Vol 17 No 7, July 2008
1674-1056/2008/17(07)/2745-08

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Analysis of precipitation characteristics of South and North China based on the power-law tail exponents*

Feng Guo-Lin(封国林)[a)b)], Gong Zhi-Qiang(龚志强)[b)c))†],
Zhi Rong(支 蓉)[b)c)], and Zhang Da-Quan(章大全)[a)]

[a)] *Department of Physics, Yangzhou University, Yangzhou* 225009, *China*
[b)] *Key Laboratory of Regional Climate-Environment for Temperate East Asia, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing* 100029, *China*
[c)] *Laboratory for Climate Studies, National Climate Center, China Meteorological Administration, Beijing* 100081, *China*

(Received 11 November 2007; revised manuscript received 15 January 2008)

Precipitation sequence is a typical nonlinear and chaotic observational series, and studies on precipitation forecasts are restricted to the use of traditional linear statistical methods, especially when analysing the regional characteristics of precipitation. In the context of 20 stations' daily precipitation series (from 1956 to 2000) in South China (SC) and North China (NC), we divide each precipitation series into many self-stationary segments by using the heuristic segmentation algorithm (briefly BG algorithm). For each station's precipitation series, we calculate the exponent of power-law tail (EPT) of the cumulative probability distribution of segments with a length larger than l for precipitation and temperature series. Our results show that the power-law decay of the cumulative probability distribution of stationary segments might be a common attribution for precipitation and other nonstationary time series; the EPT somewhat indicates the precipitation duration and its spatial distribution that might be different from area to area. The EPT in NC is larger than in SC; Meanwhile, EPT might be another effective way to study the abrupt changes in nonlinear and nonstationary time series.

Keywords: the power-law exponents, precipitation, durative, abrupt precipitation change
PACC: 9260X

1. Introduction

A time series is stationary if the mean, standard deviation, higher moments, as well as the correlation functions, are invariant under time translation.[1] Interestingly, the time series, which arise from a large number of phenomena in many systems, such as areas-including physiologic systems, economic systems, Internet, and climatic systems, are nonstationary.[2−8] Nonstationarity is a prominent feature of climatic variability that can be associated with segments of different statistical properties.

Climatic signals obtained under both natural and human influences raise serious difficulties in both technical and fundamental aspects of time series analysis. By filtering out effects of nonstationarity, much work has focused on 'intrinsic properties' of climatic signals,[9,10] This approach may reveal intrinsic correlation properties of dynamics. Indeed, important scale-invariant features in physiological processes were recently revealed after filtering out masking effects of nonstationarity.[11,12]. Besides, climatic signal is known to change from area to area and from time to time, suggesting that more than only environmental conditions are reflected in the phenomena.[13] Is there a nontrivial structure associated with nonstationarity in climatic signals, which may change with area and time? In order to solve this problem, we try to treat the precipitation series of South China (SC) and North China (NC) by using the segmentation algorithm. In such a way, a nonstationary climatic series can be divided into a number of segments that are properly stationary. The statistical properties of the segments can help us better understand the overall nonstationarity of the time series and yield practical application.[14,15]

In this paper, a heuristic segmentation algorithm (briefly BG algorithm), which was recently proposed by Bernaola-Galvan and co-workers, is applied to detecting 20 nonstationary precipitation series in SC and

*Project supported by the National Natural Science Foundation of China (Grant No 40675044) and the State Key development program for Basic Research (Grant No 2006CB400503).
†E-mail: gzq0929@126.com

NC. We try to find out the intrinsic properties of the power-law decay of the cumulative probability distribution contained in these precipitation series. In Section 2, we briefly introduce the BG algorithm and its physical meaning. In Section 3, we present the results of EPTs of 20 precipitation series. In Section 4, we consider the spatial distribution characteristics of the exponents of the power-law decay; In Section 5, we analyse the abrupt precipitation change (APC) in SC and NC with EPT. Finally we summarize our findings.

2. BG algorithm and its physical meaning

When one of traditional abrupt change detection methods, such as filter detection, moving t test, Grammer method, etc, is used to detect abrupt climate change (ACC), it is generally assumed that climate change is a stationary and linear process.[15−18] Bernaola-Galvan put forward a heuristic segmentation algorithm (BG algorithm) in 2001,[14,19] which differs from the traditional methods, and its main points can be summarized as follows.

For a time series $x(t)$ composed of N points, we move a sliding pointer from left to right along the sequence. At each position of the pointer, the mean and standard deviation of the subset of the sequence from the left of the pointer ($\mu_1(i)$ and $s_1(i)$) and from the right ($\mu_2(i)$ and $s_2(i)$) are computed, separately. Then, the pooled variance at point i is

$$s_{\rm D}(i) = \left(\frac{(N_1-1)\times s_1(i)^2 + (N_2-1)\times s_2(i)^2}{N_1+N_2-2}\right)^{1/2} \times \left(\frac{1}{N_1}+\frac{1}{N_2}\right)^{1/2}, \quad (1)$$

where N_1 and N_2 are the numbers of points to the left and to the right of the pointer. The statistical value $T(i)$ of t test is used to quantitatively express the difference between $\mu_1(i)$ and $\mu_2(i)$ as

$$T(i) = \left|\frac{\mu_1(i)-\mu_2(i)}{s_{\rm D}(i)}\right|. \quad (2)$$

Repeating the above computational procedure for each point of the time series yields the time series of the statistical value $T(t)$ of t test corresponding to $x(t)$ one by one. The statistical significant $P(T_{\max})$ of maximum value $T_{\max}$ is

$$P(T_{\max}) = \Pr\text{obability}(T \le T_{\max}), \quad (3)$$

i.e. $P(T_{\max})$ denotes the probability of obtaining the value of T equal to or lower than $T_{\max}$ within a random sequence. $P(T_{\max})$ can be approximated by means of Monte Carlo simulation, and it is generally expressed as follows:

$$P(T_{\max}) \approx [1 - I_{[v/(v+T_{\max}^2)]}(\delta v, \delta)]^{\eta}, \quad (4)$$

where $\eta = 4.19\ln N - 11.54$, $\delta = 0.40$ with N being the length of time sequence $x(t)$; $v = N-2$; $I_x(a,b)$ is the incomplete beta function. P_0 is assumed to be a threshold of P, if the difference in mean is not statistically significant i.e. if $P(T_{\max}) < P_0$, then the time series is not cut. If the difference in mean between the left and the right parts of the time series is statistically significant, i.e. if $P(T_{\max}) \ge P_0$, then the time series is cut into two segments as long as the means of the two new segments are significantly different from the means of the adjacent segments. If the time series is cut, we continue to iterate the above procedure recursively on each segment until the obtained significant value is smaller than the threshold P_0, or the length of the obtained segments is smaller than an imposed minimum segment length l_0. We will see that the value of l_0 is one of the parameters controlling the accuracy of the algorithm. After the above operation, the original sequence is divided into several self-stationary segments with statistically different means and cutting points are abrupt change points (ACP) of mean values. We use weather generator[20,21] to simulate precipitation in many places of China and obtain 10000 series with a given length of 45 years, then we segment them at different P_0 and l_0 to choose the best parameter. With l_0=40 and P_0=0.99, segment results from them are the most consistent with each other.

3. Scaling law existing in the non-stationary time series

Here we consider 20 daily average precipitation series from different stations in SC and NC, separately. The length of each precipitation series is 45 years (A.D.1956–2000). In order to remove the trend of each sequence, all these datasets have subtracted the global average. Figure 1 shows the detail information of these 20 stations. In light of the non-succession of precipitation series, we analyse a time series of 10 days' running average of each original series. Figure 2 shows a abnormal time series of 10 days' running average of Danxian in SC and Yangquan in NC; other station's running average time series are similar, and so we do not give their figures have to save the space of this paper.

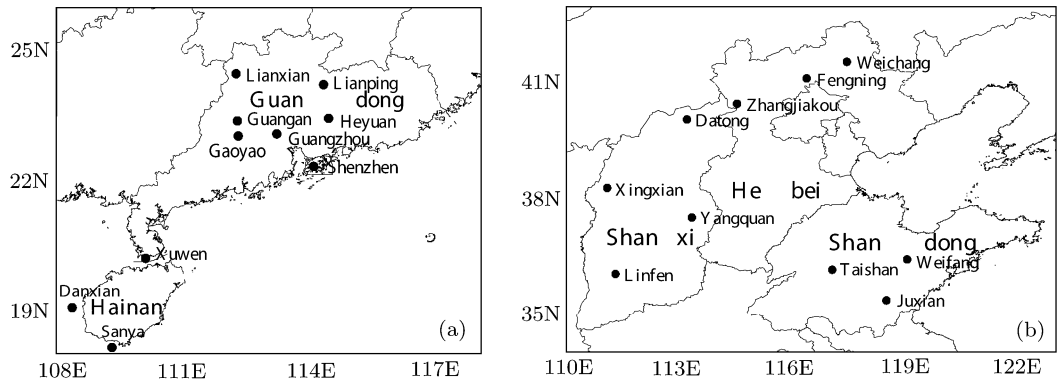


Fig.1. The analysed locations of 20 stations in SC (a) and NC (b), each with 10 stations.

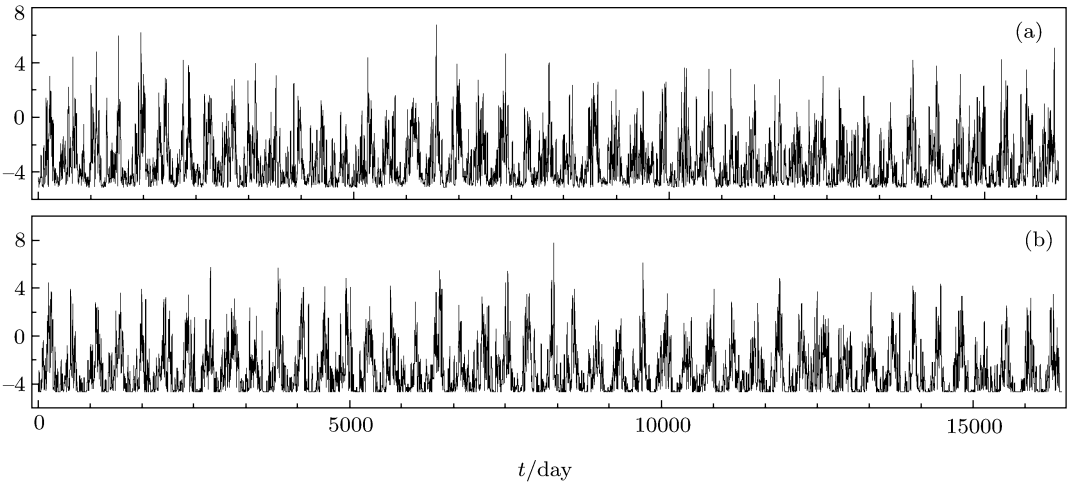


Fig.2. Abnormal time series of 10 days' running average for Danxian in SC (a) and Yangquan in NC (b).

With l_0=40 and P_0=0.99, the BG algorithm leads to the dividing of the 20 datasets into segments with well-defined means, each is significantly different from the means of its adjacent segments. Figure 3 shows the results for Yanquan and Danxian. Obviously, the lengths of segments are different from each other. To quantify the nonstationarity of each precipitation series, we study the statistical properties of the lengths of all the segments (the mean values for different segments have been analysed in other papers).

Figure 4 displays two stations' cumulative probability distributions of segment lengths, showing the probability of finding a segment with a length larger than l for precipitation and temperature series. Here the cumulative probability distribution that is widely used in economies and well known as the Pareto law,[22] is adopted to reduce the statistical errors from a limited length of time series. The cumulative probability distribution of the lengths of stationary segments cut by the BG algorithm for precipitation and temperature series is well fit to a power-law decay. For all the time series of the 20 stations, we find

$$P(L > l) \sim l^{-\gamma}. \tag{5}$$

Different time series have different values of the exponent γ of the power-law decay, indicating that the BG algorithm divides the nonstationary time series into segments with asymptotic statistical properties. This characteristic of nonstationary time series exists not only in precipitation and temperature sequences, but also in many other nonstationary signals, such as physiological signals of heart rate and internet traffic flow density, *et al.*[19,23] These findings indicate that the power-law decay of the distribution of the stationary segments may be a common attribution for nonstationary time series. From formula (5) we can know that a large EPT often corresponds to the case of more long segments and less short segments. From

this point, we can conclude that the EPT also somewhat indicates the duration of precipitation, that is to say, the smaller the EPT is, the better the duration of precipitation will be.[24]

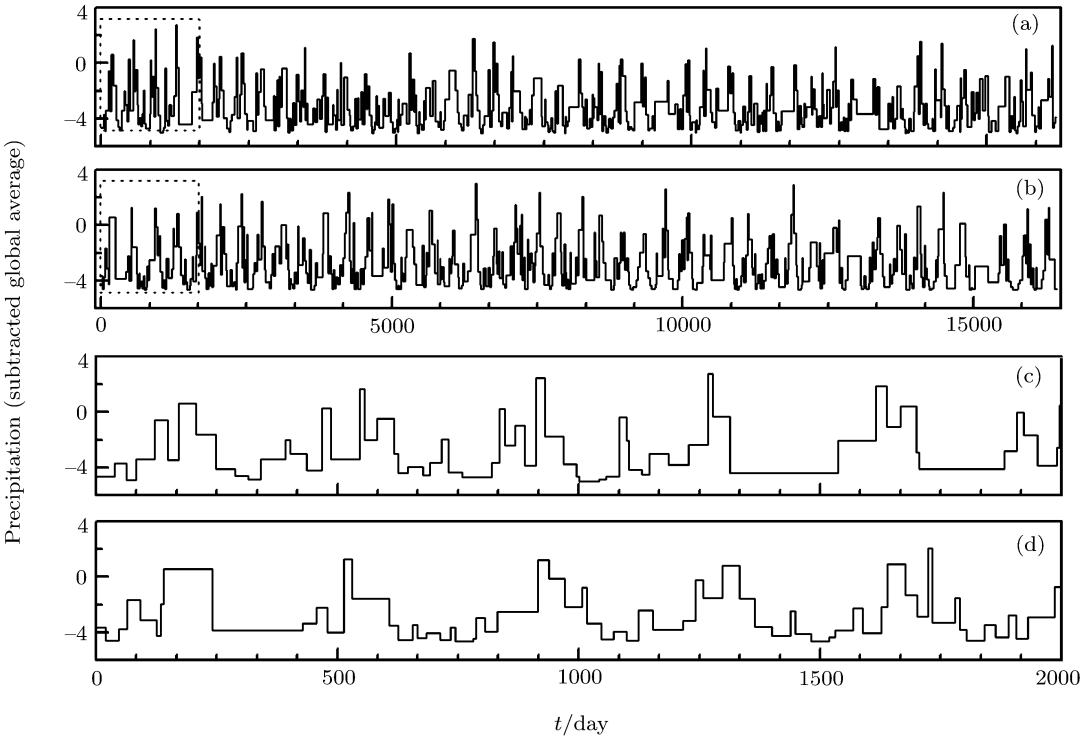


Fig.3. Mean values of segments for Danxian (a) and Yangquan (b); panels (c) and (d) show the magnifications of small parts (2000 days) of subsets in panels (a) and (b).

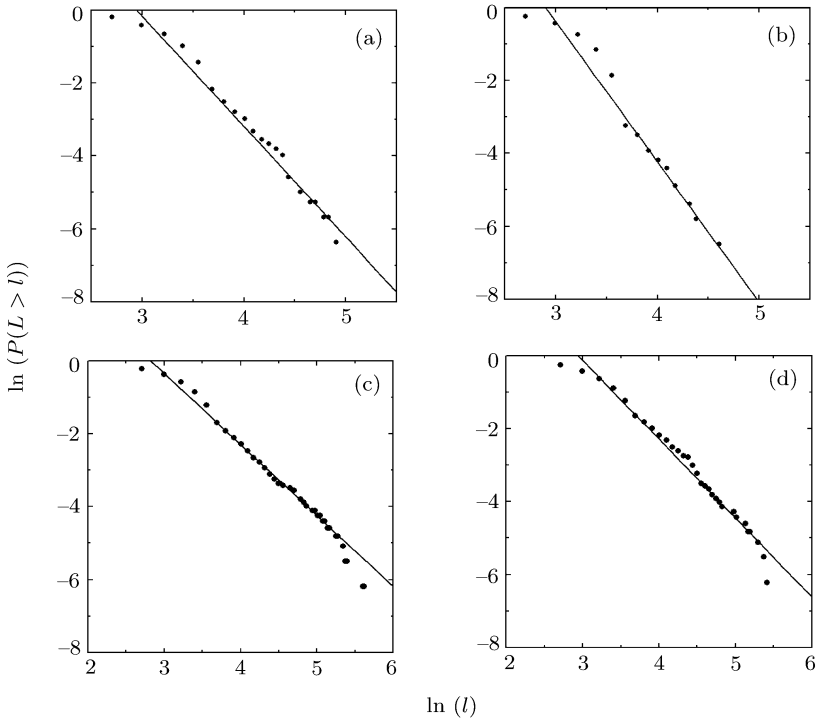


Fig.4. Cumulative probability distribution of segments with lengths larger than l for precipitation and temperature series in ln–ln scale. Panels (a) and (b) show the temperatures in Danxian and Yangquan; panels (c) and (d) display the precipitations in Danxian and Yangquan.

The power-law decay of precipitation segments is consistent with a self-organized critical process. The concept of self-organized criticality[25−27] refers to the tendency of non-equilibrium systems driven by slow constant energy input to organize themselves into critical states where all scales are relevant. The characteristic feature of self-organized critical systems, even if their dynamics is incomprehensibly complex, is that the intermediately stored energy is eventually released in sudden bursts with no typical size. That is to say, the atmosphere is the system under investigation and corresponds to the Earth's crust or the granular pile,[28,29] It is driven by a slow and constant energy input from the Sun. In particular, water is evaporated from the oceans, and the energy is stored in the form of water vapour in the atmosphere. It is then suddenly released in bursts when the vapour condenses into water drops. The power-law distribution of segments versus length is equivalent to the Gutenberg–Richter law for earthquakes and the power-law distribution of avalanche size. There is no constant drizzle accounting for the constant evaporation but segments of a wide range of lengths. One could imagine the classifying of rain process according to their lengths just as earthquakes are classified according to their Richter scale.

4. The distribution of precipitation EPTs in SC and NC

In SC and NC, the characteristics of climate are quite different, especially in precipitation. However, we find that all the precipitation series of these two regions follow the power-law decay (Fig.4). Here we calculate EPTs (i.e. the value of the exponent of the power-law decay) (Table 1) of the precipitation series of the 20 stations, separately. Besides, we also separately calculate the daily average precipitations of all the stations in SC and NC and obtain two new time series. These two new time series show daily average precipitations in SC and NC, respectively. Then we calculate EPTs of these two new series.

Table 1. The values of exponent of the power-law decay for the 20 stations in SC and NC.

SC				NC			
station	γ	station	γ	station	γ	station	γ
Xuwen	1.91	Shenzhen	1.81	Taishan	2.06	Weifang	2.10
Heyuan	1.84	Guangzhou	1.69	Juxian	2.20	Datong	2.14
Gaoyao	1.76	Guangan	1.89	Xingxian	2.25	Yangquan	2.16
Lianping	2.00	Lianxian	1.68	Linfen	2.34	Fengning	2.15
Danxian	1.95	Sanya	1.86	Weichang	2.07	Zhangjiakou	2.31

Figure 5 displays the cumulative probability distributions for 6 stations' precipitation series in SC and NC, other stations' distributions are similar. The EPTs of 10 stations in NC vary between 2.0–2.4, which are somewhat larger than those vary between 1.6–2.0 in SC (Table 1). Meanwhile, the value of the exponent of the power-law decay for average precipitation series of all the stations in SC is 1.92, and the exponent in NC is 2.54 (see Fig.6). Based on these aspects, we can conclude that the characteristic of scaling law may be a common feature for precipitation series, the exponents vary in a certain scope in different areas, and they are larger in NC than in SC. From the point of the physical meaning of EPT, it is obvious there are more long segments and less short segments of precipitation series in SC than in NC, that is to say, the duration of precipitation in SC is longer than in NC.

As is well known, the precipitation has regional characters in time and space,[30−32] so EPT vary from station to station even in the same climatic region. Meanwhile, in China there are five dynamical climatic characteristic regions, among which SC and NC belong to region II and IV, respectively. The precipitation of region II is closely related to the El Nino–Southern Oscillation (ENSO), which may be associated with the India–East Asian teleconnection pattern. While the influence of ENSO on region IV is not evident, but the influence of the ridge-line position of the subtropical high on precipitation is distinct, which may be related to the Tibetan plateau teleconnection pattern.[33,34] That might be the reason for the duration of precipitation in SC is better than in NC, i.e. the values of the exponent the power-law decay in NC are greater than 2.0 but those in SC are less than 2.0.

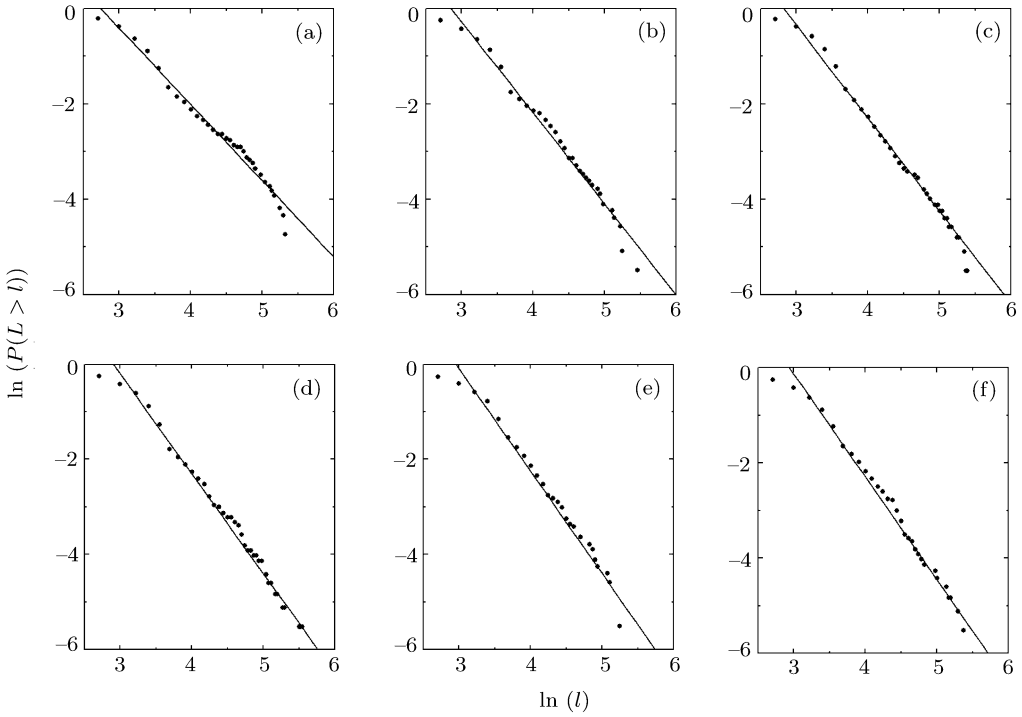


Fig.5. Cumulative probability distributions of 6 stations' precipitation series in SC and NC.

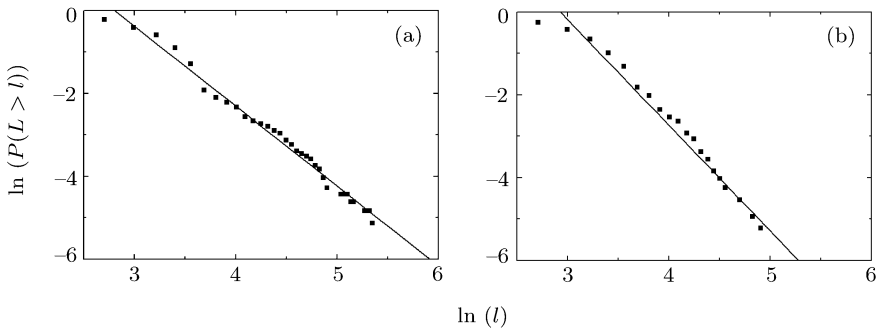


Fig.6. Cumulative probability distributions of daily average precipitation series for SC with γ=1.92, (a) and NC with γ=2.54 (b).

5. The abrupt change of precipitation in SC and NC

Abrupt precipitation change (APC) is a special manifestation of the non-linearity of climate system. The non-linearity of climate system will result in multi-scale, multi-hierarchy, and non-stationarity, which make it more difficult to analyse and detect the APC and put forward much higher requirements for detecting methods. In this section, we try to analyse the APC of SC and NC from a new aspect-EPT. We take a sliding window W with a width of 10 a and move the window separately along the two daily average precipitation series of SC and NC with a step of 1a. At each position, we calculate the value of the exponent of the power-law decay for the sub-series in a corresponding window. Rain events may happen in only one day or last several days, different places and times have different spatial and temporal characteristics, however, there are few studies on this point and there exists no specific definition about the duration of precipitation. According to recent studies,[24,35] EPT can somewhat indicate the duration of precipitation. In order to define the durative extent of precipitation, we consider the all decay exponents and divided them into four grades:

$$\gamma \sim \begin{cases} \lambda \geq 2.2, & \text{1st grade,} \\ 2.2 > \lambda \geq 2.0, & \text{2nd grade,} \\ 2.0 > \lambda \geq 1.8e, & \text{3rd grade,} \\ \lambda < 1.8, & \text{4th grade.} \end{cases} \tag{6}$$

Figure 7(a) shows that most exponents of the power-law decay for SC fall in 3rd and 4th grades, while most exponents of the power-law decay for NC fall in 1st and 2nd grades (see Fig.7(b)). Obviously, the values of the exponent of the power law decay for SC have distinct abrupt changes before or after 1973, 1980 and 1992; Distinct abrupt changes of the values of the exponent of the power law decay happening in NC before or after 1971, 1979 and 1990 are 1–2 years earlier than the formers. EPT can reflect the intrinsic properties of precipitation series, and its variation is closely related to the precipitation, so the abrupt change of the value of the exponent of the power-law decay to some extent can characterize the APC. That is to say, before or after 1970, 1980 and 1990 there might be consistently distinct APCs in SC and NC, meanwhile the APC in SC is 1–2 years later than in NC. This result is well consisted with the present conclusions.[36−38] There are also several small APCs both in SC and NC, such as 1997 in SC and 1985 and 1994 in NC, which might result from local climate changes under different external forcings. With EPTs we can quantify the extent of precipitation duration. Besides, EPTs also might be another effective way to study the abrupt changes in nonlinear and nonstationary time series i.e. ACC, APC, etc.

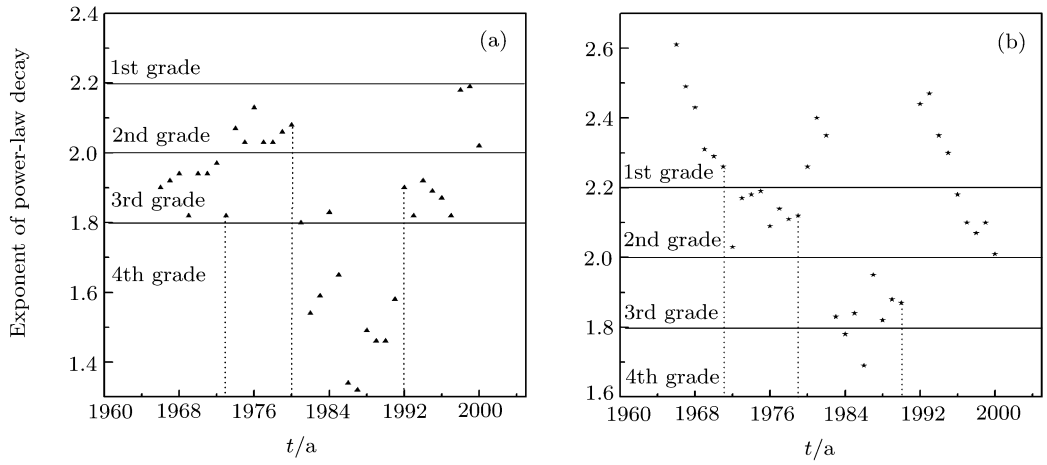


Fig.7. The exponents of power-law decay of two daily average precipitation series corresponding to each window. (Abscissa represents the last year of each window, and the ordinate represent the exponents of power-law decay in SC (a), and in NC (b)).

6. Conclusion and discussion

We first divide precipitation series into many self-stationary segments based on the BG algorithm in order to analyse the intrinsic properties of nonstationary time series and study the characteristics of precipitation in SC and NC. We find that the atmosphere system is similar to the Earth's crust or the granular pile, and that the power-law decay of the distribution of the stationary segment might be a common attribution for precipitation and other nonstationary time series. This insight might inevitably inspire a new investigation into the modelling of precipitation and atmospheric processes, and it also might be used as a benchmark for existing models and useful, for example, in drought and flooding hazard assessments. In a more general sense, our analyses show that new insights can be extracted from the very general point of view of complexity and self-organization theory. It may serve as an example to show how to use this approach in situations that seem too complex to be accessible to quantitative analysis.

The value of the exponent of power-law tail, which can somewhat indicate the durative characteristics of precipitation, is different from place to place: it is 2.54 in NC larger than that in SC (1.92); besides, before or after 1970, 1980 and 1990 there might be homochronous distinct APCs in SC and NC under the background of climate change in China, and EPT might be another effective way to study the abrupt

changes in nonlinear and nonstationary time series. However, the study of scaling law for precipitation series is just in the beginning period: the physical meaning of EPT is still not quite clear and its explanation might have practical significance to precipitation forecasts.

References

[1] Stratonovich R L 1981 *Topics in the Theory of Random Noise* (New York: Gordon and Breach) Vol. 2
[2] Shi N 2005 *Chin. Phys.* **14** 844
[3] Dai X G, Fu Z B and Wang P 2005 *Chin. Phys.* **14** 850
[4] Ivanov P Ch, Rosenblum M G, Peng C K, Mietus J E, Haclin S and Stanley H E 1996 *Nature* (London) **383** 323
[5] Ivanov P Ch, Amaral L A N, Goldberger A L, Havlin S, Rosenblum M G, Struzik Z R and Stanley H E 1999 *Nature* (London) **391** 461
[6] Bunde A, Havlin S, Kantalhardt J, Penzal T, Peter J H and Voigt K 2000 *Phys. Rev. Lett.* **85** 3736
[7] Musha T and Higuchi H 1976 *Jpn. J. Appl. Phys.* **15** 1271
[8] Takayasu M, Takayaus H and Sato T 1996 *Physics* A **233** 924
[9] Feng G L, Dong W J and Jia X J 2004 *Chin. Phys.* **13** 413
[10] Peng C K, buldyrew S V, Havlin S, Stanley H E and Goldberger A L 1995 *Chaos* **5** 82
[11] Schreiber T 1997 *Phys. Rev. Lett.* **78** 843
[12] Hausdorff J M, Purdon P L, Peng C-K, Ladin Z, Wei J Y and Goldberger A L 1996 *J. Appl. Phys.* **80** 1448
[13] Gong Z Q, Gao X Q and Dong W J 2005 *Acta Phys. Sin.* **54** 3947 (in Chinese)
[14] Bernaola-Galvan P, Ivanov P Ch, Amaral L A N and Stanley H E 2001 *Phys. Rev. Lett.* **87** 168105
[15] Feng G L, Gong Z Q and Li J P 2005 *Acta Phys. Sin.* **54** 5494 (in Chinese)
[16] Shi Y F and Zhang P Y 1996 *The Variation Trend and its Effect of the Climate in China and Sea Level Change—the Historical Climatic Change in China* (Jinan, China: Shan Dong Science and Technology Press) (in Chinese)
[17] Yang W F, Li Z Y and Li X M 1997 *Quarterly J. Appl. Meteorol.* **8** 119 (in Chinese)
[18] Yan Z W, Li Z Y and Wang X C 1993 *Scientific Atmospheric Sinica* **17** 663 (in Chinese)
[19] Fukuda K, Stanley H E and Amaral L A N 2004 *Phys. Rev.* E **69** 021108
[20] Liao Y M, Zhang Q and Chen D L 2004 *J. Geographical Sciences* **14** 417
[21] Wilks D S 1999 *Agricultural and Forest Meteorology* **93** 153
[22] Pareto V 1987 *Cours d'Economie Politique* (Lausanne: University of Lausanne Press) **2**
[23] Maritan A, Rigon R, Banavar J R and Rinaldo A 2002 *Geo. Res. Lett.* **29** 1508
[24] Gong Z Q and Feng G L 2006 *Acta Phys. Sin.* **55** 477 (in Chinese)
[25] Bak P, Tang C and Wiesenfeld K 1987 *Phys. Rev. Lett.* **59** 381
[26] Bak P 1996 *How Nature Works: The Science of Self-Organized Criticality* (New York: Springer)
[27] Jensen H J 1998 *Self-Organized Criticality: Emergent Complex Behavior in Physical and Biological Systems* (Cambridge: Cambridge University Press)
[28] Gutenberg B and Richter C F 1994 *Bull. Seismol. Soc. Am.* **34** 185
[29] Frette V, Christensen K, Malthe-Sørenssen A, Feder J, Jøssang T and Meakin P 1996 *Nature* (London) **379** 49
[30] Liu S G, Fu Z T and Liu S D 2002 *Acta Phys. Sin.* **51** 10 (in Chinese)
[31] Liu S G, Chen H and Fu Z T 2003 *Acta Phys. Sin.* **52** 1842 (in Chinese)
[32] Mo J Q and Lin W T 2004 *Acta Phys. Sin.* **53** 996 (in Chinese)
[33] Wan S Q, Feng G L, Dong W J, Li J P and Chou J F 2005 *Acta Phys. Sin.* **54** 548 (in Chinese)
[34] Feng G L, Chou J M and Dong W J 2004 *Chin. Phys.* **13** 1582
[35] Yu J H and Ding G Y 1998 *J. Nanjing Institute Metrology* **21** 145 (in Chinese)
[36] Huang R H, Xu Y H and Zhou L T 1999 *Plateau Meteorology* **18** 465 (in Chinese)
[37] Yu G M 1996 *J. Tropical Meteorology* **12** 252 (in Chinese)
[38] Dai X G, Wang P and Chou J F 2003 *Chin. Sci. Bull.* **48** 2483 (in Chinese)