Vol 17 No 8, August 2008
1674-1056/2008/17(08)/2753-06

Chinese Physics B

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On the feasibility of variable separation method based on Hamiltonian system for plane magnetoelectroelastic solids*

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(Received 1 January 2008; revised manuscript received 21 February 2008)

The eigenvalue problem of an infinite-dimensional Hamiltonian operator appearing in the isotropic plane magnetoelectroelastic solids is studied. First, all the eigenvalues and their eigenfunctions in a rectangular domain are solved directly. Then the completeness of the eigenfunction system is proved, which offers a theoretic guarantee of the feasibility of variable separation method based on a Hamiltonian system for isotropic plane magnetoelectroelastic solids. Finally, the general solution for the equation in the rectangular domain is obtained by using the symplectic Fourier expansion method.

Keywords: magnetoelectroelastic solid, variable separation method, completeness, general solution
PACC: 0200, 0340D, 0420J

1. Introduction

Magnetoelectroelastic solids possessing simultaneously piezoelectric, piezomagnetic and magnetoelectric effects, are a kind of new functional material. Their mechanical and physical behaviours have attracted more and more attention.[1−4] Because of multi-field coupling, the magnetoelectroelastic solid problem is more difficult to solve than the elasticity problem. In Ref.[5], the author investigated the isotropic plane magnetoelectroelastic solid problem by using variable separation method based on a Hamiltonian system. But, the feasibility of variable separation method in a Hamiltonian system for the isotropic plane magnetoelectroelastic solid equations has not been considered.

In this paper, the isotropic plane magnetoelectroelastic solid problem is studied. The feasibility of variable separation method in a Hamiltonian system for the isotropic plane magnetoelectroelastic solids is demonstrated. In other words, we prove the completeness of an eigenfunction system for an infinite-dimensional Hamiltonian operator. And the general solution for the isotropic plane magnetoelectroelastic solid problem with the homogeneous boundary conditions is given.

2. Basic equations and Hamiltonian system

2.1. Basic equations

The basic equations of isotropic magnetoelectroelastic solids in the xy plane are as follows:

(1) Governing equations:

$$\begin{cases} \dfrac{\partial \tau_{zx}}{\partial x} + \dfrac{\partial \tau_{zy}}{\partial y} = -f, \\ \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} = \rho, \\ \dfrac{\partial B_x}{\partial x} + \dfrac{\partial B_y}{\partial y} = 0; \end{cases} \tag{1}$$

(2) Constitutive equations:

$$\begin{pmatrix} \tau_{zx} \\ D_x \\ B_x \end{pmatrix} = C \frac{\partial}{\partial x} \begin{pmatrix} w \\ \varphi \\ \psi \end{pmatrix}$$

and

$$\begin{pmatrix} \tau_{zy} \\ D_y \\ B_y \end{pmatrix} = C \frac{\partial}{\partial y} \begin{pmatrix} w \\ \varphi \\ \psi \end{pmatrix}, \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant No 10562002), the Natural Science Foundation of Inner Mongolia, China (Grant No 200508010103), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No 20070126002) and the Inner Mongolia University Doctoral Scientific Research Starting Foundation.
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where

$$C = \begin{pmatrix} c_{11} & e_{15} & q_{15} \\ e_{15} & -\alpha_{11} & -\beta_{11} \\ q_{15} & -\beta_{11} & -\gamma_{11} \end{pmatrix} \tag{3}$$

is a positive definite matrix. Here w, φ and ψ are the elastic displacement, the electric potential and the magnetic potential, respectively; τ_{zx} and τ_{zy} are the stress components; D_x and D_y are the electric displacement components; B_x and B_y are the magnetic induction components; c_{11}, α_{11} and γ_{11} are the elastic modus, the dielectric permittivity and the magnetic permittivity, respectively; e_{15}, q_{15} and β_{11} are the piezoelectric, the piezomagnetic and the magnetoelectric constants, respectively; f and ρ are the body force and the density of free charges in the region, respectively.

2.2. Derived to Hamiltonian system

We consider a rectangular domain as shown in Fig.1 and the homogeneous boundary conditions

$$w = 0, \quad \varphi = 0, \quad \psi = 0, \text{ and } x = \pm h. \tag{4}$$

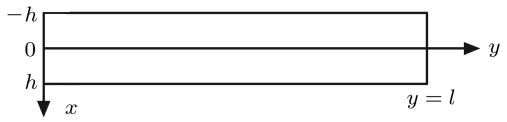


Fig.1. Rectangular domain.

The vertical coordinate y here is employed to simulate the time variable in a Hamiltonian system. Let $U = (w, \varphi, \psi)^{\mathrm{T}}$, $V = C\dfrac{\partial U}{\partial y} = (\tau_{zy}, D_y, B_y)^{\mathrm{T}}$, then Eqs.(1) and (2) are derived into the following infinite-dimensional Hamiltonian system:[5]

$$\frac{\partial}{\partial y}\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} 0 & C^{-1} \\ -C\dfrac{\partial^2}{\partial x^2} & 0 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} + \begin{pmatrix} 0 \\ f \end{pmatrix},$$

$$U = 0, \text{ and } x = \pm h, \tag{5}$$

where $f = (-f, \rho, 0)^{\mathrm{T}}$.

3. Variable separation method and the completeness of eigenfunction system

3.1. Variable separation method

In order to solve the nonhomogeneous linear partial differential equation (5), the following homogeneous equations should be solved first:

$$\frac{\partial}{\partial y}\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} 0 & C^{-1} \\ -C\dfrac{\partial^2}{\partial x^2} & 0 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix}, \tag{6}$$

$$U = 0, \text{ and } x = \pm h.$$

Equations (6) can be solved usually by using the variable separation method. Let

$$(U^{\mathrm{T}}, V^{\mathrm{T}})^{\mathrm{T}} = Q(x)T(y),$$

and substitute it into Eq.(6), then we will obtain the following eigenfunction equation:

$$HQ(x) = \lambda Q(x), \tag{7}$$

and

$$T(y) = \mathrm{e}^{\lambda y}, \tag{8}$$

where $Q(x)$ is a 6×3 matrix; $T(y)$ is a 3×1 matrix; λ is the eigenvalue to be solved; $Q(x)$ is the eigenfunction vector which must satisfy the boundary condition $Q(\pm h) = 0$. The operator H in Eq.(7) is called the infinite-dimensional Hamiltonian operator, and it is expressed as

$$H = \begin{pmatrix} 0 & C^{-1} \\ -C\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} & 0 \end{pmatrix},$$

$$\mathscr{D}(H) = \left\{ \begin{pmatrix} u \\ v \end{pmatrix} \in Z \times Z | u' \text{ is absolutely continuous}, \right.$$

$$\left. u(\pm h) = 0, u'' \in Z \right\}, \tag{9}$$

where $Z = L_2^{3\times 3}[-h, h]$, with the symbol $L_2^{3\times 3}[-h, h]$ denoting the set of all 3×3 matrices, in which each element is adopted from the function space $L_2[-h, h]$.

3.2. The completeness of eigenfunction system

By solving Eq.(7) with boundary conditions $Q(\pm h) = 0$, the eigenvalues λ_n and eigenvectors $\{Q_n\}$ of the operator H are given by

$$\lambda_{2k} = \frac{k\pi}{h}, \quad Q_{2k} = \begin{pmatrix} C^{-\frac{1}{2}} \\ \dfrac{k\pi}{h} C^{\frac{1}{2}} \end{pmatrix} \sin\frac{k\pi x}{h},$$

$$\lambda_{2k-1} = \frac{(2k-1)\pi}{2h}, \text{ and}$$

$$Q_{2k-1} = \begin{pmatrix} C^{-\frac{1}{2}} \\ \dfrac{(2k-1)\pi}{2h} C^{\frac{1}{2}} \end{pmatrix} \cos\frac{(2k-1)\pi x}{2h},$$

$$k = \pm 1, \pm 2, \ldots, \tag{10}$$

where $C^{\frac{1}{2}}$ and $C^{-\frac{1}{2}}$ are square roots of the matrices C and C^{-1}, respectively.

Lemma 1 The system of functions $\{q_n\}_{n=1}^{\infty}$, where

$$q_{2k} = \sin\frac{k\pi x}{h},$$

and

$$q_{2k-1} = \cos\frac{(2k-1)\pi x}{2h}, \quad k = 1, 2, \ldots \tag{11}$$

is a orthogonal system of functions in the function space $L_2[-h, h]$.

The next statement plays a very important role in proving our main result.

Lemma 2 The eigenfunction systems $\{Q_n\}_{-\infty}^{\infty}$ (n is non-zero integer) of infinite-dimensional Hamiltonian operator (9) are symplectic orthogonal to each other, that is,

$$\int_{-h}^{h} Q_{2k}^{\mathrm{T}} J Q_{2s}\,\mathrm{d}x = \begin{cases} 0, & k+s \neq 0, \\ 2k\pi I_3, & k+s = 0, \end{cases} \tag{12}$$

$$\int_{-h}^{h} Q_{2k-1}^{\mathrm{T}} J Q_{2s-1}\,\mathrm{d}x = \begin{cases} 0, & k+s-1 \neq 0, \\ -(2k-1)\pi I_3, & k+s-1 = 0, \end{cases} \tag{13}$$

and

$$\int_{-h}^{h} Q_{2k}^{\mathrm{T}} J Q_{2s-1}\,\mathrm{d}x = 0, \tag{14}$$

where

$$J = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}, \quad \text{and} \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The proof of Lemma 2 is simple by straightforward calculation. Therefore, it is omitted here.

Taking advantage of Lemma 1 and Lemma 2, we have the following theorem.

Theorem 1 Eigenfunction system (10) of infinite-dimensional Hamiltonian operator (9) is a complete system in the sense of Cauchy's principal value in the function space $Z \times Z$. Namely, for any $\begin{pmatrix} f \\ g \end{pmatrix} \in Z \times Z$, $\begin{pmatrix} f \\ g \end{pmatrix}$ has the following expression of symplectic Fourier series by using the eigenfunctions $\{Q_n\}(n = \pm1, \pm2, \ldots)$:

$$\begin{pmatrix} f \\ g \end{pmatrix} = \sum_{k=1}^{\infty} (Q_{2k}C_{2k} + Q_{-2k}C_{-2k} + Q_{2k-1}C_{2k-1} + Q_{-2k+1}C_{-2k+1}),$$

where

$$C_{2k} = -\frac{\int_{-h}^{h} Q_{-2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{2k\pi},$$

$$C_{-2k} = \frac{\int_{-h}^{h} Q_{2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{2k\pi},$$

$$C_{2k-1} = \frac{\int_{-h}^{h} Q_{-2k+1}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{(2k-1)\pi},$$

and

$$C_{-2k+1} = -\frac{\int_{-h}^{h} Q_{2k-1}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{(2k-1)\pi}.$$

Proof For any $\begin{pmatrix} f \\ g \end{pmatrix} \in Z \times Z$, suppose that it has the following expression of formal Fourier series:

$$\begin{pmatrix} f \\ g \end{pmatrix} = \sum_{k=1}^{\infty} (Q_{2k}C_{2k} + Q_{-2k}C_{-2k} + Q_{2k-1}C_{2k-1} + Q_{-2k+1}C_{-2k+1}),$$

multiply the formula above by $Q_{-2k}^{\mathrm{T}} J$ on the left, and integrate the obtained result from $-h$ to h, then it will follow from Lemma 2 that

$$C_{2k} = \frac{\int_{-h}^{h} Q_{-2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{\int_{-h}^{h} Q_{-2k}^{\mathrm{T}} J Q_{2k} \mathrm{d}x} = -\frac{\int_{-h}^{h} Q_{-2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \mathrm{d}x}{2k\pi}.$$

Similarly we can obtain C_{-2k}, C_{2k-1} and C_{-2k+1}.

We shall show that

$$\begin{pmatrix} f \\ g \end{pmatrix} = \lim_{N\to\infty} \sum_{k=1}^{N} (Q_{2k}C_{2k} + Q_{-2k}C_{-2k} + Q_{2k-1}C_{2k-1} + Q_{-2k+1}C_{-2k+1}). \tag{15}$$

Note that

$$\begin{aligned} & Q_{2k}C_{2k} + Q_{-2k}C_{-2k} \\ &= \frac{1}{2k\pi} \sin\frac{k\pi x}{h} \int_{-h}^{h} \left\{ \begin{pmatrix} C^{-\frac{1}{2}} \\ \frac{k\pi}{h} C^{\frac{1}{2}} \end{pmatrix} Q_{-2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} + \begin{pmatrix} -C^{-\frac{1}{2}} \\ \frac{k\pi}{h} C^{\frac{1}{2}} \end{pmatrix} Q_{2k}^{\mathrm{T}} J \begin{pmatrix} f \\ g \end{pmatrix} \right\} \mathrm{d}x \\ &= \frac{1}{2k\pi} \sin\frac{k\pi x}{h} \int_{-h}^{h} \left\{ \begin{pmatrix} C^{-1}g + \frac{k\pi}{h} f \\ \frac{k\pi}{h} g + \frac{k^2\pi^2}{h^2} Cf \end{pmatrix} + \begin{pmatrix} -C^{-1}g + \frac{k\pi}{h} f \\ \frac{k\pi}{h} g - \frac{k^2\pi^2}{h^2} Cf \end{pmatrix} \right\} \sin\frac{k\pi x}{h} \mathrm{d}x \\ &= \frac{1}{h} \begin{pmatrix} \int_{-h}^{h} f \sin\frac{k\pi t}{h} \sin\frac{k\pi x}{h} \mathrm{d}t \\ \int_{-h}^{h} g \sin\frac{k\pi t}{h} \sin\frac{k\pi x}{h} \mathrm{d}t \end{pmatrix}, \end{aligned}$$

and

$$\begin{aligned}
&Q_{2k-1}C_{2k-1}+Q_{-2k+1}C_{-2k+1}\\
&=\Delta\int_{-h}^{h}\left\{\begin{pmatrix}C^{-\frac{1}{2}}\\ \dfrac{(2k-1)\pi}{2h}C^{\frac{1}{2}}\end{pmatrix}Q_{-2k+1}^{\mathrm{T}}J\begin{pmatrix}f\\ g\end{pmatrix}\right.\\
&\quad\left.-\begin{pmatrix}C^{-\frac{1}{2}}\\ -\dfrac{(2k-1)\pi}{2h}C^{\frac{1}{2}}\end{pmatrix}Q_{2k-1}^{\mathrm{T}}J\begin{pmatrix}f\\ g\end{pmatrix}\right\}\\
&\quad\times\cos\frac{(2k-1)\pi x}{2h}\mathrm{d}x\\
&=\Delta\int_{-h}^{h}\left\{\begin{pmatrix}C^{-1}g+\dfrac{(2k-1)\pi}{2h}f\\ \dfrac{(2k-1)\pi}{2h}g+\dfrac{(2k-1)^2\pi^2}{4h^2}Cf\end{pmatrix}\right.\\
&\quad\left.-\begin{pmatrix}C^{-1}g-\dfrac{(2k-1)\pi}{2h}f\\ -\dfrac{(2k-1)\pi}{2h}g+\dfrac{(2k-1)^2\pi^2}{4h^2}Cf\end{pmatrix}\right\}\\
&\quad\times\cos\frac{(2k-1)\pi x}{2h}\mathrm{d}x\\
&=\frac{1}{h}\begin{pmatrix}\int_{-h}^{h}f\cos\dfrac{(2k-1)\pi t}{2h}\cos\dfrac{(2k-1)\pi x}{2h}\mathrm{d}t\\ \int_{-h}^{h}g\cos\dfrac{(2k-1)\pi t}{2h}\cos\dfrac{(2k-1)\pi x}{2h}\mathrm{d}t\end{pmatrix},
\end{aligned}$$

where $\Delta=\dfrac{1}{(2k-1)\pi}\cos\dfrac{(2k-1)\pi x}{2h}$.

Hence

$$\begin{aligned}
&Q_{2k}C_{2k}+Q_{-2k}C_{-2k}+Q_{2k-1}C_{2k-1}\\
&+Q_{-2k+1}C_{-2k+1}=\frac{1}{h}\int_{-h}^{h}\begin{pmatrix}f\\ g\end{pmatrix}G(x,t)\mathrm{d}t,
\end{aligned}$$

where

$$\begin{aligned}
G(x,t)&=\sin\frac{k\pi t}{h}\sin\frac{k\pi x}{h}\\
&\quad+\cos\frac{(2k-1)\pi t}{2h}\cos\frac{(2k-1)\pi x}{2h}.
\end{aligned}$$

Let $f=(f_{ij})_{3\times3}$ and $g=(g_{ij})_{3\times3}$, then (i,j)-elements Q_{ij}^k of the matrix $Q_{2k}C_{2k}+Q_{-2k}C_{-2k}+Q_{2k-1}C_{2k-1}+Q_{-2k+1}C_{-2k+1}$ will be given by

$$Q_{ij}^k=\frac{1}{h}\int_{-h}^{h}f_{ij}G(x,t)\mathrm{d}t,\ i=1,2,3,\ j=1,2,3;$$

and

$$Q_{ij}^k=\frac{1}{h}\int_{-h}^{h}g_{ij}G(x,t)\mathrm{d}t,\ i=4,5,6,\ j=1,2,3.$$

Obviously,

$$\frac{1}{h}\sum_{k=1}^{\infty}\int_{-h}^{h}f_{ij}G(x,t)\mathrm{d}t,$$

and

$$\frac{1}{h}\sum_{k=1}^{\infty}\int_{-h}^{h}g_{ij}G(x,t)\mathrm{d}t,$$

are the Fourier series of f_{ij} and g_{ij} expanded by using the orthogonal system $\{q_n\}_{n=1}^{\infty}$, respectively. This means that

$$f_{ij}=\frac{1}{h}\sum_{k=1}^{\infty}\int_{-h}^{h}f_{ij}G(x,t)\mathrm{d}t,$$

and

$$g_{ij}=\frac{1}{h}\sum_{k=1}^{\infty}\int_{-h}^{h}g_{ij}G(x,t)\mathrm{d}t.$$

Therefore, the series $\sum\limits_{k=1}^{\infty}(Q_{2k}C_{2k}+Q_{-2k}C_{-2k}+Q_{2k-1}C_{2k-1}+Q_{-2k+1}C_{-2k+1})$ converges to $\begin{pmatrix}f\\ g\end{pmatrix}$, that is, formula (15) is proved.

4. The general solutions of equations (1) and (2)

It follows from Theorem 1 that eigenfunction system (10) is a complete system in the function space $L_2^{3\times3}[-h,h]\times L_2^{3\times3}[-h,h]$. By the principle of superposition, the solution of Eq.(5) has the following representation:

$$\begin{aligned}
&\begin{pmatrix}U(x,y)\\ V(x,y)\end{pmatrix}\\
&=\sum_{m=1}^{\infty}\{Q_m(x)T_m(y)+Q_{-m}(x)T_{-m}(y)\}.\quad(16)
\end{aligned}$$

Let the inhomogeneous item $\begin{pmatrix}0\\ f\end{pmatrix}$ of Eq.(5) have the following symplectic Fourier expansion:

$$\begin{pmatrix}0\\ f\end{pmatrix}=\sum_{m=1}^{\infty}\{Q_m(x)S_m(y)+Q_{-m}(y)S_{-m}(y)\},\quad(17)$$

where the Fourier coefficients $S_m(y)$ and $S_{-m}(y)$ are given by taking the symplectic orthogonality of eigenfunctions,

$$S_m(y)=\frac{\int_{-h}^{h}Q_{-m}^{\mathrm{T}}J\begin{pmatrix}0\\ f\end{pmatrix}\mathrm{d}x}{\int_{-h}^{h}Q_{-m}^{\mathrm{T}}JQ_m\mathrm{d}x},$$

and

$$S_{-m}(y)=\frac{\int_{-h}^{h}Q_m^{\mathrm{T}}J\begin{pmatrix}0\\ f\end{pmatrix}\mathrm{d}x}{\int_{-h}^{h}Q_m^{\mathrm{T}}JQ_{-m}\mathrm{d}x},\quad m=1,2,\ldots.$$

Substitute Eqs.(16) and (17) into Eq.(5), and the following solutions can be obtained:

$$T_m(y) = K_1 \mathrm{e}^{\lambda_m y} + \int_0^y S_m(\tau)\mathrm{e}^{\lambda_m(y-\tau)}\mathrm{d}\tau, \quad (18)$$

and

$$T_{-m}(y) = K_2 \mathrm{e}^{-\lambda_m y} + \int_0^y S_{-m}(\tau)\mathrm{e}^{-\lambda_m(y-\tau)}\mathrm{d}\tau, \quad m = 1, 2, \ldots, \quad (19)$$

where K_1 and K_2 are any 3×1 constant matrices, which are determined by the boundary conditions of the lateral side y.

Substituting expressions (18) and (19) and eigenfunctions (10) into Eq.(16), it follows from Lemma 2 that

$$\begin{aligned}
&(U(x,y)^{\mathrm{T}}, V(x,y)^{\mathrm{T}})^{\mathrm{T}} \\
&= (w, \varphi, \psi, \tau_{zy}, D_y, B_y)^{\mathrm{T}} \\
&= \sum_{m=1}^{\infty} (Q_m(x)K_1\mathrm{e}^{\lambda_m y} + Q_{-m}(x)K_2\mathrm{e}^{-\lambda_m y}) + \int_0^y \sum_{m=1}^{\infty} (Q_m(x)S_m(\tau)\mathrm{e}^{\lambda_m(y-\tau)} + Q_{-m}(x)S_{-m}(\tau)\mathrm{e}^{-\lambda_m(y-\tau)})\mathrm{d}\tau \\
&= \sum_{k=1}^{\infty} \begin{pmatrix} C^{-\frac{1}{2}}[\mathcal{P}_1(\lambda_{2k}, y, 0)\sin\lambda_{2k}x + \mathcal{P}_2(\lambda_{2k-1}, y, 0)\cos\lambda_{2k-1}x] \\ C^{\frac{1}{2}}[\lambda_{2k}\mathcal{P}_2(\lambda_{2k}, y, 0)\sin\lambda_{2k}x + \lambda_{2k-1}\mathcal{P}_1(\lambda_{2k-1}, y, 0)\cos\lambda_{2k-1}x] \end{pmatrix} \\
&\quad + \int_0^y \int_{-h}^{h} \sum_{k=1}^{\infty} \begin{pmatrix} C^{-1} f\left[\frac{1}{2k\pi}\mathcal{P}_1(\lambda_{2k}, y, \tau)\sin\lambda_{2k}x\sin\lambda_{2k}t + \frac{1}{(2k-1)\pi}\mathcal{P}_1(\lambda_{2k-1}, y, \tau)\cos\lambda_{2k-1}x\cos\lambda_{2k-1}t\right] \\ \frac{1}{2h} f[\mathcal{P}_2(\lambda_{2k}, y, \tau)\sin\lambda_{2k}x\sin\lambda_{2k}t + \mathcal{P}_2(\lambda_{2k-1}, y, \tau)\cos\lambda_{2k-1}x\cos\lambda_{2k-1}t] \end{pmatrix} \mathrm{d}t\mathrm{d}\tau,
\end{aligned} \quad (20)$$

where

$$\mathcal{P}_1(\lambda, y, \tau) = K_1\mathrm{e}^{\lambda(y-\tau)} - K_2\mathrm{e}^{-\lambda(y-\tau)},$$

and

$$\mathcal{P}_2(\lambda, y, \tau) = K_1\mathrm{e}^{\lambda(y-\tau)} + K_2\mathrm{e}^{-\lambda(y-\tau)}.$$

From the first formula of Eq.(2) and Eq.(21), we have

$$\begin{aligned}
&(\tau_{zx}, D_x, B_x)^{\mathrm{T}} \\
&= \sum_{k=1}^{\infty} \{C^{-\frac{1}{2}}[\lambda_{2k}\mathcal{P}_1(\lambda_{2k}, y, 0)\cos\lambda_{2k}x \\
&\quad -\lambda_{2k-1}\mathcal{P}_2(\lambda_{2k-1}, y, 0)\sin\lambda_{2k-1}x]\} \\
&\quad + \frac{1}{2h}\int_0^y \int_{-h}^{h} \sum_{k=1}^{\infty} \{C^{-1} f[\mathcal{P}_1(\lambda_{2k}, y, \tau)\cos\lambda_{2k}x \\
&\quad \times \sin\lambda_{2k}t - \mathcal{P}_2(\lambda_{2k-1}, y, \tau)\sin\lambda_{2k-1}x \\
&\quad \times \cos\lambda_{2k-1}t]\}\mathrm{d}t\mathrm{d}\tau.
\end{aligned} \quad (21)$$

The formulae (20) and (21) describe the general solutions of Eqs.(1) and (2) with homogeneous boundary conditions (4) on the x-side, and $\lambda_{\pm(2k-1)} = \pm\frac{(2k-1)\pi}{2h}$ and $\lambda_{\pm(2k)} = \pm\frac{k\pi}{h}$, $k = 1, 2, \ldots$.

By using formulae (20) and (21), the solutions of the Eqs.(1) and (2) are exponentially decreasing along the positive direction and the negative direction of the coordinate axis y, respectively. These solutions are covered by the well-known Saint Venant principle, but their exact forms are helpful for describing the discontinuous load, etc in a rectangular domain.[6–9]

5. Conclusions

In this paper, we have proved the completeness in the sense of Cauchy's principal value of an eigenfunction system for an infinite-dimensional Hamiltonian operator. The operator comes from the isotropic plane magnetoelectroelastic solids. This result offers a theoretic guarantee of the feasibility of variable separation method based on a Hamiltonian system for isotropic plane magnetoelectroelastic solids.

The Refs.[10,11] also discussed the completeness of an eigenfunction system for an infinite-dimensional Hamiltonian operator, but the completeness was not proved in case of the function spaces(for example, the function space $L_2 \times L_2$). Furthermore, the completeness of an eigenfunction system for a specific Hamiltonian operator appearing in elasticity and relevant fields has not been studied in the references.

Although only the case of isotropic plane magnetoelectroelastic solid is studied in this paper, the approach can be used for reference to the more complex problems.

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