituting expression (8) and Eq.(5) into Eq.(7) and collecting coefficients of polynomials of ϕ, then setting each coefficient to be zero, we have

$$bgq_x^2hq_y + 2hq_yq_x^3 = 0, \tag{9}$$

$$bh^2q_x^2q_y + bg^2q_x^2q_y + 4gq_yq_x^3 = 0, \tag{10}$$

$$\begin{aligned} &- 4bhq_x^2g_y - 8bgq_xh_xq_y - 18hq_xq_{xx}q_y - 4bgq_x^2h_y - 8bg_xhq_yq_x - 2bgq_{xx}hq_y \\ &- 18h_xq_x^2q_y - 4bgq_xhq_{xy} - 18hq_x^2q_{xy} - 6h_yq_x^3 = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &- 6g_yq_x^3 - 18g_xq_yq_x^2 - 18gq_x^2q_{xy} - 2bg^2q_xq_{xy} - bg^2q_{xx}q_y - 2bh^2q_xq_{xy} - bh^2q_{xx}q_y \\ &- 4bgq_x^2g_y - 8bh_xhq_yq_x - 4bhq_x^2h_y - 8bgq_xg_xq_y - 18gq_yq_xq_{xx} = 0, \end{aligned} \tag{12}$$

$$
\begin{aligned}
&-6h_{xx}q_yq_x+2hq_xq_t-6hq_xq_{xxy}-12h_xq_xq_{xy}-6hq_{xy}q_{xx}-6h_yq_xq_{xx}-6h_xq_yq_{xx}-2hq_{xxx}q_y\\
&-bg_{xx}hq_y-2bgq_xh_{xy}-bhq_{xx}g_y-4bh_xg_yq_x-2bh_xgq_{xy}-27bgq_x^2hq_y\sigma-2bhq_xg_{xy}-bgq_{xx}h_y\\
&-bh_{xx}gq_y-2bg_xhq_{xy}-4bf_xhq_yq_x-52hq_x^3q_y\sigma-4bh_xg_xq_y-4bg_xq_xh_y-6h_{xy}q_x^2=0,
\end{aligned}\tag{13}
$$

$$
\begin{aligned}
&-6g_{xx}q_yq_x-6gq_xq_{xxy}-2gq_yq_{xxx}-6g_xq_yq_{xx}-12g_xq_xq_{xy}-6g_{xy}q_x^2-bh_{xx}hq_y-bhq_{xx}h_y\\
&-2bgq_xg_{xy}-2bh_xhq_{xy}-bgq_{xx}g_y-bg_{xx}gq_y-2bg_xgq_{xy}-2bhq_xh_{xy}-12bg^2q_y\sigma-4bf_xgq_yq_x\\
&-4bh_xh_yq_x-6gq_{xy}q_{xx}-4bg_xg_yq_x-9bh^2q_x^2q_y\sigma-40gq_yq_x^3\sigma\\
&-2bg_x^2q_y-2bh_x^2q_y+2gq_xq_t-6g_yq_xq_{xx}=0,
\end{aligned}\tag{14}
$$

$$
\begin{aligned}
&-2bh_xg_{xy}-bh_{xx}g_y-bg_{xx}h_y-2bg_xh_{xy}-11h_yq_x^3\sigma-3h_{xy}q_{xx}-3h_xq_{xxy}+hq_{xt}-3h_{xxy}q_x\\
&+h_xq_t-2bf_xh_yq_x-14bg_xhq_xq_y\sigma-16bgq_xh_xq_y\sigma-8bgq_x^2h_y\sigma-7bhq_x^2g_y\sigma-2bf_xh_xq_y\\
&-33h_xq_yq_x^2\sigma+h_tq_x-hq_{xxxy}-h_yq_{xxx}-h_{xxx}q_y-3h_{xx}q_{xy}-4bgq_{xx}hq_y\sigma\\
&-33hq_yq_xq_{xx}\sigma-33hq_x^2q_{xy}\sigma-bf_{xx}hq_y-2bf_xhq_{xy}-8bgq_xhq_{xy}\sigma=0,
\end{aligned}\tag{15}
$$

$$
\begin{aligned}
&-3g_xq_{xxy}-2bg_xg_{xy}-g_yq_{xxx}+g_tq_x-3g_{xxy}q_x-3g_{xx}q_{xy}+gq_{xt}-g_{xxx}q_y-3g_{xy}q_{xx}\\
&-gq_{xxxy}+g_xq_t-bg_{xx}g_y-24gq_x^2q_{xy}\sigma-bf_{xx}gq_y-2bh_xh_{xy}-2bf_xgq_{xy}-bh^2q_xq_{xy}\sigma\\
&-2bg^2q_{xx}q_y\sigma-4bg^2q_xq_{xy}\sigma-2bh^2q_xq_{xy}\sigma-24gq_yq_xq_{xx}\sigma-bh_{xx}h_y\\
&-8g_yq_x^3\sigma-6bgq_x^2g_y\sigma-5bhq_x^2h_y\sigma-2bf_xg_xq_y-24g_xq_yq_x^2\sigma\\
&-2bf_xg_yq_x-12bgq_xg_xq_y\sigma-10bh_xhq_xq_y\sigma=0,
\end{aligned}\tag{16}
$$

$$
\begin{aligned}
&-bf_{xx}h_y-2bf_xh_{xy}-4bgq_xh_{xy}\sigma-9h_{xy}q_x^2\sigma-3hq_{xxx}q_y\sigma-9h_xq_yq_{xx}\sigma\\
&-9hq_{xy}q_{xx}\sigma-9h_{xx}q_yq_x\sigma-18h_xq_xq_{xy}\sigma-9hq_xq_{xxy}\sigma-2bgq_{xx}h_y\sigma\\
&-2bh_{xx}gq_y\sigma-2bg_xhq_{xy}\sigma-2bhq_xg_{xy}\sigma-4bh_xgq_{xy}\sigma-bg_{xx}hq_y\sigma-bhq_{xx}g_y\sigma\\
&-9h_yq_xq_{xx}\sigma-h_{xxxy}+h_{xt}-33hq_x^3q_y\sigma^2-6bh_xg_yq_x\sigma-6bh_xg_xq_y\sigma\\
&-6bg_xq_xh_y\sigma-6bf_xhq_xq_y\sigma-18bgq_x^2hq_y\sigma^2+3hq_xq_t\sigma=0,
\end{aligned}\tag{17}
$$

$$
\begin{aligned}
&-bg_{xx}gq_y\sigma-2bgq_xg_{xy}\sigma-2bg_xgq_{xy}\sigma-6gq_{xy}q_{xx}\sigma-6gq_xq_{xxy}\sigma-2bh_xhq_{xy}\sigma\\
&-bf_{xx}g_y-12g_xq_xq_{xy}\sigma-bh_{xx}hq_y\sigma-g_{xxxy}+g_{xt}-16gq_yq_x^3\sigma-2bh_x^2q_y\sigma\\
&-6bg^2q_x^2q_y\sigma^2-6g_{xy}q_x^2\sigma-4bf_xgq_yq_x\sigma+2gq_xq_t\sigma-4bg_xg_yq_x\sigma-3bh^2q_x^2q_y\sigma^2\\
&-2bg_x^2q_y\sigma-4bh_xh_yq_x\sigma-2bf_xg_{xy}-bhq_{xx}h_y\sigma-bgq_{xx}g_y\sigma-6g_{xx}q_yq_x\sigma\\
&-6g_yq_xq_{xx}\sigma-2bhq_xh_{xy}\sigma-2gq_yq_{xxx}\sigma-6g_xq_yq_{xx}\sigma=0,
\end{aligned}\tag{18}
$$

$$
\begin{aligned}
&h_xq_t\sigma-3h_xq_{xxy}\sigma-2bgq_{xx}hq_y\sigma^2-2bg_xh_{xy}\sigma-2bh_xg_{xy}\sigma-bh_{xx}g_y\sigma+hq_{xt}\sigma\\
&-15hq_yq_xq_{xx}\sigma^2-3h_{xy}q_{xx}\sigma-bf_{xx}hq_y\sigma-3h_{xx}q_{xy}\sigma-hq_{xxxy}\sigma-3h_{xxy}q_x\sigma\\
&-bg_{xx}h_y\sigma-2bf_xhq_{xy}\sigma-4bgq_xhq_{xy}\sigma^2-15hq_x^2q_{xy}\sigma^2+h_tq_x\sigma-h_{xxx}q_y\sigma\\
&-h_yq_{xxx}\sigma-2bf_xh_yq_x\sigma-15h_xq_yq_x^2-2bf_xh_xq_y\sigma^2-2bf_xh_xq_y\sigma-4bgq_x^2h_y\sigma^2\\
&-6bg_xhq_xq_y\sigma^2-8bgq_xh_xq_y\sigma^2-3bhq_x^2g_y\sigma^2-5h_yq_x^3\sigma^2=0,
\end{aligned}\tag{19}
$$

$$
\begin{aligned}
&-bf_{xx}gq_y\sigma+g_xq_t\sigma-2bh_xhq_xq_y\sigma^2-2g_yq_x^3\sigma^2-2bf_xg_yq_x\sigma-2bgq_x^2g_y\sigma^2\\
&-2bf_xg_xq_y\sigma-4bgq_xg_xq_y\sigma^2-6g_xq_yq_x^2\sigma^2+g_tq_x\sigma-3g_xq_{xxy}\sigma-bhq_x^2h_y\sigma^2\\
&+gq_{xt}\sigma-2bgf_xq_{xy}\sigma-6gq_yq_xq_{xx}\sigma^2-2bg^2q_xq_{xy}\sigma^2-bg^2q_{xx}q_y\sigma^2\\
&-6gq_x^2q_{xy}\sigma^2-bh_{xx}h_y\sigma-2bh_xh_{xy}\sigma+f_{xt}-g_yq_{xxx}\sigma-g_{xxx}q_y\sigma\\
&-gq_{xxxy}\sigma-3g_{xxy}q_x\sigma-3g_{xy}q_{xx}\sigma-3g_{xx}q_{xy}\sigma=0,
\end{aligned}\tag{20}
$$

$$
\begin{aligned}
&hq_xq_t\sigma^2 - 2bh_xg_xq_y\sigma^2 - 2bg_xq_xh_y\sigma^2 - bgq_{xx}h_y\sigma^2 - h_{xxxy}\sigma - 2bf_xh_{xy}\sigma \\
&- 3h_{xx}q_xq_y\sigma^2 - bf_{xx}h_y\sigma - hq_{xxx}q_y\sigma^2 - 3h_xq_yq_{xx}\sigma^2 - 3hq_xq_{xxy}\sigma^2 \\
&- 5hq_x^3q_y\sigma^3 - 2bh_xg_yq_x\sigma^2 - 3bgq_x^2hq_y\sigma^3 - 2bhf_xq_xq_y\sigma^2 - 2bgq_xh_{xy}\sigma^2 \\
&- 2bgh_xq_{xy}\sigma^2 - 6h_xq_xq_{xy}\sigma^2 - 3h_yq_xq_{xx}\sigma^2 - 3h_{xy}q_x^2\sigma^2 \\
&- bgh_{xx}q_y\sigma^2 + h_{xt}\sigma - 3hq_{xy}q_{xx}\sigma^2 = 0.
\end{aligned} \tag{21}
$$

According to Eqs.(9) and (10), we have

$$
g = -2\frac{q_x}{b}, \quad \text{and} \quad h = 2\frac{q_x}{b}. \tag{22}
$$

Substituting expression (22) into Eq.(13) yields

$$
f = -\frac{1}{2b}\int \frac{-2q_x^2q_{xxy} - 2q_xq_{xxx}q_y + 2q_xq_{xy}q_{xx} + q_x^4q_y\sigma + q_{xx}^2q_y + q_x^2q_t}{q_x^2q_y}\mathrm{d}x. \tag{23}
$$

Substituting expressions (22) and (23) into Eqs.(9)–(21) yields the following single equation:

$$
\begin{aligned}
&-3[-2q_x62q_yq_{xxxy} + q_x^2q_yq_{xt} - 2q_{xy}^2q_xq_{xx} + 2q_{xy}q_x^2q_{xxy} - q_{xy}q_x^2q_t - 2q_yq_{xx}^2q_{xy} \\
&+ 2q_xq_yq_{xxy}q_{xx} + 2q_{xy}q_xq_{xxx}q_y]/(bq_xq_y) = 0.
\end{aligned} \tag{24}
$$

Obviously, it is very difficult to obtain the general solution to the above equation. Fortunately, in this special case, one of particular solutions can be expressed as

$$
q = \chi(x) + \varphi(y - ct), \tag{25}
$$

where $\chi \equiv \chi(x)$, and $\varphi \equiv \varphi(y - ct)$ are two arbitrary variable separation functions of x and of $(y - ct)$, respectively. According to the solution of Eq.(5), one thus obtains an explicit solution of Eq.(2).

Case 1 For $\sigma = -1$, we can derive the following solitary wave solution of Eq.(2):

$$
\begin{aligned}
u_1 =& \frac{1}{2b}\Bigg[-\frac{2\chi_x\chi_{xxx} + \chi_x^4 - \chi_{xx}^2 + \chi_x^2c}{\chi_x^2} + 4\chi_{xx}(\coth(\chi+\varphi) + \operatorname{csch}(\chi+\varphi)) \\
&- 4\chi_x^2(\operatorname{csch}(\chi+\varphi)^2 + \operatorname{csch}(\chi+\varphi)\coth(\chi+\varphi))\Bigg],
\end{aligned} \tag{26}
$$

and

$$
v_1 = -\frac{1}{2b}[4\varphi_y\chi_x(\operatorname{csch}(\chi+\varphi)^2 + \operatorname{csch}(\chi+\varphi)\coth(\chi+\varphi))]. \tag{27}
$$

Case 2 For $\sigma = 1$, we can obtain the following periodic wave solution of Eq.(2):

$$
\begin{aligned}
u_2 =& \frac{1}{2b}\Bigg[-\frac{2\chi_x\chi_{xxx} - \chi_x^4 - \chi_{xx}^2 + \chi_x^2c}{\chi_x^2} + 4\chi_{xx}(\cot(\chi+\varphi) + \csc(\chi+\varphi)) \\
&- 4\chi_x^2(\csc(\chi+\varphi)^2 + \csc(\chi+\varphi)\cot(\chi+\varphi))\Bigg],
\end{aligned} \tag{28}
$$

and

$$
v_2 = -\frac{1}{2b}[4\varphi_y\chi_x(\csc(\chi+\varphi)^2 + \csc(\chi+\varphi)\cot(\chi+\varphi))]. \tag{29}
$$

Case 3 For $\sigma = 0$, we can derive the following variable separation solution of Eq.(2):

$$
u_3 = -\frac{1}{2}\frac{-\chi_{xx}^2 + \chi_x^2c + 2\chi_x\chi_{xxx}}{b\chi_x^2} + 4\frac{\chi_{xx}}{b(\chi+\varphi)} - 4\frac{\chi_x^2}{b(\chi+\varphi)^2}, \tag{30}
$$

and

$$
v_3 = -4\frac{\varphi_y\chi_x}{b(\chi+\varphi)^2}. \tag{31}
$$

3. Dromion-lattice and complex wave excitations in the (2+1)-dimensional GKdV system

Owing to the arbitrariness of the functions $\chi(x)$ and $\varphi(y-ct)$ included in the above cases, the physical quantities u and v may possess rich structures. In this section, we mainly discuss some localized coherent excitations in the (2+1)-dimensional GKdV system. For simplicity, we discuss only the field v_3 of expression (31), namely

$$V = v_3 = -4\frac{\varphi_y \chi_x}{b(\chi+\varphi)^2}. \tag{32}$$

3.1. Dromion-lattice excitation

According to the solution V of expression (32), we first discuss its dromion-lattice excitation. If we choose χ and φ as

$$\chi = \sum_{n=-N}^{N} 0.1\mathrm{sech}(x+6n)$$

and

$$\varphi = \sum_{m=-M}^{M} 0.1\mathrm{sech}(y-ct+6m), \tag{33}$$

$$M = N = 2,$$

we can obtain a 5×5 'dromion-lattice' for the physical quantity V under condition (33) presented in Fig.1 with fixed parameters $b=-0.5$, $c=1$, and $t=0$.

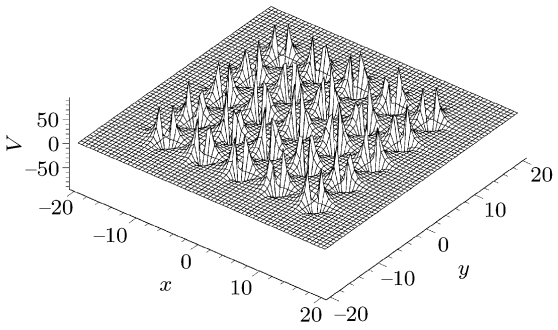


Fig.1. A dromion-lattice plot of the solution V from expression (32) under condition (33) at time $t=0$.

3.2. Complex wave excitation: dromion in a background wave

Now we focus our attention on the intriguing evolution of a dromion in a background wave for the solution V. For instance, if we choose χ and φ as

$$\chi = 2 + 0.18\tanh(0.5x-t) + 0.04\mathrm{sn}(0.7x, 0.3),$$

and

$$\varphi = 2+0.18\tanh(0.5(y-ct))+0.04\mathrm{sn}(0.7y, 0.3), \tag{34}$$

where, sn is the Jacobi sine function, we can obtain an evolutional profile of single-dromion in a background wave for the physical quantity V of expression (32) presented in Fig.2 with fixed parameters $b=-0.5$, and $c=1$, at different times $t=-8$ (a), -4 (b), 4 (c) and 10 (d). From Fig.2, we can see that the dromion moves in a background wave, and the amplitude of dromion changes with time because of the interaction between the dromion and the background wave.

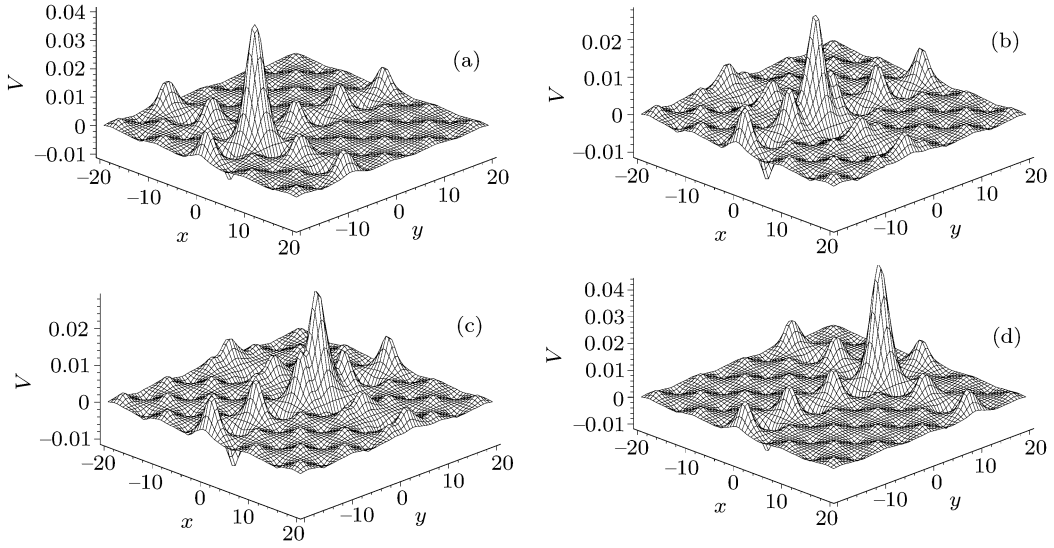


Fig.2. Evolutional profile of a single-dromion in a background wave for the solution V from expression (32) with condition (34) at different times $t=-8$ (a), -4 (b), 4 (c) and 10 (d).

4. Chaotic patterns in the (2+1)-dimensional GKdV system

In this section, we mainly discuss some localized coherent excitations with chaotic behaviour in the (2+1)-dimensional GKdV system. For simplicity, we discuss only the field v_1 of expression (27), namely

$$V = v_1 = -\frac{1}{2b}[4\varphi_y\chi_x(\mathrm{csch}(\chi+\varphi)^2 + \mathrm{csch}(\chi+\varphi)\coth(\chi+\varphi))]. \quad (35)$$

4.1. Chaotic line soliton

If the functions χ and φ are assumed to be solutions of a chaotic dynamical system, we can derive some localized excitations with chaotic behaviour. For example, χ is determined to be a solution of the following nuclear spin generator (NSG) system,[5]

$$m_\xi = -1.3m + n, \quad n_\xi = -m - 1.3n(1-3l),$$

and

$$l_\xi = 0.15 - 0.52l - 3.9n^2, \quad (36)$$

where m, n, and l are functions of ξ ($\xi = x$ or $\xi = y - ct$). The NSG system is a high frequency oscillator which generates and controls the oscillations of a nuclear magnetization vector in a magnetic field. One of typical chaotic attractors for the NSG system (36) is depicted in Fig.3 when

$$m(0) = 1, \quad n(0) = 2 \ \ and \ l(0) = 0. \quad (37)$$

Now we take

$$\chi(x) = 1 + m(x) \ \ and \ \ \varphi = 1 + \exp(y - ct), \quad (38)$$

where $m(x)$ is the solution of NSG system (36) with initial conditions (37). Through this choice, the dromion localized in all directions is changed into a chaotic line soliton, which presents chaotic behaviour in the x-direction though it is still localized in the y-direction. Figure 4 shows the corresponding plot of the chaotic line soliton for the field V of expression (35) with fixed parameters $b = 1$ and $c = 1$ at time $t = 0$.

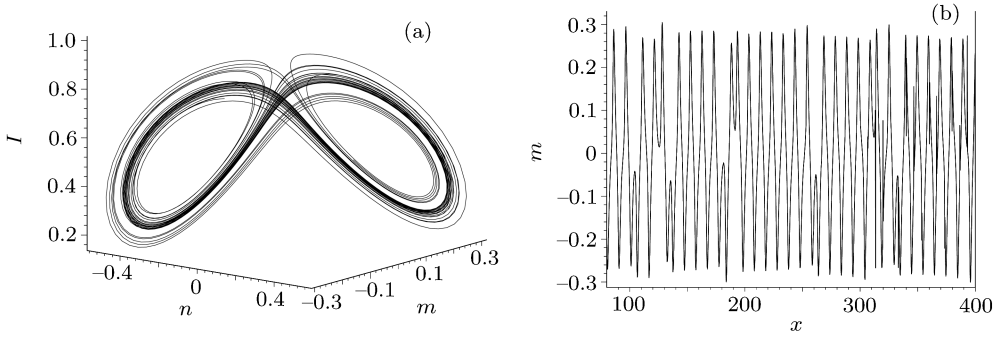


Fig.3. (a) A typical attractor plot of the chaotic NSG system (36) with initial condition (37), and (b) a typical plot of the chaotic solution m of system (36) related to (a).

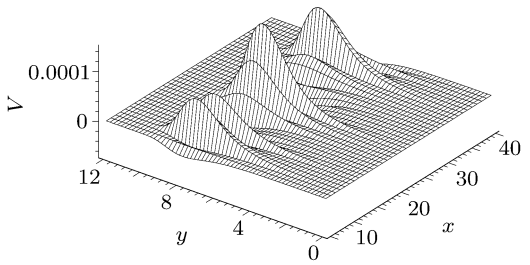


Fig.4. A plot of the chaotic line soliton for the field V determined from expression (35) with condition (38), $b = 1$, $c = 1$ and $t = 0$.

4.2. Chaotic pattern

Furthermore, if χ and φ are both selected as chaotic solutions of some lower-dimensional non-integrable model, the field V of expression (35) will behave chaotically in all directions and yield a chaotic pattern. For example, χ and φ may be chosen as

$$\chi = 1 + m(x) \ \ and \ \ \varphi = 1 + m(y - ct), \quad (39)$$

where $m(x)$ and $m(y - ct)$ are the solutions of NSG system (36). Figure 5(a) shows a plot of the special chaotic pattern for field V expressed by expression (35) with condition (39), $b = 1$ and $c = 1$ at time $t = 0$. In order to show that the chaotic behaviour is due to the peak values of solitons, we enlarge small region in Fig.5(a) ($x \in [10, 11]$, $y \in [10, 11]$), and the result is shown in Fig.5(b), which presents clearly a kind of dromion with a chaotic structure.

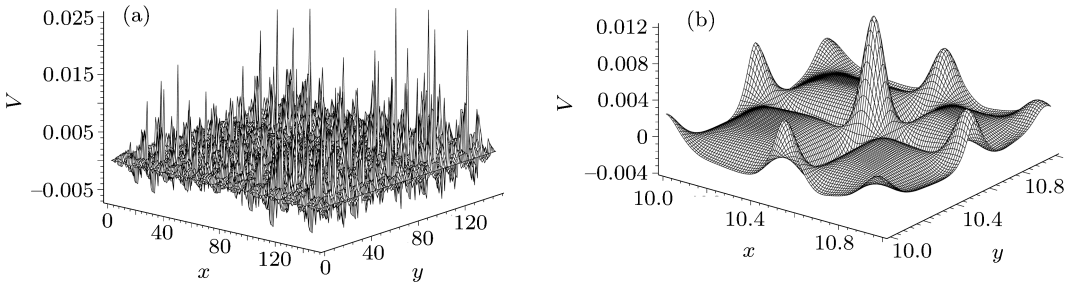


Fig.5. (a) A plot of the chaotic patten for the field V expressed by expression (35) with condition (39) at $t = 0$, and (b) an enlargement of the central area of (a).

5. Summary and discussion

The mapping method is a kind of classic, efficient and well-developed method to solve nonlinear evolution equations, of which the remarkable characteristics are that we can infinitely have different ansatzes and thus end up with the abundance of solutions. In this paper, by using the modified mapping approach and a linear variable separation approach, we find some new exact solutions of the general (2+1)-dimensional Korteweg–de Vries system. According to the derived solutions, we obtain a 5×5 dromion-lattice excitation and a complex wave excitation—a dromion moves in a background wave, which are different from the results of the previous work. Additionally, by using the NSG system, Fang *et al* recently obtained some chaotic solitons of the (2+1)-dimensional Generalized Broer–Kaup system.[5] Along with the above line, we use the NGS system to obtain new chaotic patterns of the (2+1)-dimensional GKdV system. Especially, we enlarge the small region in Fig.5(a) to show that the chaotic behaviour is due to the peak values of solitons.

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